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HANDBOOK
ON
THE GAS ENGINE

HAEDER & HUSKISSON

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A HANDBOOK

ON THE

GAS ENGINE

COMPRISING A PRACTICAL TREATISE ON
INTERNAL COMBUSTION ENGINES

FOR THE USE OF ENGINE BUILDERS, ENGINEERS, MECHANICAL DRAUGHTSMEN
ENGINEERING STUDENTS, USERS OF INTERNAL COMBUSTION
ENGINES, AND OTHERS

BY

HERMAN HAEDER

CIVIL ENGINEER, WIESBADEN

TRANSLATED FROM THE GERMAN AND EDITED BY

W. M. HUSKISSON, A.M.I.E.E.

(9½ YEARS WITH MESSRS. CROSSLEY BROTHERS, OPENSHAW)

*WITH THE ADDITION OF NUMEROUS USEFUL TABLES
AND OTHER MATTER*

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AUTHOR'S PREFACE TO THE FIRST EDITION.

THE question whether, for industrial purposes, steam or power gas * (internal combustion) engines are the most economical, has been much discussed of late years in technical periodicals, but unfortunately mostly from a partisan point of view. Those interested in the construction both of gas and of steam engines commit themselves to exaggerated statements, which darken rather than throw light upon the real subject.

In estimating the size of a gas engine plant, the following points must be kept in view :—

THE OUTPUT.

GAS ENGINES.

It is not possible to increase the output, because the effect of the explosion stroke cannot be enhanced. With the output corresponding to the most favourable, that is the most economical, fuel consumption, the power developed by every explosion engine is therefore exhausted, and if a greater output be demanded, the engine simply comes to a stand. If, for instance, a so-called 10 HP gas or other internal combustion engine is run on the brake and develops 12 HP, erroneous conclusions may be drawn from the trial. Since at 10 HP no explosion strokes are supposed to be missed,† it naturally follows that the engine has intentionally been made larger than is needed to develop that power. It would be as reasonable to rate it at 8 HP and to specify among its advantages that it would give a maximum of 12 HP, that is a " reserve of power " of 50 per cent.

As a rule, the normal output has hitherto been taken as corresponding to 85 to 90 per cent. of the maximum, thus giving from 10 to 15 per cent. " reserve of power."

STEAM ENGINES.

In steam engines a larger output can be obtained by a later cut-off. This is effected automatically by the governor. If, for instance, a 10 *HP engine is required to run under a load of* 14 *HP, the governor adjusts the valve gear to give a later cut-off, and the engine will then develop virtually* 14 *HP, the maximum output being* 40 *per cent. more than the so-called normal output. For this increased output the consumption of steam is about* 10 *per cent. more than at normal load.*

It will be seen from the above that it is necessary to take the so-called normal output of a gas engine relatively higher than that of a steam engine doing the same work. In other words, the

Maximum output of a gas engine should be = 1·4 × normal output of a steam engine.

The gas engine will then be of the same value in regard to the output or work done, as the steam engine.

If, for instance, a 30 HP engine is sufficient to supply the power required to drive a work-

* Suction and pressure gas plants are generally classed as power gas producers.

† In quality and quantity governing, less work is done during the explosion strokes, with a smaller load, and results in a larger consumption of fuel.

shop or mill, and a gas engine has been selected for the purpose, it will be necessary to have an engine developing $1{\cdot}25 \times 30 = 37$ HP and giving a maximum of $1{\cdot}4 \times 30 = 42$ HP.

To denote the size and power of a gas engine, therefore, and to make a correct comparison with a steam engine, the maximum HP on the brake should also always be stated. For gas engines it is best to rate the power developed at

$$\text{Normal output} = 0{\cdot}7 \times \text{maximum output},$$

because this method of computation has become the rule for steam engine work, and 40 per cent. reserve of power is desirable in nearly all power plants. The consumption of coal should at the same time be calculated for this normal output. At the present time it is the almost invariable rule to state the power in gas engines at

$$\text{Normal output} = 0{\cdot}85 \times \text{maximum output}.$$

AUTHOR'S PREFACE TO THE SECOND EDITION.

THE matter in this book, and especially the Tables, have been thoroughly revised and checked, in accordance with the latest information obtainable.

The endeavour to build much lighter gas engines for export, and thus effect a saving in weight and freight charges, is laudable, but should not be carried too far, because the dimensions exercise considerable influence on the smooth running, freedom from accidents, and life of an engine. The cast-iron parts (the frame, cylinder, and piston) can be kept relatively light, if they are very carefully constructed, and of the best material, judiciously distributed. On the other hand, if the flywheel is too light, it has always a disadvantageous effect on the steadiness of the engine. This is especially noticeable in engines supplying electric light, and in installations where the power required varies considerably, as fully explained at p. 176.

My book on "Die Kranke Gasmotor" (Defects in Gas Engines) was collated with this book during its revision, and data taken from it were used in the parts relating to gas engine construction. Thus previous omissions were as far as possible made good.

I shall be glad to have my attention drawn to any errors, or suggestions for fuller details in later editions.

TRANSLATOR'S PREFACE.

THE English Edition of this work has been prepared with a view to assist those engaged in the application of scientific principles to internal combustion engine design, and especially for those who cannot read the German text with ease, and have neither the time nor the desire to convert the German units into English. If the English edition proves as useful to the reader as "Die Gasmotoren" has been to me, the book will have fulfilled its object.

The tabulated dimensions of the various engines are expressed in millimetres as they relate to continental engines, but tables of the nearest equivalent in inches are also given, to save time and labour to those who are not accustomed to work in millimetres.

In order to render the book as complete as possible, and to save the reader the time and trouble of referring to other works, a full set of tables has been added; for granting me permission to use these tables, etc., I take the opportunity of thanking the publishers, and the authors of the works from which these tables, etc., have been taken, viz. Mackrow's "Naval Architect's Pocket-book," Kempe's "Engineer's Year Book," Hutton's "Practical Engineer's Handbook," Clark's "Mechanical Engineer's Pocket-book," and "The Mechanical World Pocket Diary"; my thanks are also due to Mr. H. A. Humphrey, and Mr. Edwin Kenyon, for their papers.

In conclusion, I have to acknowledge my indebtedness to Mrs. E. M. Donkin in helping me with the translation, and to Mr. Guy E. Lloyd, Assoc.M.I.C.E., in checking all the formulas and calculations throughout the book.

WILLIAM M. HUSKISSON.

45, CAMBRIDGE GARDENS,
NORTH KENSINGTON,
LONDON,
July, 1911.

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GAS ENGINES

CHAPTER I.

Method of Working, Type, Description, and Diagrams.

ENGINES driven with gas, petroleum, alcohol, benzine, etc., are usually termed "*Internal Combustion Engines*," because the power generated in them is produced by the combustion of a mixture of gas, oil, etc., and air in the cylinder. The customary term, "Explosion or Ignition Engines," is derived from the sudden, almost instantaneous, combustion of the gases.

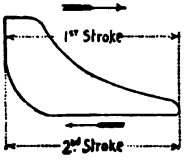
Method of Working.

If a mixture of gas and air is enclosed in a chamber and ignited, it explodes with violence ; this explosion is a result of the rapid combustion of the gas. The speed and violence of the explosion depend upon the proportions in which the gas and air are combined, and upon the pressure of the mixture. When ignition takes place, there is a rise of both temperature and pressure in the combustion chamber, and this pressure and the subsequent expansion of the products of combustion drive the piston of the cylinder forward, and useful work is done.

There are two main working systems or cycles, *i.e.*—

1. The *Four-cycle* (Otto).
2. The *Two-cycle.*

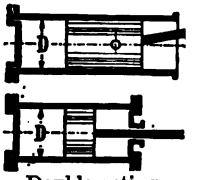
The word *cycle* here means a single stroke to and fro of the piston, and the expressions *four-cycle* and *two-cycle* denote respectively the number of single strokes required to complete one cycle of operations.



Steam Engine.

(Steam engines work on the *two-cycle* principle, that is to say, two strokes are required on one face of the piston to complete one working cycle, as shown in the accompanying diagram. The beginner is usually inclined to consider the steam engine as a *one-cycle* engine.)

Single acting.



Double acting.
Gas Engine.

Single- and Double-acting Engines.

The term *single-acting* denotes an engine in which work is done at one end of the cylinder only, whereas in a "*double-acting*" engine work is done at both ends.

In a single-acting engine the piston usually serves as the crosshead ; in double-acting engines a separate crosshead and stuffing box for the piston rod

are necessary. The difference will be readily understood from the outline drawings. Care must be taken not to confuse the expressions "two-cycle" and "four-cycle," and "single-acting" and "double-acting," with each other.

Thus we have—

Single-acting and double-acting *Four*-cycle engines, and
" " " " " *Two*-cycle engines.

A careful study of Table I., p. 3, will make this clear.

Short Description of Various Types of Engines.

From the classification on p. 3 it will be seen that a lengthy description is required to explain the distinctive features of each type. The construction and method of working

FIG. 1.—Single-acting Four-cycle.

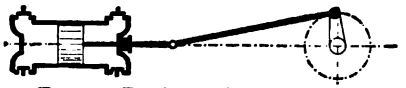


FIG. 2.—Double-acting Four-cycle.

differ so much in the various engines that shortened forms and lettering have been adopted to describe them, as shown in the first column of Table I. In the outline drawings of four-cycle engines, Fig. 1 will be used to denote single-acting, and Fig. 2 double-acting engines.

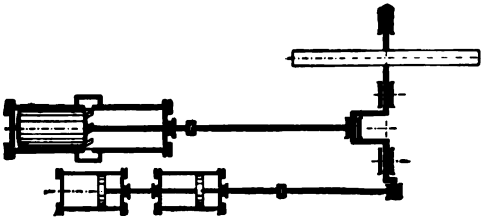


FIG. 3.—Double-acting Two-cycle (Koerting).

Two-cycle double-acting engines (Fig. 3), Koerting type, require special charging pumps.*

The Oechelhäuser engine (Fig. 4), has two pistons in one cylinder working in opposite directions; its distinguishing features are the triple crank, and charging pumps.

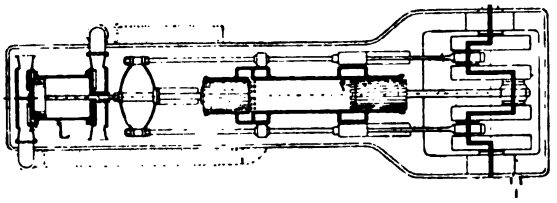


FIG. 4.—Oechelhäuser Engine (Two-cycle).

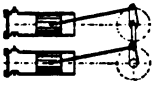
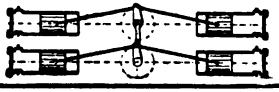
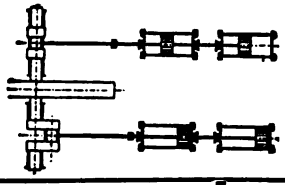
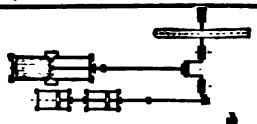
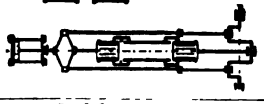
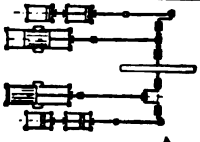
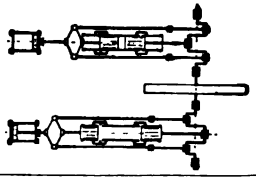
Two-cycle engines will be fully described in the chapter on "Large Gas Engines."

The usual types of gas engines are tabulated in Table I., and should be carefully studied by beginners, as we shall often have occasion to refer to them.

* Single-acting two-cycle engines are seldom used, and will not be considered in this book.

Table I.—Type and Description of the Thirteen best-known Engines.

The thick capital letters of the right-hand column correspond to the first letters of the (German) descriptions in the left-hand column (to avoid confusion the German lettering has been retained, but it does not correspond to the first letters of the English terms).

	Symbol.	Type.	Description.
Single-acting four-cycle.	E 4, 1 Z		Single-acting Four-cycle engine (One Cylinder).
	G Z 4, 2 Z		Vis-à-vis Four-cycle engine (Two Cylinders).
	Z 4, 2 Z		Twin Four-cycle engine (Two Cylinders).
	R 4, 2 Z		Tandem Four-cycle engine (Two Cylinders).
	Z G Z 4, 4 Z		Double Vis-à-vis Four-cycle engine (Four Cylinders).
Double-acting four-cycle.	D 4, 1 Z		Double-acting Four-cycle engine (One Cylinder).
	Z D 4, 2 Z		Twin Double-acting Four-cycle engine (Two Cylinders).
	R D 4, 2 Z		Tandem Double-acting Four-cycle engine (Two Cylinders).
	R Z D 4, 4 Z		Twin Tandem Double-acting Four-cycle engine (Four Cylinders).
Double-acting two-cycle.	Kör. D 2, 1 Z		Koerting Double-acting Two-cycle engine (One Cylinder). (Necessitating separate charging pumps for gas and air.)
	Oech. D 2, 1 Z		Oechelhäuser Double-acting Two-cycle engine (One Cylinder). (Charging pump for gas and air is shown to the left of the drawing.)
	Kör. Z D 2, 2 Z		Koerting Twin Double-acting Two-cycle engine (Two Cylinders). (Each motor cylinder requires separate gas and air pumps as shown.)
	Oech. Z D 2, 2 Z		(Oechelhäuser Twin Double-acting Two-cycle engine (Two Cylinders). (Each motor cylinder requires a separate charging pump as shown to the left.)

Thus the symbol **D 4** always denotes double-acting four-cycle engines.
" " **D 2** " " " two-cycle "
Engines without the symbol **D** (the first group in Table I.) are all single-acting.

Four-cycle engines are by far the most extensively used. By combining several single-cylinder four-cycle engines, a number of other types have been evolved (compare Table I.), and it seems desirable first to examine and describe the working method of these various systems. This plan will also enable us to understand the action of the combustible mixture in every position of the crank, and thus to obtain a clear idea of the processes taking place inside the engine.

Combustion Chamber.

This part of a gas engine is best compared with the clearance of a steam engine. In the latter (Figs. 6, 6A) the clearance space, the effect of which is wholly injurious, is kept

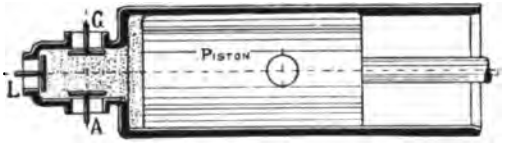


FIG. 5.
Gas Engine. Combustion Chamber indicated by Dots.

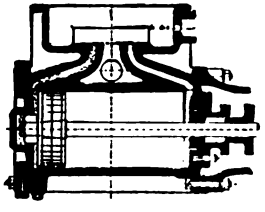


FIG. 6.
Steam Engine. Clearance indicated by Dots.

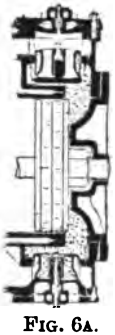


FIG. 6A.

as small as possible, while in a gas engine (Fig. 5) this space must be of a certain size, since to work without it is impossible.

The size of the combustion chamber depends on the degree of compression (*i.e.* on the compression pressure, C, Fig. 7).

We shall in a subsequent chapter return to the determination of the cubic capacity required in this chamber.

Rise of the Explosion Line in the Diagram (Fig. 7).

Shortly before explosion the gaseous mixture is at the final compression pressure, C (Fig. 7). Ignition raises the pressure to p atmospheres.

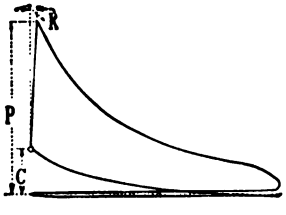


FIG. 7.—Diagram.

The question of the speed of explosion of a gas in a closed chamber, in other words, the length of time which elapses between ignition and complete combustion, cannot be mathematically determined without further study.

In Fig. 7 the travel of the piston corresponding to the time of combustion is denoted by k, and varies from 0·02 to 0·05 of the stroke.* For present purposes we will take $k = 0·03$ in the diagram.

The degree of *final compression* giving the best results, from which, as already mentioned, the size of the combustion chamber is determined,

* The effects of pre-ignition and after-combustion are here neglected.

depends on the composition of the gas, and of the explosive gaseous charge. Too high a compression gives rise to pre-ignition, whilst too low a compression is uneconomical. We may take—

With lighting gas C = 6 atm. (= 85 lbs. per sq. in.). Suction gas C = 9 atm. (= 130 lbs. per sq. in.). Blast furnace gas C = 13 atm. (= 185 lbs. per sq. in.) above atmospheric pressure. These compressions correspond to an ignition pressure for lighting gas p = 20 atm. (= 285 lbs. per sq. in.). Suction gas p = 23 atm. (= 328 lbs. per sq. in.). Blast furnace gas p = 20 atm. (= 285 lbs. per sq. in.) above atmosphere.*

Gas Engine Indicator Diagram.

A thorough knowledge of the theory of the steam engine, and especially of indicator diagrams, is taken for granted, and is essential to any useful study of our subject. Before, however, proceeding to a closer examination of the working method, we must obtain some idea of the behaviour of the mixture of gas and air in the engine cylinder, and it will be advisable to make a comparison between steam and gas. For this purpose we will take a steam engine with an initial pressure of 6 atmospheres (absolute) (= 88 lbs. per sq. in.) and a gas engine with an initial pressure of 27 atmospheres (absolute) (= 397 lbs. per sq. in.), and we get the following diagrams :—

FIG. 8.

FIG. 9.

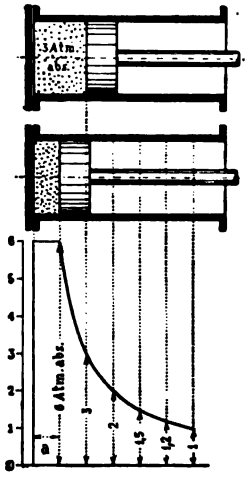


FIG. 10.—Steam Engine. At *a* is shown the cut-off, including the clearance.

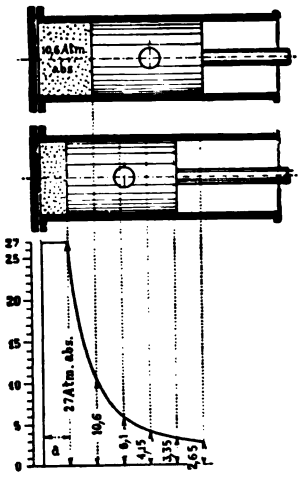


FIG. 11.

FIG. 12.

FIG. 13.—Gas Engine. At *a* is shown the combustion and compression chamber.

Assume that, behind the piston as shown, at the distance *a* from the cylinder cover there is—

In **Fig. 9**, Steam at 6 atmospheres (abs.) pressure,
In **Fig. 12**, Gas at 27 atmospheres (abs.) pressure,

both cylinders being steam- and air-tight. Assume further that the piston moves out

* [TRANSLATOR'S NOTE.—Gas-engine makers have recently increased the compression pressures; and now make C = 120 lbs. per sq. inch for lighting gas, and C = 150 lbs. per sq. inch for producer or suction gas.]

the distance a; then, as indicated in Figs. 8 and 11, the pressure in the steam cylinder will fall to 3 atmospheres, and in the gas cylinder to 10·6 atmospheres. The further the pistons move out, the lower will the pressures fall, as shown in Figs. 10 and 13.

The curve in Figs. 10 and 13 is called the *expansion curve*. This is obtained by plotting the pressures corresponding to given positions of the piston vertically on a horizontal line (the so-called atmospheric line). Any given scale is chosen for this purpose, and any given height to represent one atmosphere; in Fig. 10, for instance, 5 mm. ($\frac{3}{16}$ inch) = 1 atmosphere for steam; and in Fig. 13, 1·1 mm. ($\frac{1}{20}$ inch) = 1 atmosphere for gas.

As shown in these figures, the pressure falls more rapidly during expansion in gas than in steam engines; in Fig. 14 the two diagrams are superimposed.

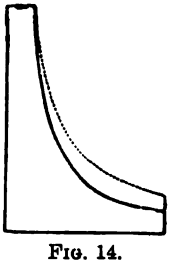


FIG. 14.

— Gas Engine Indicator Diagram.
...Steam Engine Indicator Diagram.

The same thing is expressed in equations 1 to 3.

Let p be initial pressure in atmospheres (absolute),
„ v be volume of steam at pressure p,
„ p_1 be pressure at any given point of the stroke (or position of piston),
„ v_1 be volume of steam at pressure p_1.

The fall in pressure will therefore follow the law—

$$p \cdot v = p_1 \cdot v_1 = \text{constant (for steam)} \quad \ldots \quad (1)$$

$$p \cdot v^{\chi} = p_1 \cdot v_1^{\chi} = \text{constant (for gas)} \quad \ldots \quad (2)$$

χ varies from 1·2 to 1·5, and may be taken at a mean of = 1·35 . . . . . (3)

Although a knowledge of the meaning of the different curves in the steam engine diagram, Fig. 15, is taken for granted, it is necessary clearly to understand the curves of the gas-engine diagram, Fig. 16. That part of the travel of the piston marked "cut-off"

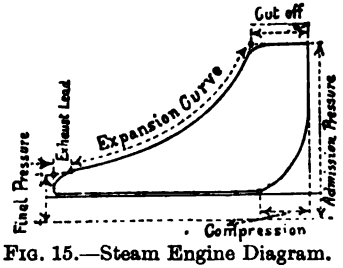


FIG. 15.—Steam Engine Diagram.

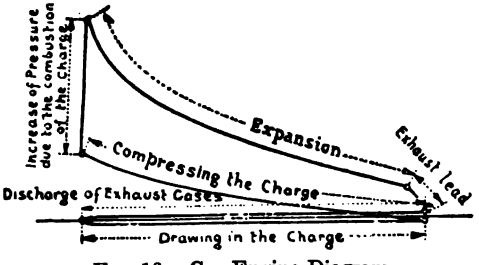


FIG. 16.—Gas Engine Diagram.

in the steam diagram is in a gas engine = zero, and the part marked "compression" is in the latter = one whole stroke.

This point will be considered more in detail in a subsequent chapter.

Indicator Diagram of a Four-cycle Gas Engine.

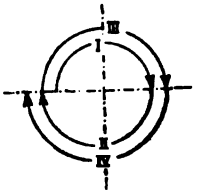
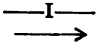


FIG. 17.

This diagram forms a closed figure, in which the various phases of work in the cylinder are shown. The numbers I. to IV. in Fig. 17 mark the different strokes, each corresponding to half a revolution of the crank shaft.

(The admission and exhaust lines in the diagram (Fig. 16) lie closely one over the other. This is explained in the purposely distorted diagram at the side.) The sequence of working strokes, represented by the different lines of the indicator diagram, is as follows :—



1*st Stroke.*—Drawing in the mixture of gas and air, combined in definite proportions. In the diagram this line lies rather below the atmospheric line, as a fall in pressure is produced by the frictional resistance of the mixture in the passages and valves.*

2*nd Stroke.*—Compression of the mixture drawn in, therefore increase of pressure due to reduction of the cylinder volume as the piston moves in.

3*rd Stroke.*—Increase of pressure through ignition of the compressed mixture. Expansion follows, with fall in pressure of the burnt charge, since the piston moves out, and the volume of the cylinder is enlarged. As a result of the opening in advance of the exhaust, the expanded gases are discharged, till the piston reaches the dead point.

4*th Stroke.*—Disharge of remaining gases, as a result of the reduction of the cylinder volume by the piston moving in. As there is a rise in pressure, due to frictional resistance of the gases in the exhaust valve, this line in the diagram lies somewhat higher than the atmospheric.*

The above explanation will suffice for a cursory study of the indicator diagram. In a later chapter the actual plotting of the diagram will be considered; meanwhile the standard diagram (p. 23) should be consulted.

Working Method of Four-cycle Engines.

In studying these, the lines of the indicator diagram plotted between the drawings of cylinders must not be overlooked. The line to which a given stroke of the piston marked above it corresponds, is drawn full, the lines corresponding to the rest of the cycle are dotted. Thus the evolution of the diagram can be followed.

* See the developed diagrams (pp. 8–17). This line, for the sake of simplicity, is shown as = the atmospheric.

Single-cylinder Four-cycle E 4, One Cylinder (Usual Type).

The term "four-cycle" denotes, as already explained on p. 1, the number of single strokes of the piston required to complete one cycle of operations. The four positions of the piston in the illustrations below show this graphically and clearly.

The Roman numerals I. to IV. give the different piston strokes for one cycle.

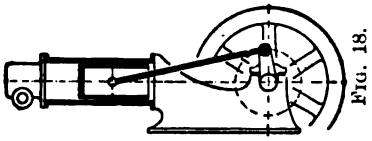
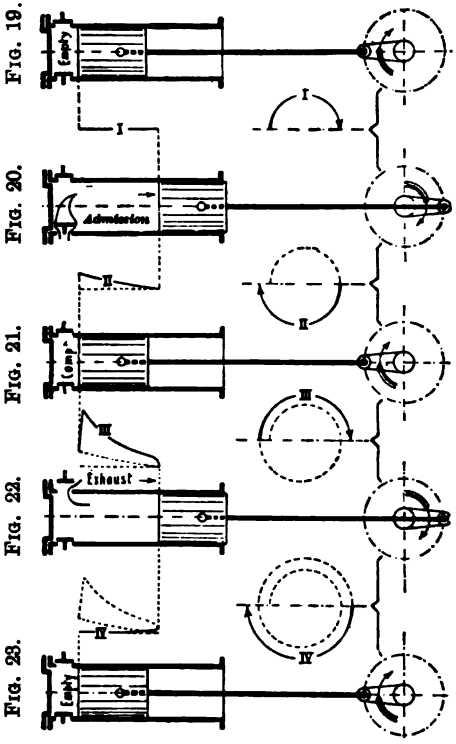


FIG. 18.



STROKE I.

Outstroke of piston towards bearings. Drawing in a charge of combustible mixture during the whole stroke.

STROKE II.

Instroke.—Compression of the mixture, ignition taking place at end of stroke (left-hand position of piston).

STROKE III.

Outstroke.—Explosion and expansion of the burning gases. Shortly before piston reaches end of cylinder (right-hand) discharge of exhaust begins.

STROKE IV.

Instroke.—Exhausting the products of combustion. At the end of this stroke (left-hand position of piston) all gases are exhausted, and the cycle of operations is completed.

Fresh charge drawn in, as in Stroke I.

The term "outstroke" signifies the moving of the piston towards the crank bearings. At full load, work is done by the piston once in every two revolutions of the crank shaft, as against four working strokes per two revolutions in a steam engine. At half load, or when there is a miss-fire, ignition only takes place once in every four revolutions, and the piston and connecting rod must be carried along during seven piston strokes by the energy stored up in the flywheel.

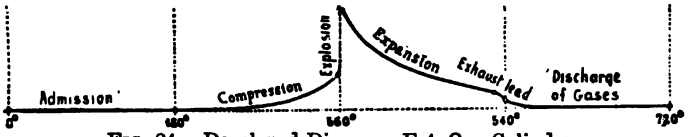


FIG. 24.—Developed Diagram, E 4, One Cylinder.

The developed diagram (Fig. 24) is a so-called "continuous" diagram of the cycle of operations, the numbers give in degrees the path described by the crank pin.

180° *is equivalent to one piston stroke.*

Double-acting Four-cycle Engine, D 4, One Cylinder.

The working method of this engine is the same as that of *two* single-cylinder *four-cycle* engines, the ignitions in which take place first at 180° then at 540° apart, in the circle described by the crank.

Back End of Cylinder.	Front End of Cylinder.
OUTSTROKE.	
1st Stroke. Drawing in the combustible mixture.	*2nd Stroke.* Compression of charge (ignition at extreme right-hand position of piston).
INSTROKE.	
2nd Stroke. Compression of charge (ignition at extreme left-hand position of piston).	*3rd Stroke.* Explosion and expansion of the burning charge.
OUTSTROKE.	
3rd Stroke. Explosion and expansion of the burning charge.	*4th Stroke.* Discharge of exhaust gases.
INSTROKE.	
4th Stroke. Discharge of exhaust gases.	*1st Stroke.* Drawing in the combustible mixture.

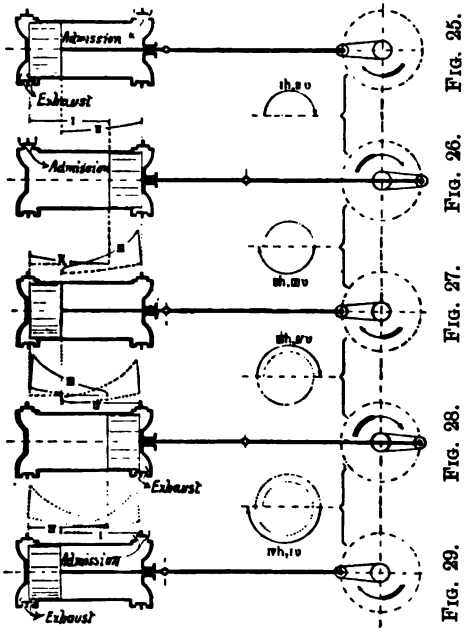


FIG. 25. FIG. 26. FIG. 27. FIG. 28. FIG. 29.

The output of such an engine is thus double that of a single-acting four-cycle engine of the same cylinder dimensions.

Here again the Roman numerals show the sequence of piston strokes for one cycle, *v* denoting the front, and *h* the back end of the piston.

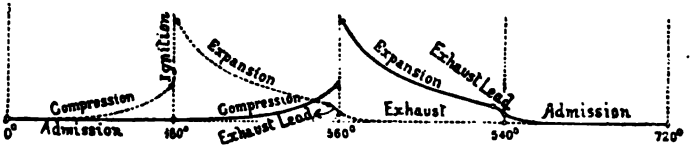
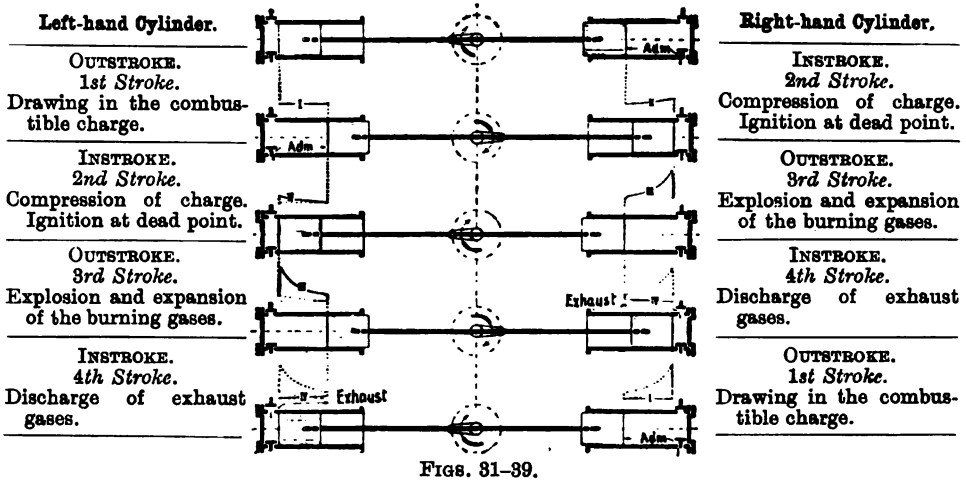


FIG. 30.—Developed Diagram for D 4, One-cylinder Engine.

The developed diagram for a double-acting four-cycle engine is shown at Fig. 30. The work done at the back end of the cylinder is marked ——, that at the front ..... Each developed diagram represents the work done during four strokes, or two revolutions of the crank shaft.

Vis-à-Vis Four-cycle Engine, G Z 4, Two Cylinders (Deutz).



FIGS. 31–39.

The working method is the same, as regards the sequence of operations, as that of the double-acting four-cycle engine (p. 9). Both pistons act upon the *same* crank. The pistons pass successively over the dead points, front and back. The sequence of the working strokes is as follows:—

	Left-hand Cylinder.	*Right-hand Cylinder.*
One revolution	1st stroke Admission.	2nd stroke Compression.
	2nd „ Compression.	3rd „ Explosion and expansion.
One revolution	3rd „ Explosion and expansion.	4th „ Exhaust.
	4th „ Exhaust.	1st „ Admission.

The developed diagram (Fig. 40) is given, as before, to explain the working operations. The work done in each cylinder is shown, and comprises four strokes, or two revolutions of the crank shaft.

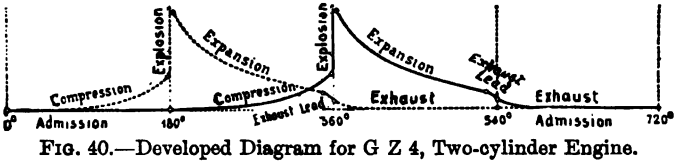


FIG. 40.—Developed Diagram for G Z 4, Two-cylinder Engine.

Besides this engine there are also *vis-à-vis* engines with cranks set at an angle of 180° to each other. With this arrangement the two pistons pass at the same time over the dead point, back or front. The developed diagram is the same as that of engine **Z 4** (p. 11).

Twin Four-cycle Engine, Z 4, Two Cylinders.

The cranks are at the same angle, and the two pistons are always in the same positions (with reference to the dead point) at the same moment.

Ignition and the working operations in both cylinders succeed each other at an angle of 360°, that is, ignition is obtained, in one or other cylinder, at each revolution of the crank shaft.

Cylinder A.	Cylinder B.
INSTROKE.	
2nd Stroke. Compression of charge. Ignition at dead point.	*4th Stroke.* Discharge of exhaust gases.
OUTSTROKE.	
3rd Stroke. Explosion and expansion of the burning gases.	*1st Stroke.* Drawing in the combustible charge.
INSTROKE.	
4th Stroke. Discharge of exhaust gases.	*2nd Stroke.* Compression of charge. Ignition at dead point.
OUTSTROKE.	
1st Stroke. Drawing in the combustible charge.	*3rd Stroke.* Explosion and expansion of the burning gases.

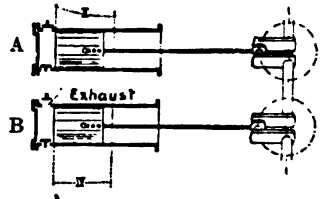


FIG. 41.

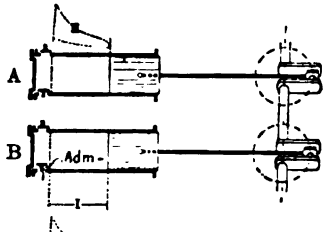


FIG. 42.

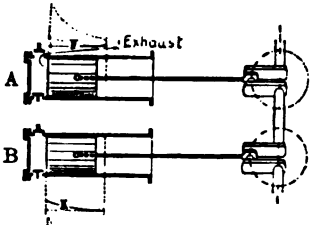


FIG. 43.

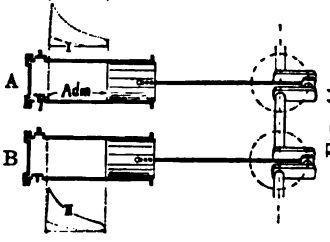


FIG. 44.

The developed diagram for this type is shown at Fig. 45; the work done in cylinder A is marked ———, and that in cylinder B .....

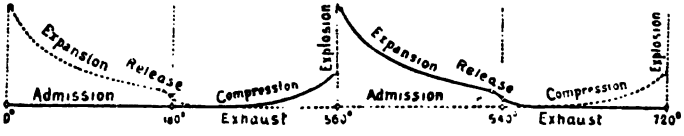


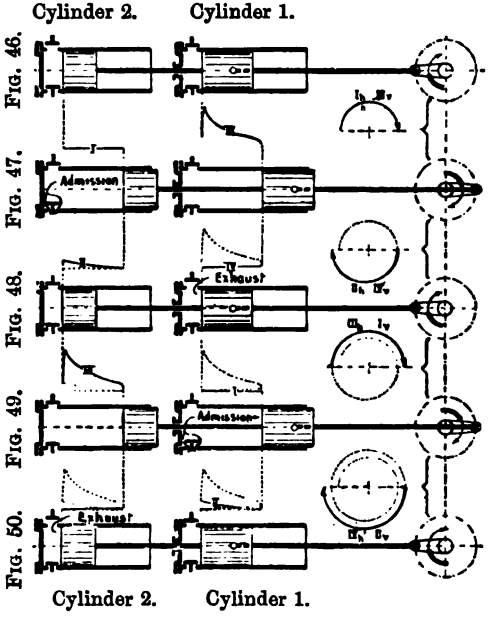
FIG. 45.—Developed Diagram for Z 4, Two-cylinder Engine.

Tandem Four-cycle Engine, R 4, Two Cylinders (Nürnberg).

The cylinders are arranged one behind the other, and both pistons work upon the same crank. The sequence of operations in the two cylinders takes place at an angle of 360° apart; thus there is one ignition for each revolution of the crank shaft. The working method is the same as that of the last engine.

The sequence of the different working strokes is as follows :—

	First Cylinder.	*Second Cylinder.*
One revolution	3rd stroke Explosion and expansion.	1st stroke Admission.
	4th ,, Exhaust.	2nd ,, Compression.
One revolution	1st ,, Admission.	3rd ,, Explosion and expansion.
	2nd ,, Compression.	4th ,, Exhaust.



Cylinder 2.	Cylinder 1.
OUTSTROKE.	
1st Stroke. Drawing in the combustible charge.	*3rd Stroke.* Explosion and expansion of the burning gases.
INSTROKE.	
2nd Stroke. Compression of charge. Ignition at dead point.	*4th Stroke.* Discharge of exhaust gases.
OUTSTROKE.	
3rd Stroke. Explosion and expansion of the burning gases.	*1st Stroke.* Drawing in the combustible charge.
INSTROKE.	
4th Stroke. Discharge of exhaust gases.	*2nd Stroke.* Compression of charge. Ignition at dead point.

The developed diagram for both cylinders is shown at Fig. 51; the work done in cylinder 2 is marked ———, that in cylinder 1 .....

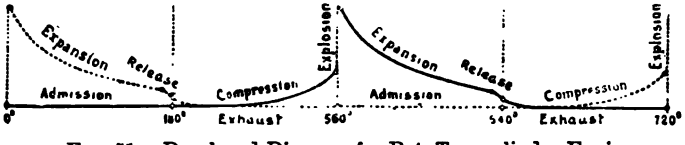


Fig. 51.—Developed Diagram for R 4, Two-cylinder Engine.

Double Vis-à-Vis Four-cycle Engine, Z G Z 4, Four Cylinders (Deutz).

Type especially intended for engines driven with blast furnace gas (now little made, as it has several disadvantages). As all four pistons move in the same direction, the working strokes in each cylinder succeed each other at an angle of 180° apart, that is to say, an ignition is obtained in one or other cylinder at every half revolution of the crank shaft.

Left End of Engine.		Cyl.		Cyl.	Right End of Engine.	
Cylinder 1.	*Cylinder* 3.				*Cylinder* 2.	*Cylinder* 4.
INSTROKE.					OUTSTROKE.	
2nd Stroke. Compression of charge. Ignition at dead point.	*4th Stroke.* Discharge of exhaust gases.	1 / 3	Exhaust / Adm	2 / 4	*3rd Stroke.* Explosion and expansion of the burning gases.	*1st Stroke.* Drawing in the combustible charge.
OUTSTROKE.					INSTROKE.	
3rd Stroke. Explosion and expansion of the burning gases.	*1st Stroke.* Drawing in the combustible charge.	1 / 3	Exhaust / Adm	2 / 4	*4th Stroke.* Discharge of exhaust gases.	*2nd Stroke.* Compression of charge. Ignition at dead point.
INSTROKE.					OUTSTROKE.	
4th Stroke. Discharge of exhaust gases.	*2nd Stroke.* Compression of charge. Ignition at dead point.	1 / 3	Exhaust / Adm	2 / 4	*1st Stroke.* Drawing in the combustible charge.	*3rd Stroke.* Explosion and expansion of the burning gases.
OUTSTROKE.					INSTROKE.	
1st Stroke. Drawing in the combustible charge.	*3rd Stroke.* Explosion and expansion of the burning gases.	1 / 3	Adm / Exhaust	2 / 4	*2nd Stroke.* Compression of charge. Ignition at dead point.	*4th Stroke.* Discharge of exhaust gases.
		Cyl.		Cyl.		

FIGS. 52–55.

Fig. 56 shows the developed diagram of the sequence of work. The diagrams of the different cylinders cover two revolutions of the crank shaft, and are marked ——, —·—·—, ·····, — — —.

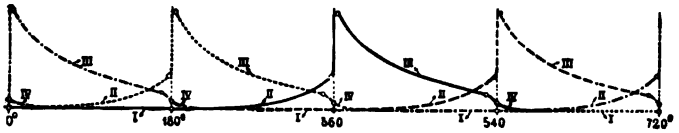
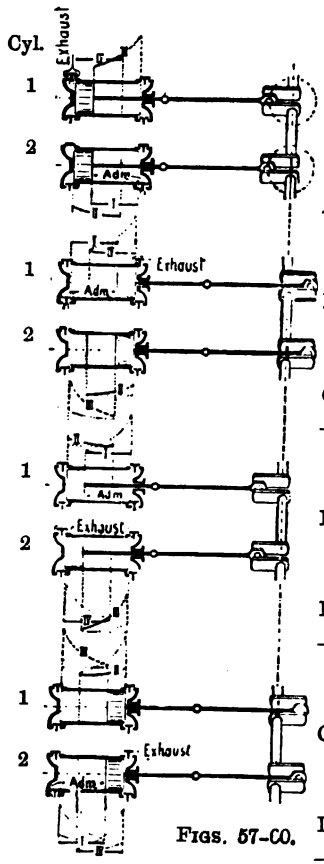


FIG. 56.—Developed Diagram for Z G Z 4, Four-cylinder Engine.

Twin Double-acting Four-cycle Engine, Z D 4, Two Cylinders.

Type for large engines, especially when driven with blast furnace gas.

The working strokes at either end of the cylinders succeed each other at an angle of 180° apart, both pistons moving in the same direction, that is, ignition takes place at every half revolution of the crank shaft. The cranks may also be set 180° apart, the strokes will then follow each other at the same interval as before.



FIGS. 57-60.

	Front End of Cylinder.	Back End of Cylinder.
	INSTROKE.	
	Cylinder 1.	
	3rd Stroke. Explosion and expansion of the burning gases.	*4th Stroke.* Discharge of exhaust gases.
	Cylinder 2.	
	1st Stroke. Drawing in the combustible charge.	*2nd Stroke.* Compression of charge. Ignition at dead point.
	OUTSTROKE.	
	Cylinder 1.	
	4th Stroke. Discharge of exhaust gases.	*1st Stroke.* Drawing in the combustible charge.
	Cylinder 2.	
	2nd Stroke. Compression of charge. Ignition at dead point.	*3rd Stroke.* Explosion and expansion of the burning gases.
	INSTROKE.	
	Cylinder 1.	
	1st Stroke. Drawing in the combustible charge.	*2nd Stroke.* Compression of charge. Ignition at dead point.
	Cylinder 2.	
	3rd Stroke. Explosion and expansion of the burning gases.	*4th Stroke.* Discharge of exhaust gases.
	OUTSTROKE.	
	Cylinder 1.	
	2nd Stroke. Compression of charge. Ignition at dead point.	*3rd Stroke.* Explosion and expansion of the burning gases.
	Cylinder 2.	
	4th Stroke. Discharge of exhaust gases.	*1st Stroke.* Drawing in the combustible charge.

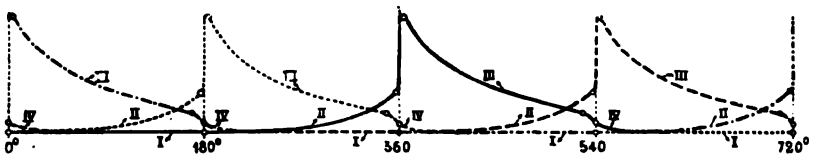


FIG. 61.—Developed Diagram for Z D 4, Two-cylinder Engine.

The developed diagram is exactly the same as that of **Z G Z 4**, four cylinders (p. 13). The diagrams of work at either end of the cylinders are marked——, — · — · —, ·····, — — — —.

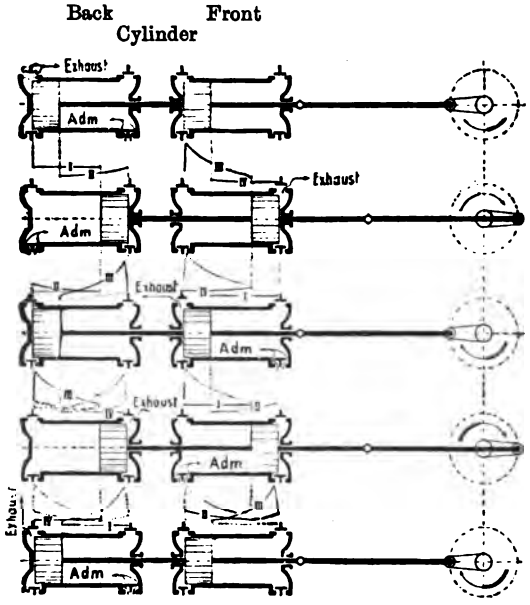
Tandem Double-acting Four-cycle Engine, R D 4, Two Cylinders.

Type for large engines, especially for work with blast furnace gas.

The working strokes at either ends of the cylinders follow each at 180° distance, that is, an ignition is obtained at every half revolution.

Back Cylinder.

Back End of Cylinder.	*Front End of Cylinder.*
OUTSTROKE.	
1st Stroke. Drawing in the combustible charge.	*2nd Stroke.* Compression of charge. Ignition at dead point.
INSTROKE.	
2nd Stroke. Compression of charge. Ignition at dead point.	*3rd Stroke.* Explosion and expansion of the burning gases.
OUTSTROKE.	
3rd Stroke. Explosion and expansion of the burning gases.	*4th Stroke.* Discharge of exhaust gases.
INSTROKE.	
4th Stroke. Discharge of exhaust gases.	*1st Stroke.* Drawing in the combustible charge.



Front Cylinder.

Back End of Cylinder.	*Front End of Cylinder.*
OUTSTROKE.	
3rd Stroke. Explosion and expansion of the burning gases.	*4th Stroke.* Discharge of exhaust gases.
INSTROKE.	
4th Stroke. Discharge of exhaust gases.	*1st Stroke.* Drawing in the combustible charge.
OUTSTROKE.	
1st Stroke. Drawing in the combustible charge.	*2nd Stroke.* Compression of charge. Ignition at dead point.
INSTROKE.	
2nd Stroke. Compression of charge. Ignition at dead point.	*3rd Stroke.* Explosion and expansion of the burning gases.

FIGS. 62–66.

The developed diagram is the same as that of **Z G Z 4**, one cylinder, p. 13. The diagrams for the two ends of both cylinders are marked ——, — · — · —, ·····, — — — —, each representing two revolutions of the crank shaft. The strokes are so arranged that the working (or motor) stroke cannot take place in both cylinders at the same time.

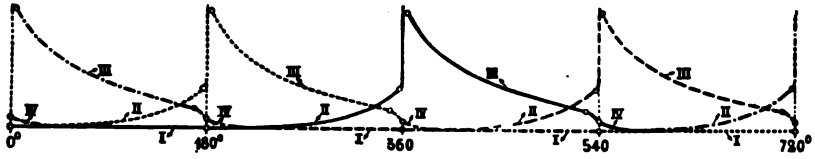


FIG. 67.—Developed Diagram for R D 4, Two-cylinder Engine.

Remarks on the Developed Diagrams.

To explain the developed diagrams given at pp. 8 to 17, let us suppose the four single strokes of the piston required for one complete cycle of operations to be set out side by side, thus :—

Outstroke.	Instroke.	Outstroke.	Instroke.
I	II	III	IV

If the corresponding curves of the indicator diagram are plotted vertically, a graphic representation of the sequence of operations in the engine is obtained. The pressure in the cylinder during admission and exhaust is in these diagrams taken as the same as the line of atmospheric pressure.

Classification of Types yielding the same Developed Diagrams.

The numbers below the diagrams give the angle described by the crank pin, in degrees, thus—

0° to 180° corresponds to a single stroke, or half a revolution of the crank shaft.
0° to 360° „ „ a double stroke (to and fro), or one complete revolution of the crank shaft.
0° to 540° „ „ three single strokes, or three half-revolutions of the crank shaft.
0° to 720° „ „ two double strokes (to and fro), or two complete revolutions of the crank shaft.

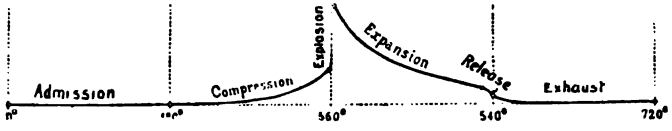
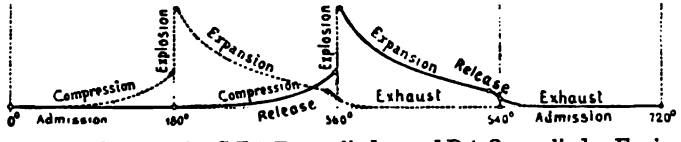


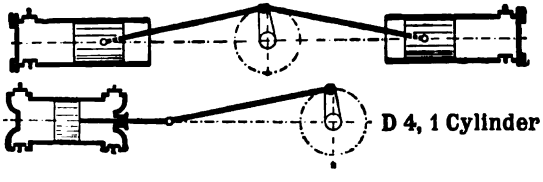
FIG. 68.—Diagram for E 4, One-cylinder Engine.



IG. 69.—Diagram for G Z 4, Two-cylinder, and D 4, One-cylinder Engine.

E 4, 1 Cylinder.

G Z 4, 2 Cylinders.



D 4, 1 Cylinder

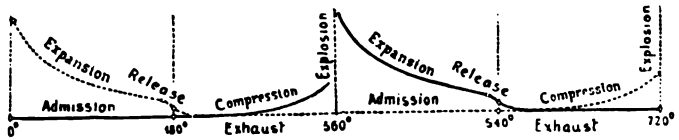


FIG. 70.—Diagram for Z 4, Two-cylinder, and R 4, Two-cylinder Engines.

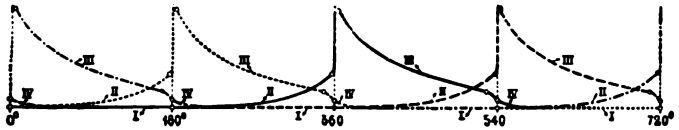
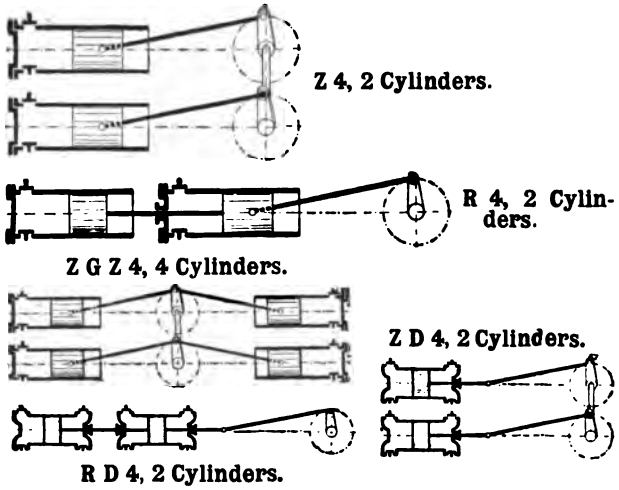


FIG. 71.—Diagram for Z G Z 4, Four-cylinder, Z D 4, Two-cylinder, and R D 4, Two-cylinder Engines.



Remarks on diagram, Fig. 71.

The working operations in the different cylinders (in double-acting engines at both ends of the cylinder) are marked ——, — · — · —, ····· — — — — ·

The cycle requires four strokes, or two revolutions of the crank shaft. The words on the developed diagrams denote—

1st Stroke	*Admission:*	Drawing in the charge of gas and air.
2nd „	*Compression:*	Of the charge drawn in.
3rd „	*Explosion:*	Ignition of the compressed mixture which burns with rapid increase of pressure.
	Expansion:	Of the exploded charge as a consequence of the increase of cylinder volume due to the piston moving out.
	Release:	Escape of gases and products of combustion before the end of the stroke.
4th „	*Exhaust:*	Discharge of gases still remaining in cylinder (the volume of the cylinder being reduced by the piston moving in).

Thus for a complete cycle of work at either end of the cylinder, a four-cycle engine requires two revolutions or four piston strokes, but in the last three types, described above, these are so arranged that work is done at each stroke of the piston (compare Fig. 71).

Remarks on the Developed Diagrams.

To explain the developed diagrams given at pp. 8 to 17, let us suppose the four single strokes of the piston required for one complete cycle of operations to be set out side by side, thus :—

Outstroke.	Instroke.	Outstroke.	Instroke.
I	II	III	IV

If the corresponding curves of the indicator diagram are plotted vertically, a graphic representation of the sequence of operations in the engine is obtained. The pressure in the cylinder during admission and exhaust is in these diagrams taken as the same as the line of atmospheric pressure.

Classification of Types yielding the same Developed Diagrams.

The numbers below the diagrams give the angle described by the crank pin, in degrees, thus—

0° to 180° corresponds to a single stroke, or half a revolution of the crank shaft.
0° to 360° „ „ a double stroke (to and fro), or one complete revolution of the crank shaft.
0° to 540° „ „ three single strokes, or three half-revolutions of the crank shaft.
0° to 720° „ „ two double strokes (to and fro), or two complete revolutions of the crank shaft.

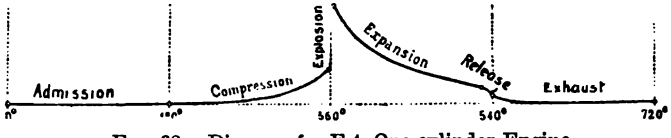
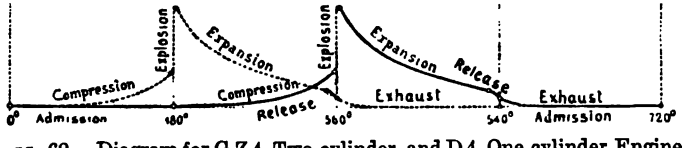


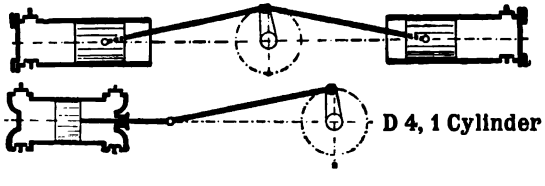
FIG. 68.—Diagram for E 4, One-cylinder Engine.



IG. 69.—Diagram for G Z 4, Two-cylinder, and D 4, One-cylinder Engine.

E 4, 1 Cylinder.

G Z 4, 2 Cylinders.



D 4, 1 Cylinder

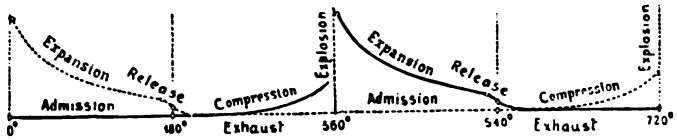
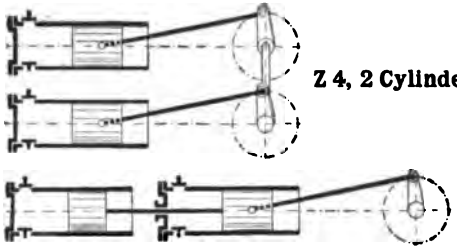


FIG. 70.—Diagram for Z 4, Two-cylinder, and R 4, Two-cylinder Engines.



Z 4, 2 Cylinders.

R 4, 2 Cylinders.

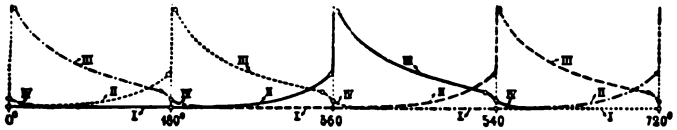
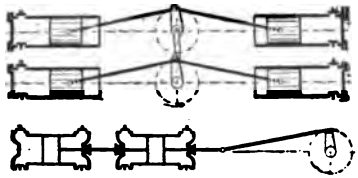
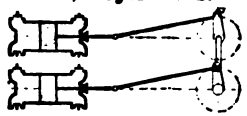


FIG. 71.—Diagram for Z G Z 4, Four-cylinder, Z D 4, Two-cylinder, and R D 4, Two-cylinder Engines.



Z G Z 4, 4 Cylinders.



Z D 4, 2 Cylinders.

R D 4, 2 Cylinders.

Remarks on diagram, Fig. 71.

The working operations in the different cylinders (in double-acting engines at both ends of the cylinder) are marked ——, —·—·—·, ····· — — — — ·

The cycle requires four strokes, or two revolutions of the crank shaft. The words on the developed diagrams denote—

1st Stroke	*Admission:*	Drawing in the charge of gas and air.
2nd „	*Compression:*	Of the charge drawn in.
3rd „	*Explosion:*	Ignition of the compressed mixture which burns with rapid increase of pressure.
	Expansion:	Of the exploded charge as a consequence of the increase of cylinder volume due to the piston moving out.
	Release:	Escape of gases and products of combustion before the end of the stroke.
4th „	*Exhaust:*	Discharge of gases still remaining in cylinder (the volume of the cylinder being reduced by the piston moving in).

Thus for a complete cycle of work at either end of the cylinder, a four-cycle engine requires two revolutions or four piston strokes, but in the last three types, described above, these are so arranged that work is done at each stroke of the piston (compare Fig. 71).

CHAPTER II.

Output of the Engine.

Indicated and Effective (or Brake) Horse-power.

LET (IHP) be the whole of the work done in the motor cylinder; IHP the available indicated work; (IHP) — IHP the energy absorbed by the engine, namely, in four-cycle engines the energy absorbed in drawing in the charge; in two-cycle engines the energy absorbed by the resistance of the pumps (as shown by the indicator); RHP frictional resistance of the engine; BHP actual output, or work given off at the end of the crank shaft. Then—

IHP = RHP + BHP; (IHP) = IHP + the indicated work required for the pumps or for drawing in the charge . . . . . (1)

The Mechanical Efficiency η

is the ratio the Brake Horse-power bears to the Indicated Horse-power, therefore—

$$\eta = \frac{\text{BHP}}{\text{IHP}}\ ;\ \text{BHP} = \text{IHP}\,\eta\ ;\ \text{IHP} = \frac{\text{BHP}}{\eta} \quad . \quad . \quad . \quad . \quad . \quad (2)$$

The mechanical efficiency varies with differences in the load on the same engine, and is greatest at maximum and lowest at minimum load.

Table I.—Approximate Values of η (Mechanical Efficiency).

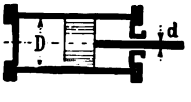
Load.	Maximum.	Rated $\epsilon^* = 85$.	$\frac{3}{4}$	$\frac{1}{2}$	$\frac{1}{4}$
Single-acting Four-cycle, 1 cylinder $\eta =$	0·84	**0·78**	0·70	0·60	0·48
" " " " 2 " $\eta =$	0·86	**0·80**	0·74	0·62	0·49
" " " " 4 " $\eta =$	0·88	**0·82**	0·79	0·68	0·52
Double-acting Four-cycle, 1 cylinder $\eta =$	0·86	**0·78**	0·75	0·65	0·50
" " " " 2 " $\eta =$	0·88	**0·80**	0·78	0·68	0·54
" " " " 4 " $\eta =$	0·90	**0·82**	0·80	0·70	0·58
Two-cycle (with separate charging pump) $\eta =$	0·90	**0·82**	0 80	0·70	0·58
Steam Engine . . . . . $\eta =$	0·90	**0·89**	0·88	0·82	0·70

The Effective Piston Area, A,

is equal to the sectional area of the motor cylinder, less the area of the piston rod, therefore—

FIG. 1.

FIG. 2.



For a single-acting engine, $A = \frac{\pi}{4} D^2$ . . . . . (3)

" double-acting engine, $A = \frac{\pi}{4}\left(D^2 - \frac{d^2}{2}\right)$ . . (4)

For engines with a piston rod passing through both covers $\Big\}$ $A = \frac{\pi}{4}(D^2 - d^2)$ . . . (4A)

Mean Pressure on the Piston p_m.

This pressure varies with the different positions of the piston during a cycle of work, that is, in a four-cycle engine, during two revolutions of the crank shaft).

* For ratio ϵ, see next page.

The mean or average pressure, p_m, for one cycle therefore represents the mean pressure exerted on the piston during the motor stroke, less the mean pressure during the three other strokes, when the piston receives its motion (or impulse) from the flywheel.*

The Ratio ϵ.

In a single-cylinder *four-cycle* engine, as already explained, there is one impulse or motor stroke for every two revolutions of the crank shaft. Therefore if n = the number of revolutions per minute, $\frac{n}{2}$ will be the maximum number of impulses obtained with this type of engine, that is, the maximum power developed. It is customary with engine builders to rate their engines at a lower or *nominal horse-power*, and to assume that they will be run with a lesser number of ignitions or impulses per minute than the maximum $\frac{n}{2}$.

We will denote the ratio $\frac{\text{actual number of impulses per minute}}{\text{maximum number of impulses per minute}}$ by ϵ . . (5)

The maximum theoretical number of impulses depends upon the type of engine. For this, Table II., p. 20, may be consulted, which gives—

For *four-cycle engines*, maximum number of impulses $= \frac{z}{4} 2n$ . . . . (6)

For *two-* „ „ „ „ „ „ $= \frac{z}{2} 2n$ . . . . (6A)

z being the number of working ends of the cylinder (in a single-acting engine, $z = 1$; in a double-acting engine, $z = 2$, for each cylinder).

The ratio ϵ can also be applied to engines which are governed by varying the *proportions* of the charge; in these, as in other types, the mean pressure on the piston may be taken as $= \epsilon p_m$.

Three systems of governing are chiefly employed in gas engines, namely, the "*hit-and-miss*," the "*qualitative*" and the "*quantitative*," and also a combination of the two latter. In the first, the gas valve is closed from time to time during the suction stroke, if the governor runs at too high a speed, but when the valve is opened, the quantity of gas reaching the cylinder is *always the same*. Governing by "cut-off," or varying the quantity or quality of the charge, may be effected in various ways; we shall recur to it later.

Approximate Determination of the Mean Pressure on the Piston p_m.

As various kinds of combustible (rich and poor gases) are used in internal combustion engines, naturally the curves obtained in the indicator diagrams also vary † (see Fig. 9). It is necessary, however, to have a standard, or *normal diagram*, as a basis for rough *approximate* calculations, and this is shown at Fig. 10, p. 21.

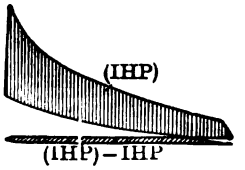


FIG. 3.

* In four-cycle engines, if the diagram of work be planimetered, it will give p_m direct for the IHP, because when this is done the lower area ////// (work of drawing in the charge (IHP)−IHP is already deducted from the positive work |||||| (Fig. 3).

† The theoretical explanation of these diagrams will be found in a later chapter.

A = effective area of motor cylinder in sq. inches, according to equations 3 and 4.

S = stroke, or travel of piston in feet.

n = number of revolutions per minute.

V = $2Sn$ = mean piston speed in feet per minute. . . . (7)

p = mean pressure on the piston in lbs. per sq. inch during the working stroke.

p_i = mean pressure on the piston or on the sectional area of the cylinder during the cycle, *c.c.* during four strokes in an E 4 engine. By formula AVp_i this gives the maximum work of the engine in ft.-lbs. per minute.

$\varepsilon = \dfrac{\text{actual number of impulses per minute}}{\text{maximum number of impulses per minute}}$ comp. p. 19.

εp_i = mean pressure on piston, or on sectional area of the cylinder, taking count of governing on hit-and-miss principle, or by quantity or quality variation of the charge. By formula $AV\varepsilon p_i$ this gives the indicated rated work of the engine in ft.-lbs. per minute.

$\varepsilon \frac{z}{T} 2n$ = number of impulses per minute. Comp. equations 5 and 6 and Table II. (at maximum power $\varepsilon = 1$).

$100\,(1-\varepsilon)$ = percentage of miss-fires. For instance—

If the miss-fires = 0%, 10%, 15%, 25%, 50%, 100%.

ε = 1, 0·9, 0·85, 0·75, 0·5, 0.

z = ends of the cylinder at which work is done.

T = number of strokes per cycle. Thus $p_i = \frac{z}{T} p_m$.

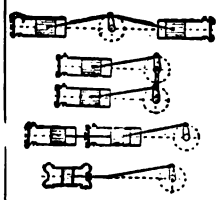
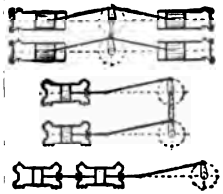
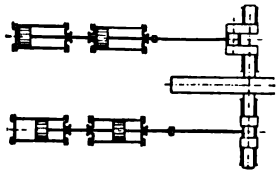
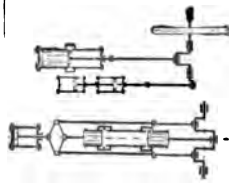
$AV\varepsilon p_i$ = indicated work of the engine in ft.-lbs. per minute.

η = mechanical efficiency according to Table I., p. 18.

Therefore the *Indicated* Horse-power, $\text{IHP} = \dfrac{AV}{33{,}000}\,\varepsilon p_i$; and the *Brake* Horse-power, $\text{BHP} = \dfrac{\eta AV}{33{,}000}\,\varepsilon p_i = \text{IHP}\eta$ . . (8)

For *approximate calculations* we will take p_m = 82·5 lbs. per sq. inch; and the rated horse-power, or working load, ε = 0·85.

Table II.—Approximate Values for Calculating the Output.

	Four-cycle, *i.e.* T = 4.				Two-cycle, *i.e.* T = 2.	
	Type a	b	c	d	e	f
						Twin Körting. Twin Oechelhäuser.
z =	1	2	4	8	2	4
p = 82·5 lbs. p_i =	$\frac{1}{4}\cdot p_m$	$\frac{2}{4}\cdot p_m$	$\frac{4}{4}\cdot p_m$	$\frac{8}{4}\cdot p_m$	$\frac{2}{2}\cdot p_m$	$\frac{4}{2}\cdot p_m$
p_i =	20·6	41·25	82·5	165	82·5	165 lbs. per sq. in.
ε = 0·85 $\varepsilon . p_i$ =	17·5	35	70	140	70	140 ,, ,,
	FIG. 4.	FIG. 5.	FIG. 6	FIG. 7.	FIG. 8.	

In two-cycle engines we may write—

$$\left.\begin{array}{ll}\text{Instead of } p_m \ldots\ldots\ldots & 0{\cdot}9\, p_m \\ \text{,, ,, } p_i \ldots\ldots\ldots & 0{\cdot}9\, p_i\end{array}\right\} \text{approx.}$$

This value may be more accurately determined by subtracting the mean resistance of the air-pumps from the work done in the motor cylinder (compare Part B., "Larger Gas Engines").

The mean pressure p_m calculated from the standard diagram (Fig. 10) is available for determining the IHP of the engine.

In most cases the diagram (Fig. 10) corresponds to the work done by an engine with lighting or coal gas.

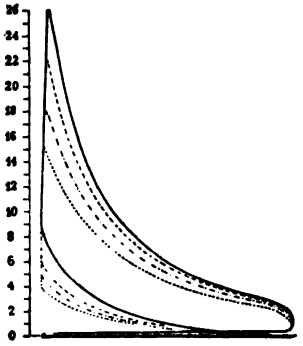


FIG. 9.—Various Diagrams.

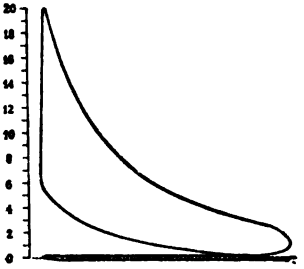


FIG. 10.—Standard Diagram.

Calculation of Indicated Horse-power, IHP, and the Brake Horse-power, BHP.

(See Table I., p. 18, and Table II., p. 20.)

Example 1 (Table II., p. 20).—Taking the rated horse-power of an engine at 80 per cent. of the maximum, then $\epsilon = 0{\cdot}8$, that is to say, out of every possible 100 impulses or ignitions there are twenty miss-fires, or the governor must so regulate the admission of gas that the indicator diagram shows that only 80 per cent. of the total power has been developed.

Example 2 (Table II.).—If a single-acting four-cycle engine has a cylinder diameter of D = 16·5 inches, V = 472 feet per minute piston speed, and mechanical efficiency $\eta = 0{\cdot}78$ (Table I., p. 18), then by equation 8, and the last line of column *a* in Table II. ($\epsilon = 0{\cdot}85$), the rated HP

$$= 0{\cdot}78\,\frac{\pi}{4}\,\frac{16{\cdot}5^2 \times 472}{33{,}000} \times 17{\cdot}5 = 42 \text{ BHP, working load.}$$

Example 3 (Table II.).—Let A = 108·5 sq. inches, V = 550 feet per minute piston speed, for a four-cylinder, double-acting four-cycle engine (for type, see column *d*, Table II.). According to Table I., the mechanical efficiency $\eta = 0{\cdot}82$, and by equation 8 and Table II.,

$$\text{Maximum HP} = \eta\,\frac{\text{AV}}{33{,}000}\,p_i = 0{\cdot}85\,\frac{108{\cdot}5 \times 550}{33{,}000} \times 165 = \mathbf{254} \text{ HP.}$$

$$\text{Rated or Brake HP at working load} = \eta\,\frac{\text{AV}}{33{,}000}\,\epsilon p_i = 0{\cdot}82\,\frac{108{\cdot}5 \times 550}{33{,}000} \times 140$$
$$= \mathbf{208} \text{ BHP at working load.}$$

We will take this engine as developing 200 HP.

Thus to *calculate the work done by any engine* it is necessary to know—

1. Whether the engine is of the *four-cycle* or *two-cycle* type.
2. Whether the engine is *single-* or *double*-acting.
3. *Number* of *motor cylinders*.
4. Nature of *combustible* to be used.

Rapid approximate calculation of the normal power developed by an engine. Taking $\epsilon = 0{\cdot}85$ (p. 20), and η from Table I., p. 18, we get Table III., Values of Constant C.

D = diameter of cylinder in feet.
S = stroke of piston in feet.
n = revolutions per minute.

$$\text{Rated HP} = C \times D^2 \times S \times n\,;\ D^2 = \frac{\text{rated HP}}{CSn}\ ;\ n = \frac{\text{rated HP}}{CD^2S} \quad . \quad . \quad . \quad . \quad (9)$$

Table III.

Combustible.	Mean calorific value in British thermal units per cubic foot, or per lb.	p_m lbs. per sq. in.	Single-acting four-cycle. $\eta = 0{\cdot}78$ to $0{\cdot}82$.			Double-acting four-cycle. $\eta = 0{\cdot}78$ to $0{\cdot}82$.			Double acting two-cycle. $\eta = 0{\cdot}82$.	
			1 cylinder.	2 cyl.	4 cyl.	1 cyl.	2 cyl.	4 cyl.	1 cyl.	2 cyl.
Lighting or Coal Gas	563 B.T.U. per cub. ft.	82·5	C = **0·094**	0·196	0·392	—	—	—	—	—
Pressure or Suction Gas	124 ,, ,, ,, ,,	65·5	,, = **0·077**	0·156	0·312	**0·147**	0·312	0·653	**0·312**	0·625
,, rich .	140 ,, ,, ,, ,,	71	,, = **0·0825**	0·170	0·341	**0·162**	0·341	0·710	**0·341**	0·682
Blast Furnace Gas	113 ,, ,, ,, ,,	57	,, = **0·0655**	**0·136**	0·278	**0·130**	0·270	0·568	**0·278**	0·540
Benzine . .	19,800 B.T.U. per lb.	74	,, = **0·0825**	0·176	—	—	—	—	—	—
Petroleum .	18,000 ,, ,, ,,	54	,, = **0·0625**	0·128	—	—	—	—	—	—
Alcohol . .	10,250 ,, ,, ,,	54	,, = **0·0625**	0·128	—	—	—	—	—	—

The above values give the so-called *Rated HP* with 15 per cent. reserve. In exceptional cases, as when the engine is worked with rich gas, or has a high mechanical efficiency, insufficient cooling of the parts, etc., the *maximum* HP may be 25 *per cent.* higher.

Example 1 (Table III.).—*Blast Furnace Gas Engine.*
An R D 4, two-cylinder gas engine (double-acting, four-cycle). Diameter of cylinders D, 24 inches, stroke S = 28 inches, n = 120 revolutions per minute; taking the constant C = 0·27 from Table III. for blast furnace gas, we get—

$$\text{Rated HP} = 0{\cdot}27 \times 2^2 \times \frac{28}{12} \times 120 = 300 \text{ HP}\,;\ \text{Maximum HP } 1{\cdot}25 \times 300 = 375 \text{ HP.}$$

Example 2 (Table III.).—*Suction Gas Engine.*
An engine of the Z 4, type (two-cylinder, single-acting, four-cycle). Diameter of cylinder D = 18 inches = 1·5 foot, stroke S = 24 inches = 2 feet. It is required to develop 100 BHP, at what speed must the engine run? Taking from Table III. the value of the constant C for suction gas (for this type of engine), C = 0·156, we get—

$$n = \frac{100}{0{\cdot}156 \times 1{\cdot}5^2 \times 2} = 143 \text{ revolutions per minute.}$$

For lighting or coal gas, Table III. gives C = 0·196, therefore—

$$n = \frac{100}{0{\cdot}196 \times 1{\cdot}5^2 \times 2} = 114 \text{ revolutions per minute.}$$

The *thick figures* of values of C in Table III. can always be used for one motor cylinder, and the work of the engine calculated from it, and multiplied by the *number* of cylinders.

CHAPTER III.

Main Dimensions of the Engine.

To lengthen the stroke of the piston gives a better division of the pressure on the connecting rod, and therefore greater uniformity in running the engine; but theoretical determinations of the *ratio S : D* (stroke to diameter of cylinder), have led to no practical results, and it will be advisable to adhere to such dimensions as practical experience has shown to be best. In the diagram Fig. 1, this ratio is plotted for a number of different types of engine, and varies from 1·05 to 2.

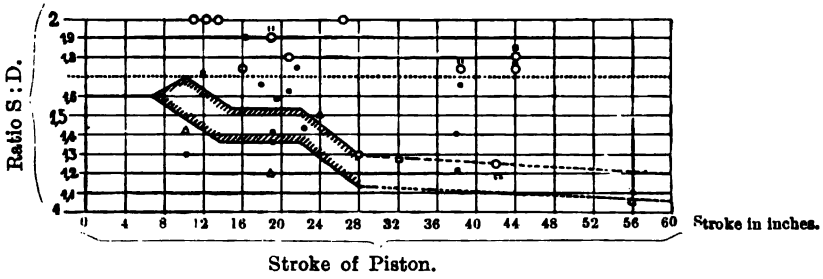


FIG. 1.—Ratio S : D for Horizontal Engines.

Engines made by the same firm are denoted by the same symbol in the diagram, as, for instance—

Deutz, ○. Nuremberg, □. Swiderski, △. Körting (two-cycle), Ö.

The same ratio has not been adopted in the different types, as will be seen, nevertheless S : D = 1·6 for small engines (with 5-inch stroke), decreasing to 1·2 for larger engines (up to 28-inch stroke) appears to be the best, and we will take this as the *normal* ratio.

(The ratio S : D of the standard types of Haeder steam engines is, for the sake of comparison, indicated in Fig. 1, by the dotted line .....)

The shaded area /////// represents the ratio S : D for types of engines the dimensions of which are given in the table overleaf.

For gas engines, as for the standard types of Haeder steam engines, the following rule holds good :—

To build as many different types of engines, of uniformly increasing powers, with as few patterns as possible.

On referring to Table I., it will be noticed that two different diameters of cylinders are given for engines above 3 BHP. ; the larger cylinder gives a more powerful engine, with the same stroke. The alternative dimensions may be used in order to save the expense of patterns, etc.

Table I.—Types of Engines. Four-cycle up to 28-inch Stroke and 115 BHP per Cylinder.

Stroke (or travel) of piston S =	5″	6½″	7½″	8½″	10½″		12″		13¾″		16″		17¾″		20″		21½″		24″		28″		ins.
Diameter of cylinder D =	3″	4″	5″	5¾″	6″	7½″	8″	8½″	9″	10¼″	10½″	12″	12¼″	13″	13¼″	14″	14½″	16″	17″	18¾″	21¼″	24	ins.
Ratio S : D =	1·67	1·62	1·5	1·48	1·75	1·4	1·5	1·4	1·53	1·34	1·52	1·33	1·45	1·37	1·51	1·43	1·48	1·34	1·41	1·28	1·32	1·17	
Number of revolutions n =	240	240	235	230	230		225		220		210		195		185		175		165		150		per min.
Speed of piston V =	200	260	294	326	402		450		505		560		578		618		626		660		700		ft. per min.
Area of cyl. A =	7	12½	19½	26	28¼	44	50¼	56¾	63¾	82½	86½	113	118	132¾	138	154	165	201	227	276	354¾	452½	sq. ins.
Rated HP * =	0·5	1	2	3	4	6	8	9	11½	15	17	22½	24	28	30	34	37	45	53	65	88	113	BHP.
Maximum HP (at 25%) . . .	0·625	1·25	2·5	3·75	5	7·5	10	11·25	14·3	18·7	21	28	30	35	37·5	42·5	46	56	66	81	110	140	HP.
Flywheel diameter	2′ 7½″	3′ 3⅜″	3′ 9¼″	4′ 3¼″	5′ 3″		5′ 10⅞″		6′ 6¾″		7′ 2¾″		7′ 10½″		8′ 6⅜″		9′ 0¾″		9′ 6¼′		10′ 10″		ft. & ins.
„ breadth	3½″	3¾″	4″	4½″	4¾″		5⅜″		6″		6¾″		7½″		8″		9⅜″		13¼″		17⅛″		ins.
Wgt. of flywheel—																							
For power † . .	485	585	870	1147	1047	1543	1675	2050	2205	2820	2780	3920	4640	5250	5740	6620	7600	9050	11550	14500	21500	23800	lbs.
For electric light ‡	773	939	1390	1830	1675	2470	2690	3310	3530	4410	4410	6290	7390	8390	9150	10600	12100	14300	18600	23200	34500	38400	lbs.

Number of Revolutions.—For the number of revolutions here taken as a standard, see Fig. 11, p. 130. The powers developed vary in direct ratio to the number of revolutions.

* The *Rated HP* are calculated for single-acting four-cycle engines, worked with *lighting gas*, $p_m = 82{\cdot}5$ lbs. per sq. inch; $\epsilon = 0{\cdot}85$; $\eta = 0{\cdot}74$. Compare pp. 18, *et seq.*; with *suction gas* (see p. 22) 25 per cent. less power is developed.

† Mean values, calculated for a variation of $\delta_0 = \frac{1}{50} = 0{\cdot}02$
‡ „ „ „ „ „ $\delta_0 = \frac{1}{80} = 0{\cdot}0125$
} For further details, see chapter on "Flywheels."

These dimensions form the basis of the standard types given in Chapter VII., "Parts of the Engine."

CHAPTER IV.

Effect of the Inertia of the Reciprocating Parts.

BEFORE we proceed to consider the diagram of tangential pressures for calculating the weight of the flywheel and the construction of the various parts of the engine, it is necessary to take into account the inertia of the moving parts. Although here as elsewhere it is assumed that the student has a knowledge of the effect of the inertia of the moving parts in a steam engine, we will briefly recapitulate the fundamental rules, and reduce the equations embodying them to their simplest expression. Let—

A be the sectional area of the working cylinder in sq. inches.

W = „ weight of the reciprocating parts (piston with piston pin, connecting rod, etc.) in lbs. according to Tables II., III.

D = „ diameter of the cylinder in inches.

S = „ stroke in feet.

r = „ radius (or throw) of the crank (*i.e.* half the stroke) in feet.

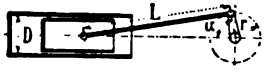


FIG. 2.

n = „ revolutions per minute.

ϕ = „ acceleration of the reciprocating parts in feet per second.^{2}

(ϕ) = „ negative acceleration or retardation of the reciprocating parts in feet per sec.^{2}

P = „ pressure in lbs. required to impart to the reciprocating parts having weight W, the exact degree of acceleration determined by a given speed and length of stroke.

v = „ $\frac{r\pi n}{30}$ circumferential velocity in circle described by crank in feet per second.

L = „ length of connecting rod in feet.

α = „ 0° to 180° angle of inclination of the crank in relation to the dead points, see Figs. 5–6.

Thus for any given angle α of the crank:

During the outstroke of the piston towards the crank shaft.

Outstroke.—Acceleration $\phi = \frac{v^2}{r}\left(\cos\alpha + \frac{r}{L}\cos 2\alpha\right)$ (1)

During the return stroke towards the back end of the cylinder.

Instroke.—Acceleration $\phi = \frac{v^2}{r}\left(\cos\alpha - \frac{r}{L}\cos 2\alpha\right)$ (2)

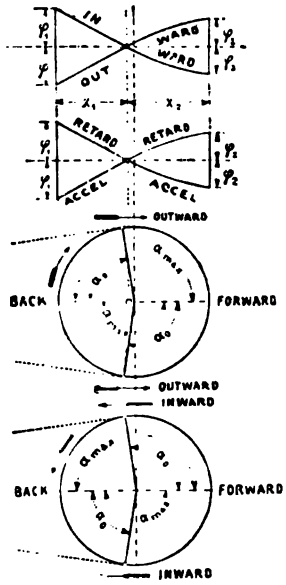
From these equations for the positions of the cranks at the dead points we get—

Outstroke.—Back dead point. Front dead point.

Acceleration $\phi_1 = \frac{v^2}{r}\left(1 + \frac{r}{L}\right)$; $(\phi_2) = -\frac{v^2}{r}\left(1 - \frac{r}{L}\right)$ (3)

Instroke.—Front dead point. Back dead point.

Acceleration $\phi_2 = \frac{v^2}{r}\left(1 - \frac{r}{L}\right)$; $(\phi_1) = -\frac{v^2}{r}\left(1 + \frac{r}{L}\right)$ (4)*



FIGS. 3–6.

The degrees of acceleration can be plotted as ordinates along a straight line (Figs. 3 and 4), the + (plus) values (acceleration) below, and the − (minus) values (retardation) above.

* Acceleration with a minus sign − (equations 3 and 4) denotes the same as retardation.

The direction in which the crank rotates has no effect on the values of ϕ (Figs. 3 to 6). If we take the usual ratio $\frac{r}{L} = \frac{1}{5{\cdot}5}$ to $\frac{1}{6}$, or a mean value of $\frac{1}{5{\cdot}8}$, we obtain for the crank positions at the dead points—

Outstroke.—Back dead point.

$$\text{Acceleration } \phi_1 = \frac{v^2}{r} \times \frac{6{\cdot}8}{5{\cdot}8} \sim \frac{n^2 \times r}{78}$$

Instroke.—Front dead point.

$$\text{Acceleration } \phi_2 = \frac{v^2}{r} \times \frac{4{\cdot}8}{5{\cdot}8} \sim \frac{n^2 \times r}{110}$$

Front dead point.

$$\text{Retardation } (\phi_2) = \frac{v^2}{r} \times \frac{4{\cdot}8}{5{\cdot}8} \sim \frac{n^2 \times r}{110} \qquad (5)$$

Back dead point.

$$\text{Retardation } (\phi_1) = \frac{v^2}{r} \times \frac{6{\cdot}8}{5{\cdot}8} \sim \frac{n^2 \times r}{78} \qquad (6)$$

and also—

Outstroke.—Back dead point.

$$\text{Pressure due to acceleration, } P_1 = \phi_1 \times \frac{W}{g}$$

Instroke.—Front dead point.

$$\text{Pressure due to acceleration, } P_2 = \phi_2 \times \frac{W}{g}$$

Front dead point.

$$\text{Pressure due to retardation, } P_2 = (\phi_2) \times \frac{W}{g} \qquad (7)$$

Back dead point.

$$\text{Pressure due to retardation, } P_1 = (\phi_1) \times \frac{W}{g} \qquad (8)$$

This pressure, due to acceleration or retardation, may be considered as distributed over the whole area of the piston, then the pressure, or inertia, per unit of area will be—

At the back dead point $q_1 = P_1 \div A$; and $(q_1) = (P_1) \div A$ respectively (9)

" front " " $q_2 = P_2 \div A$; and $(q_2) = (P_2) \div A$ " (10)

The inertia diagram (per unit of piston area) can be plotted on ordinates along a straight line, in the same way as the acceleration itself in Figs. 3 and 4. See Fig. 7. Here the axis of the abscissa is cut at the point k, where the inertia is = zero.

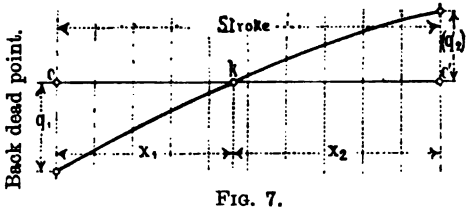


FIG. 7.

Fig. 7 shows the diagram of inertia of the moving parts (per sq. inch of piston area) for one stroke S of the piston, namely, the outward stroke towards the crank shaft. The distance x_1 of point k, from the dead point is dependent on the ratio $\frac{r}{L} = \frac{\text{radius of crank}}{\text{length of connecting rod}}$.

The angle α_0 of the crank (in which ϕ = zero) corresponds to the distance x_1 (Fig. 7) traversed by the piston, and according to Radinger * is determined thus—

$$\cos \alpha_0 = \frac{-1 + \sqrt{1 + 8\left(\frac{r}{L}\right)}}{4\frac{r}{L}} \qquad (11)$$

* Radinger, "Maschinen mit hoher Kolbengeschwindigheit" ("Engines with High-Piston Speeds").

The travel of the piston x (Fig. 8) corresponding to any given angle of the crank α is calculated thus—

$$x = r\,(1 - \cos\alpha) + \mathrm{L}\,(1 - \cos\beta) \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (12)$$

The angle of the connecting rod β can be determined by the equation—

$$\sin\beta = \frac{r\,.\,\sin\alpha}{\mathrm{L}} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (13)$$

There is no necessity to work out equations 12 and 13, because q_1 and q_2 can be determined from equations 9 and 10, and x_1 from Table I. Given these three points, the general shape of the curve can be determined. The values in Table I. are calculated from equations 11 to 13.

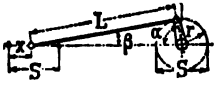


FIG. 8.

Table I.—Values of α_0, x_1, and x_2 (Figs. 5 and 6).

Ratio	$r : \mathrm{L} =$	0	$\frac{1}{7}$	$\frac{1}{6}$	$\frac{1}{5\cdot5}$	$\frac{1}{5}$	$\frac{1}{4}$
Angle	$\alpha_0 =$	90°	82°	81°	**80°**	79°	77°
Travel of piston . .	$x_1 = r$		$0\cdot94\,.\,r$	$0\cdot93\,.\,r$	$\mathbf{0\cdot92}\,.\,r$	$0\cdot91\,.\,r$	$0\cdot9\,.\,r$
" " . .	$x_2 = r$		$1\cdot06\,.\,r$	$1\cdot07\,.\,r$	$\mathbf{1\cdot08}\,.\,r$	$1\cdot09\,.\,r$	$1\cdot1\,.\,r$

Fig. 9.

The inertia diagram, or diagram of acceleration (Fig. 7), also shows that where the piston begins to move out during its travel along the distance x_1, part of the available effort is absorbed in starting or accelerating the motion of the reciprocating parts. Therefore the ordinates for this part of the diagram lie below the line of no pressure (= zero). During the travel of the piston along x_2 (Fig. 7) there is retardation in the motion of the moving parts, and therefore, instead of absorbing, they release energy. Therefore the ordinates of the diagram (Fig. 9) for this part of the stroke lie above the atmospheric line.

(It should be noted here that in single-cylinder four-cycle engines, the necessary acceleration can only be imparted to the moving parts of the piston during the third stroke (explosion and expansion). During the first stroke (admission), the second (compression), and the fourth (exhaust), this energy must be communicated by the flywheel, but it has no effect on the graphic representation of the pressure curves of the reciprocating parts.)

The effect of the reciprocating parts of the engine on the pressure diagram of the piston will be most clearly seen by developing the different parts of the diagram, or by drawing a *continuous* diagram. Fig. 9 shows, for instance, the sequence of the in- and out-strokes for one revolution. For a four-cycle engine, however, the diagram must be drawn for four strokes, as at Fig. 13.

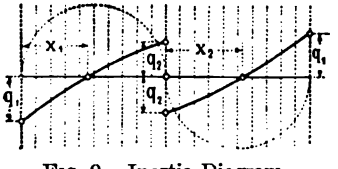


FIG. 9.—Inertia Diagram.

When designing an engine, the weights of the reciprocating parts are not generally known. Table II. gives these weights approximately for the piston and connecting rod of the standard types of engine (see Table I., p. 24).

Table II.—Weights of Reciprocating Parts for Single-acting Four-cycle Engines.*

Stroke S . . . . =	180 (5⅛″)	160 (6¼″)	190 (7½″)	220 (8¾″)	270 (10⅝″)	310 (12¼″)	350 (13¾″)	400 (15¾″)	450 (17¾″)	500 (19¾″)	550 (22″)	600 (23¾″)	700 mm. (27½″)
Diam. of cylinder D =	80 (3¼″)	100 (4″)	125 (5″)	145 (5¾″)	185 (7½″)	220 (8¾″)	260 (10½″)	300 (12″)	330 (13″)	360 (14½″)	400 (16″)	480 (19″)	600 mm. (24″)
Piston including bolts =	12	20	29	37½	71	120	190	272	345	445	550	830	1320 lbs.
Connecting rod . . =	10	15½	26½	35½	78	133	210	300	395	485	620	930	1500 ,,
Total weight W . . =	22	35½	55½	73	149	253	400	572	740	930	1170	1760	2820 ,,
Weight per sq. in. of cyl. area = W ÷ A . .	2·7	2·8	2·8	2·8	3·4	4·2	4·6	5	5·6	5·65	5·8	6·2	6·25 lbs. per sq. in.
Weight per HP = W ÷ HP	37·5	28·6	27·8	21	21	22	22	22	22	22	22	22	22 lbs.

We can also write—

Table III.—Approximate values for large gas engines (stroke more than 32-inch). W and HP apply to one cylinder only.

Single-acting four-cycle	$W = 6{\cdot}43 \times A$ or	$W = 22{\cdot}3 \times HP$
Double- ,, ,, ,,	$W = 8{\cdot}55 \times A$,,	$W = 15{\cdot}6 \times HP$
,, ,, two- ,,	$W = 8{\cdot}55 \times A$,,	$W = 7{\cdot}8 \times HP$
Large steam engine	$W = 4{\cdot}98 \times A$,,	$W = 5{\cdot}1 \times HP$

Engines for Motor Cars run at very high speeds, and therefore the moving parts must be kept extremely light, otherwise the forces due to inertia would be too great.

Table IV.—Approximate Values for Motor-car Engines.

Power developed per cylinder	3	4	4	5	6	8 HP
D × S =	3⅝″ × 4″		4″ × 4⅜″		4⅜″ × 4¾″	
n =	1000	1410	900	1200	800	1100 revs. per min.
Permissible values of W =	7·7 lbs.		11·2 lbs.		15·9 lbs.	
W ÷ A =	0·75 lbs.		0·89 lbs.		1·06 lbs. per sq. in.	
q_1 =	51	102	55·5	102	55·5	102 lbs. per sq. in.
q_2 =	34	68	37	68·3	37	68 ,, ,,

Example.—Assuming a large single-acting four-cycle engine to have a cylinder diameter of 40 inches, what should be the weight of the reciprocating parts?

According to Table III., for a cylinder diameter of 40 inches (or 1260 sq. inches area)—

$$W \div A = 6{\cdot}43 \text{ lbs. per sq. inch.; therefore } W = \frac{\pi}{4} \times 40^2 \times 6{\cdot}43 = 8100 \text{ lbs.}$$

Example.—The reciprocating parts of a single-acting four-cycle gas engine, having a cylinder diameter of 13 inches, and 18 inches stroke, weigh according to Table II., W = 740 lbs. Taking the speed at n = 180 revolutions per minute, what are the pressures exerted by the moving parts?

We have—

Outstroke.

From equation 5—

$$\text{Acceleration } \phi_1 = \frac{180^2 \times 0{\cdot}75}{78} = 312 \text{ ft. per sec.}$$

From equation 7—

$$\text{Inertia } P_1 = 312 \times \frac{740}{32} = 7200 \text{ lbs.}$$

Instroke.

From equation 6—

$$\phi_2 = \frac{180^2 \times 0{\cdot}75}{110} = 220 \text{ ft. per second.}$$

From equation 8—

$$\text{Inertia } P_2 = 220 \times \frac{740}{32} = 5100 \text{ lbs.}$$

or reduced to the area of the piston: (area of 13 inches = 133 sq. inches).

* If the engine be properly constructed, and good material used, the weights of the reciprocating parts may be somewhat less, say, 0·8 of the values given in this table. Special attention should be given to reducing the weight of the piston.

According to equation 9—

At the dead point back end of cylinder, $q_1 = \frac{7200}{133} = 54$ lbs. per sq. inch.

According to equation 10—

At the dead point, front end of cylinder, $q_2 = \frac{5100}{133} = 38{\cdot}4$ lbs. per sq. inch.

The values are plotted to scale in Fig. 13, 1 inch = 180 lbs.

In order that the beginner may be able to detect errors in calculating q_1 and q_2, the values for the three sizes of engines (viz. ½ HP, 30 HP, and 500 HP) are calculated in Table V. The first half of each column gives the *normal speed* for these powers, the second shows how q increases with the number of revolutions. (The spaces left in Table V. are for the insertion of further examples.) The change of load on the rods takes place exactly at the dead point, if q = C (Fig. 10), because the pressure of explosion p is only developed at the distance k from the dead point.

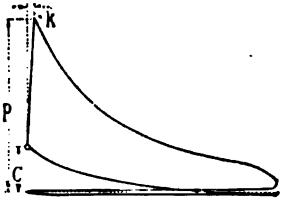
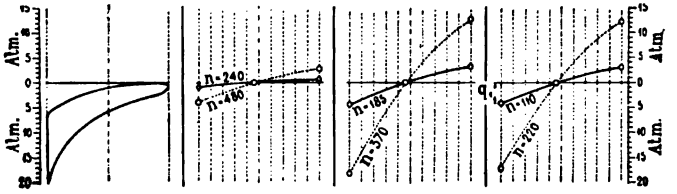


FIG. 10.

The diagrams at the bottom of Table V. show the difference in the pressures between the standard diagram and the acceleration q.

Table V.—Acceleration for Engines of Different Powers (Single-acting, Four-cycle).

Stroke =	5¼″		20″		47¼″		
Diameter D =	3¼″		14½″		43½″		
W =	22		930		9500		lbs.
n =	240	480	185	370	110	220	revs. per min.
ϕ_1 =	158	630	362	1445	306	1230	ft. per sec.²
ϕ_2 =	112	447	256	1022	217	870	ft. per sec.²
P_1 =	108	431	10,420	41,800	90,300	363,000	lbs.
P_2 =	76·6	306	7,400	29,500	64,300	256,000	lbs.
q_1 =	18·1	52·2	63	254	55	244	lbs. per sq. in.
q_2 =	9·3	37·2	44·9	179	43	172	,, ,,



This table shows that engines with 19 inches and 47 inches stroke cannot be run at double the usual speed, the rule being that—

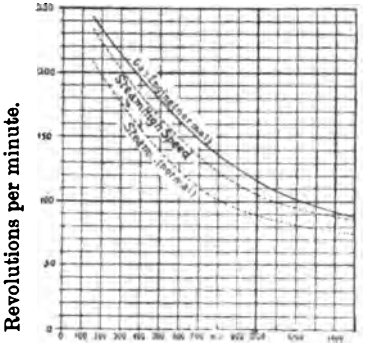


FIG. 11.

Radial acceleration $q < C$ . . . . . (14)

(otherwise there would be two successive changes of pressure in the rods).

The inertia increases as the *square of the number of revolutions*, in other words, q increases with n^2. It will be well, therefore, to compare the speeds of gas and of steam engines.

These are plotted in Fig. 11. As gas engines require much larger cylinder dimensions than steam engines, to develop the same power, it was necessary from the first *to run them at much higher speeds*. On the other hand, the speed of steam engines has of late years considerably increased.

A Combination of the Inertia and Indicator Diagrams,

gives the energy diagram, which we shall use to plot the diagram of tangential pressures (Chapter V.). The method of combining these two diagrams is as follows :—

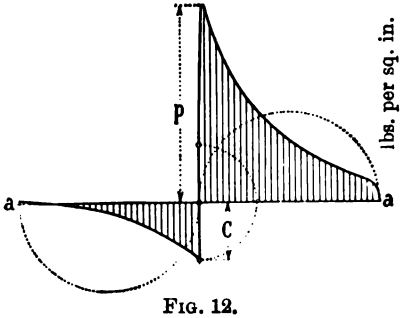


FIG. 12.

We will suppose the area of work shown in the diagram is obtained without compression, and therefore without counterpressure on the back end of the piston, and is shown to the right in Fig. 12, down to the atmospheric line *aa*; the area of work lost (absorbed) by compression is plotted below this line, to the left; the difference of the two areas gives the actual diagram of work. In the same way admission (drawing in the charge, and exhaust, both at a pressure of $\frac{1}{10}$ inch of an atmosphere (= say, $1\frac{1}{2}$ lbs. per sq. inch) are plotted below this line (Fig. 14).

A numerical example will best explain the further development of the diagram.

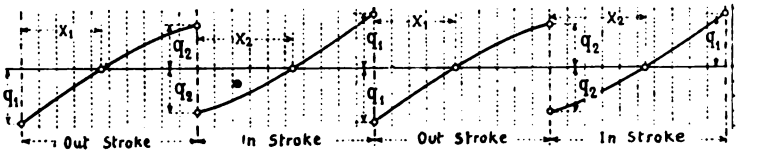
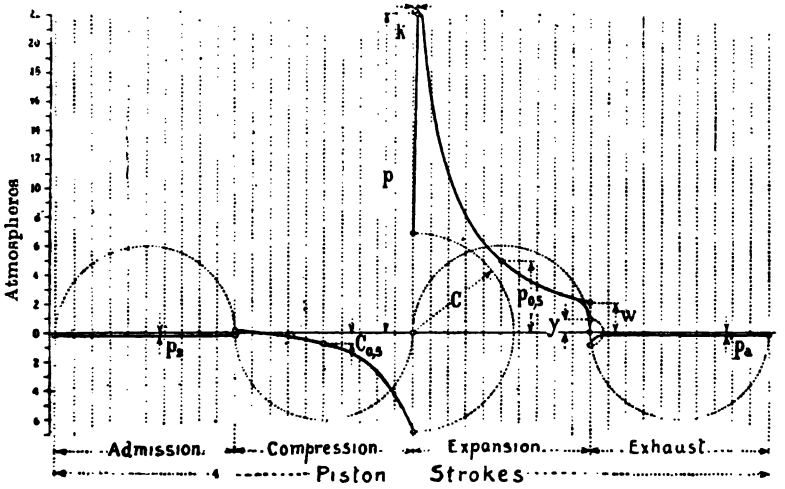


FIG. 13.—Continuous Inertia Diagram.

Example.—We will take $p = 22$ atmospheres (323 lbs. per sq. inch) as the pressure of explosion, and for the inertia the figures given in the example on the previous page, in which $q_1 = 52$ lbs. per sq. inch, $q_2 = 37$ lbs. per sq. inch. Scale = 1 inch = 190 lbs.

The diagram of acceleration is then drawn, and developed for one cycle of work, or four strokes (Fig. 13), and the degrees of acceleration, or inertia, plotted as ordinates,



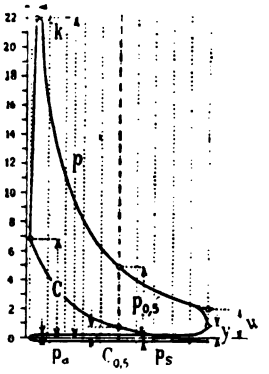


FIG. 15.

FIGS. 14, 15.—Illustration of the Continuous Diagram.

as in Fig. 9. The developed diagram of work is then plotted. Figs. 14, 15 show how this is done, and require no further explanation.

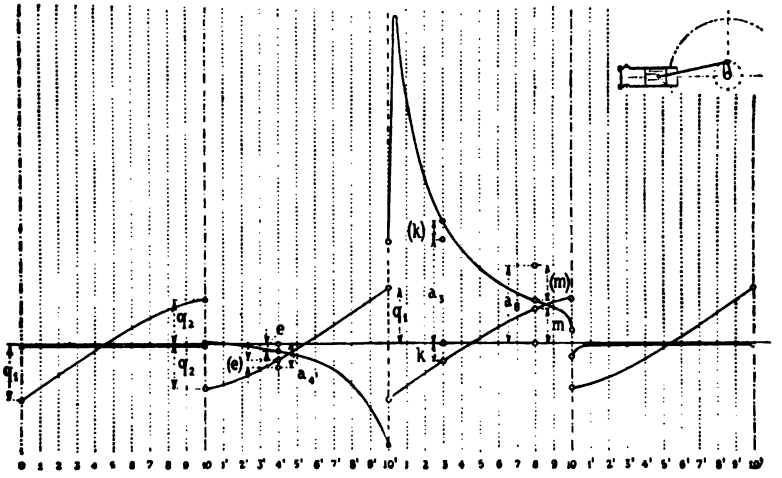


FIG. 16.—Inertia and Indicator Diagram Combined.

The two diagrams, Figs. 13 and 14, must then be combined, and drawn one over the other, as shown at Fig. 16, in order to plot the resulting diagram of pressures.

The pressures resulting from this combination have been determined in Fig. 16, for several positions of the piston. In the second stroke, for instance, (e) is drawn $= e$, and a_4' is the resulting pressure. Again, in the third stroke, (k) is plotted $= k$, and $(m) = m$, therefore the resulting pressures $= a_3$ and a_8 respectively.

The pressures for each position of the piston are determined in the same way; the diagram thus produced is shown at Fig. 17. The beginner should refer back from Fig. 17 to Fig. 16, in order to understand both diagrams clearly.

The pressure diagram thus evolved yields in the first place the corresponding pressure on the crank pin for every position of the piston, expressed in lbs. per sq. inch of piston

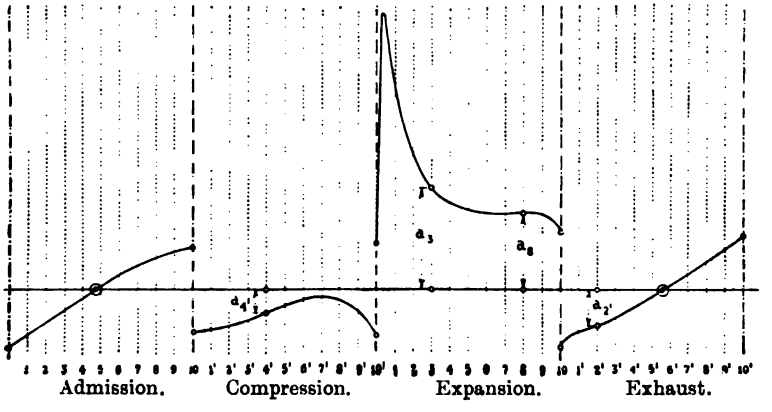


FIG. 17.—Diagram corrected for Inertia.

area. It also shows where the changes of pressure in the rods during the various strokes occur, namely, at the points marked O, where the energy curve cuts the neutral lines. Thus, according to Fig. 17, we have in this example two changes of pressure during one cycle of work, viz.—

During the *first* stroke (admission, or drawing in the charge); and
During the *fourth* stroke (exhaust).

In the second stroke (compression) and the third (explosion and expansion) there are no similar changes in pressure (*i.e.* the pressure is either wholly above, or wholly below, the neutral line).

The inertia diagram, as well as the resultant pressure diagrams, or diagram corrected for inertia, can be plotted for other engines in the same way; the method of drawing them should be clear, without further explanation, to the student.

The developed or continuous diagrams may be classified under four heads, as follows:—

The *Inertia* diagram shows the effect of the reciprocating parts (Fig. 13).
The diagram of *work* is plotted from the indicator diagram (Fig. 14).
The *pressure* diagram is a combination of the diagrams of work and inertia (Fig. 17).
} In relation to the stroke.

The diagram of *tangential* pressure, commonly called the turning moment diagram, or the pressure diagram reduced to the circle described by the crank forms the subject of the next chapter.

CHAPTER V.

Tangential Pressure or Turning Moment Diagram for Calculating the Flywheel.

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Double Vis-à-Vis Four-cycle Engine, **Z G Z 4**, 4 cyl.	45–48
Twin Double-acting Four-cycle Engine, **Z D 4**, 2 cyl.	48–49
Tandem Double-acting Four-cycle Engine, **R D 4**, 2 cyl.	49–50

A shortened method for determining the weight of the Flywheel, with the necessary Tables, will be found in the chapter on "Flywheels." But it is desirable that the beginner should study the following chapter thoroughly, and work out a few examples for himself.

CHAPTER V.

Tangential Pressure or Turning Moment Diagram for Calculating the Flywheel.

THE energy of the engine fluctuates periodically, the work of the flywheel consists in acting as an accumulator or reservoir of energy, storing up the excess taken from and restoring the deficiency of energy to the engine, so as to secure as far as possible a uniform, angular velocity of the crank shaft. As already explained, there is, in single-cylinder four-cycle engines, only one working or motor stroke per two revolutions, or four strokes, and in these engines the flywheel must carry the engine forward during the three remaining strokes by the kinetic energy stored up in its rim. During the working stroke (Stroke III.) the motion is accelerated, and the flywheel absorbs energy; during the other strokes it is retarded, and energy is liberated.

If the engine had no flywheel it would come to a stop at the dead point, since the whole pressure of the piston is transferred to the head of the crank pin, and cannot act upon and drive round the crank.

To *calculate the revolving masses*, or determine approximate values for the necessary equations, the graphic method and the turning moment, or tangential pressure diagram, will be found of most use. With the help of this diagram, the complicated conditions arising during a cycle of work can easily be represented.

The resistance acts *tangentially* to the circumference of the flywheel, and also, if reduced, to the circle described by the crank. The force of the piston is therefore divided into two components—a *tangential* and a *radial*. The tangential component, at the points where it cannot overcome the whole resistance, and where it acts counter to it, is aided and sustained by the kinetic energy stored up in the flywheel.

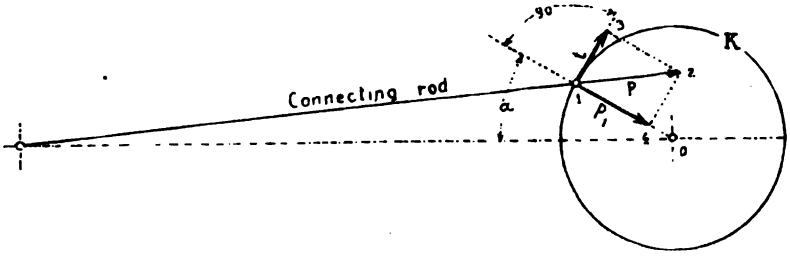


FIG. 1.—Determination of the Tangential and Radial Pressure.

Determination of the Tangential Pressure and of the Radial Component.

$$\text{Let } P = \frac{\text{thrust on the connecting rod in lbs.}}{\text{area of the piston in sq. inches}} \text{ in lbs. per sq. inch} \quad . \quad . \quad . \quad . \quad (1)$$

= the load on the connecting rod reduced to 1 sq. inch of piston for any given position of the crank.

a = the angle formed by the crank with the dead point, in any given position.

Draw the crank circle K (of any given diameter, Fig. 1).

Produce the centre line of the connecting rod beyond point 1, where it intersects the crank circle.

Mark off the load P on the connecting rod along this line, beyond point 1, to scale (for instance, 1 inch = 400 lbs., or 2 inches = 400 lbs.).

Draw a line 1–3 at right angles to the crank position 0–1.

Draw the parallelogram of forces 1–4–2–3. Then the line—

1–3 gives the tangential pressure
1–4 „ „ radial pressure } to the same scale as P.

Example.—Determine the tangential pressure when the crank is at an angle $\alpha = 30°$, P = 100 lbs. per sq. inch.

Taking 180 lbs. to the inch as the scale; we carry out (from Fig. 1) P = 0·55 inch. Draw the parallelogram of forces, and measure from the diagram t = 0·33 inch. Calculating this to the above scale, the tangential pressure works out at t = 0·33 inch = 60 lbs. per sq. inch. (This example is shown to scale in Fig. 1.)

(The *radial pressure* P_1, also measured from Fig. 1, is = 0·44 inch = 79·5 lbs. per sq. inch; for the line P_1 is 0·44 inch long, and the scale is 1 inch = 180 lbs. per sq. inch. In the following chapters, however, we have only to deal with the tangential pressure (or turning moment); the radial pressure need not be taken into account, except when determining the stress upon the crank shaft (comp. p. 83).)

We will now calculate the flywheel for the different types of engines.

I. Single-acting Four-cycle Engine, E 4, One Cylinder (pp. 35–40).

A cursory representation of the working method in a four-cycle engine might be afforded by the continuous diagram, Fig. 2, in which the direction and amount of the

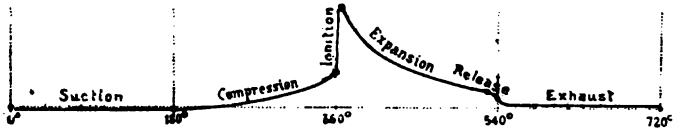


FIG. 2.—Continuous Diagram (not applicable).

pressures are plotted from the indicator diagram. But this simple method cannot be utilized, because a distinction must be made between—

Positive work, that is, the work done in the motor cylinder, and
Negative work „ „ „ given off by the flywheel.

To make this clearer, we will now proceed as follows :—

1. Divide the *indicator diagram* (Fig. 3) into ten or more equal parts.
2. Draw the continuous (developed) diagram of *work*, as at Figs. 14 and 15, p. 31.
3. Draw the *inertia* diagram, as at Fig. 13, p. 30.
4. Draw the last two diagrams one over the other, as at Fig. 3A, and combine them to form the *energy* diagram (Fig. 4).

In this *energy* diagram, from which the diagram of tangential pressure (or the turning moment diagram corrected for inertia) must be plotted, the *area above the atmospheric*

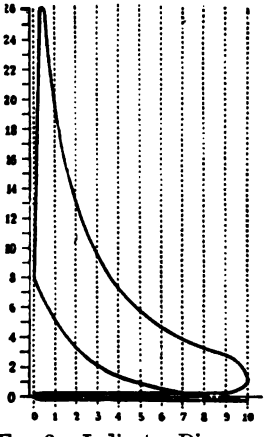


FIG. 3.—Indicator Diagram.

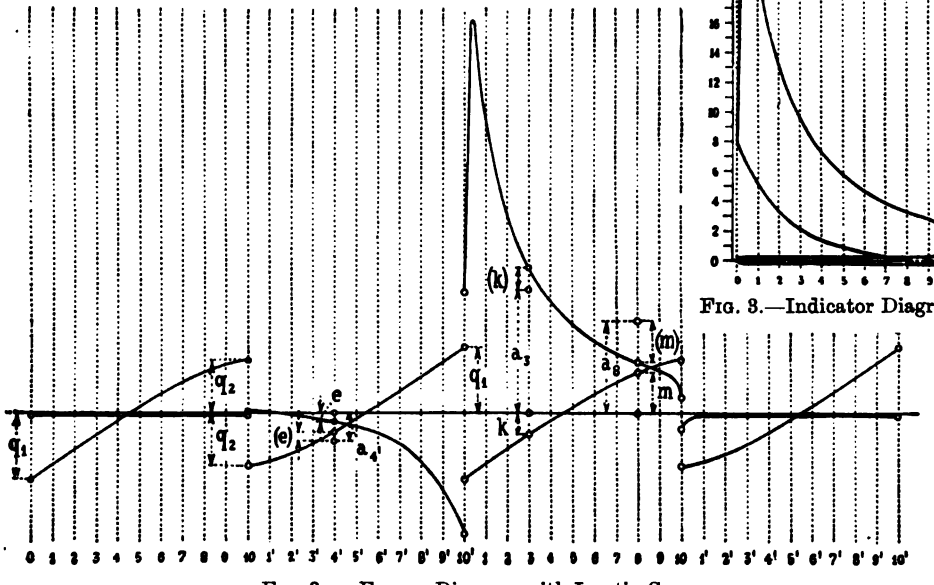


FIG. 3A.—Energy Diagram with Inertia Curves.

line represents the work done in the motor cylinder, and the *area below the atmospheric line* the work done by the flywheel.

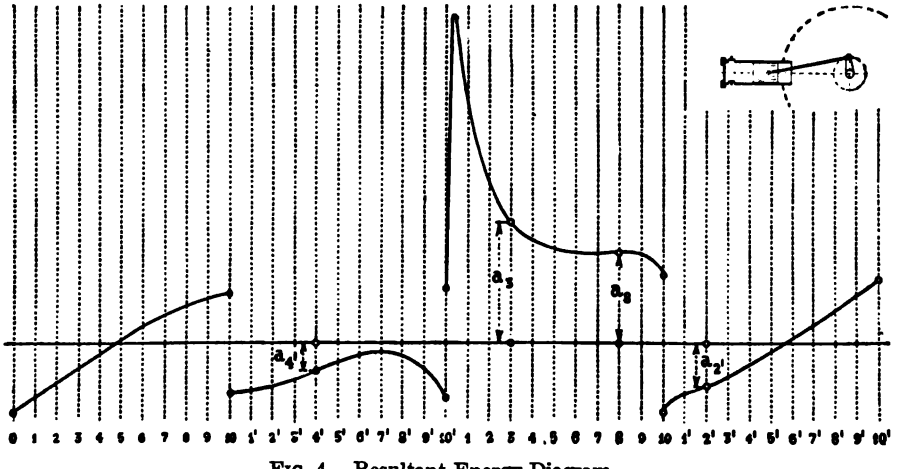


FIG. 4.—Resultant Energy Diagram.

If we designate the first as plus and the second as minus work, then in a single-acting four-cycle engine there will be plus work during one stroke (Stroke III.) and minus work during the other strokes (Strokes I., II., and IV.).

We will now plot the turning moment or tangential pressure diagram by means of a *numerical example*. We take, to start with, a diagram showing an explosive pressure of 27 atmospheres or, say roughly, 400 lbs. per sq. inch (Fig. 3). The inertia of the reciprocating parts we take approximately from p. 29, at $q_1 = 64$ lbs. per sq. inch $q_2 = 50$ lbs. per sq. inch. Scale of energy, 1 inch = 180 lbs. The lines of the diagram (Fig. 3A) are combined, according to Figs. 16, 17 (pp. 31, 32), to form the resultant energy diagram corrected for inertia (Fig. 4).

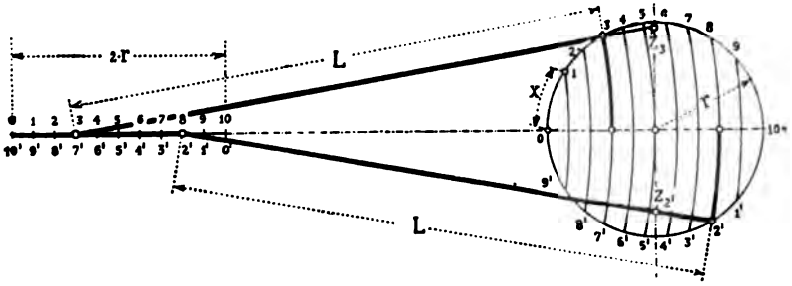


FIG. 5.—Movements of the Crank.

To determine the tangential pressure we will not use the method given at p. 34 for plotting the diagram, but an *auxiliary process* taken from Radinger.*

Draw out the path described by the crank (Fig. 5). It will be best to make the travel of the piston 0 to 10 equal to the length of a stroke in the diagram (Fig. 4). Divide the stroke into the same number of equal parts, as the diagram, and from L (length of the connecting rod) describe radii cutting the crank circle (Fig. 5). Mean of $L : r = 5{\cdot}8$.

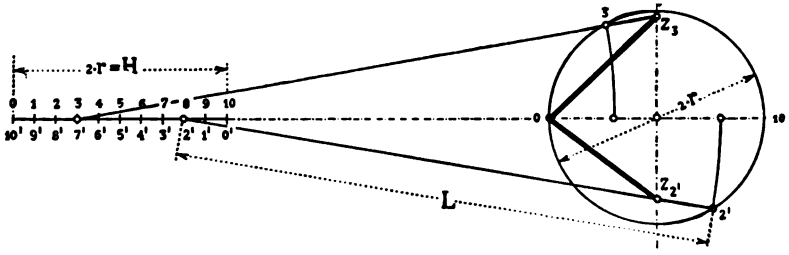


FIG. 6.—Determination of z_3 and $z_{2'}$.

Next, produce the line of direction of the connecting rod; for instance, for 0·3 travel of the piston (three-tenths of the stroke) (Fig. 6) to cut the vertical axis of the crank at z_3. This point is then connected to O (Fig. 6), the pressure on the piston corresponding

* Radinger, "Maschinen mit hoher Kolbengeschwindigkeit" ("Engines with High-Piston Speeds").

to this position of the crank, which, according to Fig. 4, is a_3; it is then laid off on a horizontal line from O to the right, and a vertical line is drawn at right angles to it, up to the connecting line z_3O (Fig. 7). *This vertical t_3 gives the turning moment at the third position of the piston (three-tenths of the stroke).*

In Fig. 7 the turning moment t_2' is also determined for position 2′ of the piston. For the pressure exerted by the piston a_2' at this point, see Fig. 4.

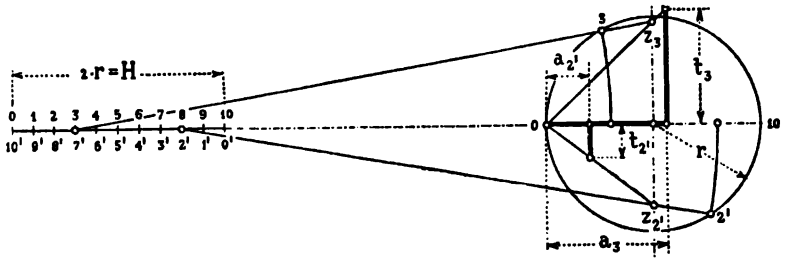


FIG. 7.—Turning Moment for Crank Positions 3 and 2′ (the Turning Moments here plotted are of the same value as those from Fig. 1).

The neutral line BB′ is marked off into horizontal divisions 1 to 10; *the sections measured on the circle described by the crank* (x in Fig. 5) are then plotted for the different crank positions during the out- and in-strokes of a working

FIG. 8.—Development of the Crank Circle. Plotting the Turning Moment t.

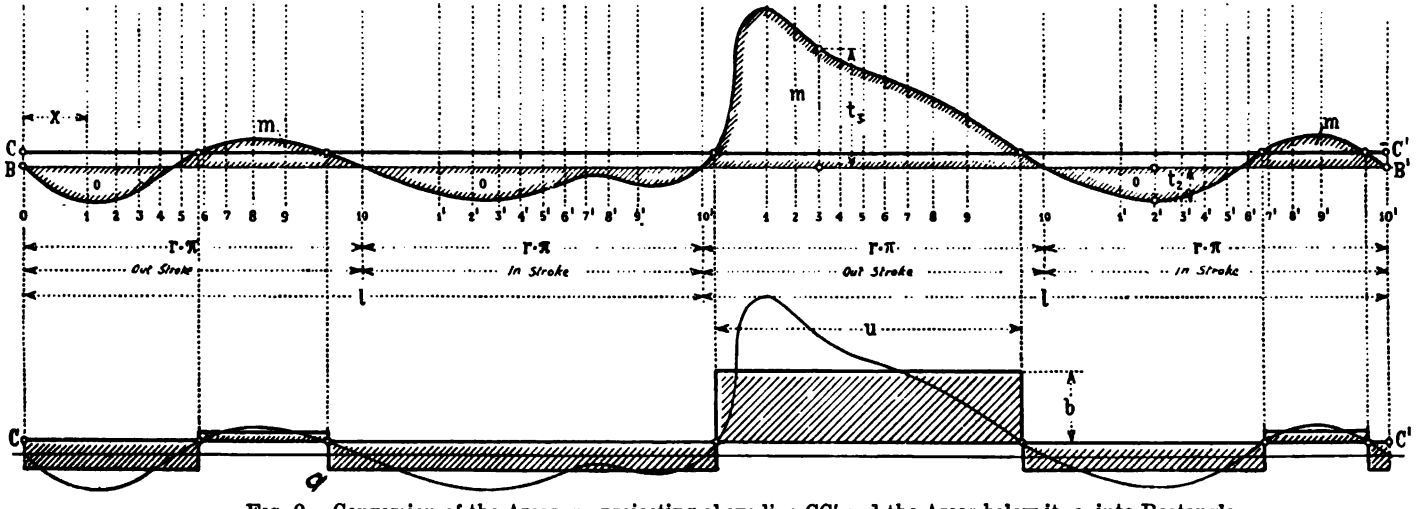


FIG. 9.— Conversion of the Areas, m, projecting above line CC′ and the Areas below it, o, into Rectangles.

cycle, and therefore for two revolutions. (As these sections are measured on the circumference, they vary in length.)

The tangential pressures, or turning moment, t, from Fig. 7, are plotted over the ordinates of the corresponding crank positions, and the points thus obtained are joined (Fig. 8).

The whole of the shaded area is converted into a rectangle having the same length (in which the areas m lying above the neutral line BB′ are taken as plus, and the areas o lying below it are taken as minus areas), and line CC′ is drawn parallel to BB′ at the distance BC thus found. The height of the rectangles BCC′B′ thus obtained represents the mean tangential effort on the crank-pin, and hence the difference between the plus and minus areas. The same scale must be used as that taken for the diagram, Figs. 3 and 4. The resultant line CC′ cuts the turning moment curve at several points. Thus the areas which lie above line CC′ denote the energy *absorbed by the flywheel*, and the areas below it the energy *given off by the flywheel*, showing plainly that the *larger* the *areas projecting above or below the line*, the greater must the *weight* of the flywheel be.

The areas above and below the line CC′ are then converted by the planimeter, the Simpson formula, or simply by measuring with the eye, into rectangles (Fig. 9), and their size determined in sq. inches. The largest must be taken as the basis of the following calculation. Let—

A = sectional area of motor cylinder in sq. inches.
g = gravity = 32 feet per second².
δ_0 = permissible degree of irregularity in speed.
R = radius of centre of gyration of rim section in feet.*
n = revolutions per minute.

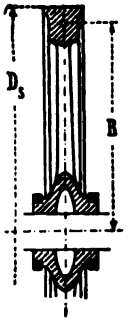
$$v = \frac{2 \times R \times \pi \times n}{60} = \frac{R \times \pi \times n}{30} = \text{circumferential velocity}$$

around path described by centre of gravity of rim section in feet per second . . . . . (2)

u = length of *greatest* area above the line of mean tangential effort, measured from the diagram, in feet.
b = height of this area in lbs. per sq. inch (therefore height in inches multiplied by the scale).
l = height of a double stroke, to and fro, measured from the diagram in feet.
S = stroke of the piston in feet.

$u \times \frac{S \times \pi}{l}$ = length of greatest area reduced to the crank circle in feet.

1·1 W = weight of rim section, plus effect of weight of arms at the centre of gravity of the rim.



Thus the value of the rotating parts will be †—

$$1{\cdot}1\,\frac{W}{g} = u \times \frac{S \times \pi}{6} \times b \times \frac{A}{v^2} \times \frac{1}{\delta_0} = \frac{u}{l} \times b \times \frac{A \times S\pi}{v^2} \times \frac{1}{\delta_0} \quad (3)$$

Hence weight of the rim section in lbs.—

$$W = 9{\cdot}2 \times \frac{u}{l} \times b \times \frac{A \times S}{v^2} \times \frac{1}{\delta_0} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (4)$$

---

* Radius of centre of gyration of rim section is about 0·45 × external diameter of flywheel D_s (see small drawing at side) . (5)

† Deduced from the original equation :—

$$\underbrace{u \times \frac{b}{l} \times S \times \pi \times A}_{\text{Excess of energy shown in diagram.}} = \underbrace{1{\cdot}1\,\frac{W}{2g} \times (v_{max}^2 - v_{min}^2)}_{\text{Energy absorbed by the flywheel.}} \quad . \quad . \quad . \quad (6)$$

If we write $\delta_0 = \frac{v_{max} - v_{min}}{v}$; $v = \frac{v_{max} + v_{min}}{2}$;

$$\delta_0 \times v = \frac{v^2_{max} - v^2_{min}}{2v}; \quad \delta_0 \times v^2 = \frac{v^2_{max} - v^2_{min}}{2};$$

then

$$u \times \frac{b}{l} \times S \times \pi \times A = 1{\cdot}1\,\frac{W}{g} \times \delta_0 \times v^2 \quad . \quad . \quad . \quad (7)$$

In the above equation v_{max} and v_{min} denote the greatest and least circumferential velocity in the path of the circle described by the centre of gyration of the rim section.

Since in four-cycle engines the work done during one cycle or two revolutions of the crank is always the same, the equations and formulas given above may be used for all standard types of engine, because the tangential pressures, and therefore the turning moment diagram corrected for inertia, are practically the same in all.

Example.—We take a single-acting four-cycle engine with—

Cylinder diameter = 12 inches.
Stroke = 16 inches or 1·33 feet.
Number of revolutions = 210 per minute.
Diameter of flywheel Ds = 7 feet 3 inches.

The engine is supposed to work with δ_0 degree of irregularity = 1/80. It is required to determine the weight of the flywheel.

For calculating the turning moment diagram corrected for inertia, we will use the diagram at p. 38. (Explosion pressure 27 atmospheres = say, 400 lbs. per sq. inch, p = 90 lbs. per sq. inch.)

Measured from Fig. 9, we get—

Length of greatest area above mean TM line u = 0·131 foot.
Height " " " b = 0·36 foot.
= 0·36 inch × 180 lbs. = 65 lbs. per sq. inch.

(since in the diagram 1 inch = 180 lbs.).

Circumference of circle described by the crank, measured from Fig. 9, l = 0·29 foot.

Then $$u \times \frac{b}{l} = 0{\cdot}131 \times \frac{65}{0{\cdot}29} = 29{\cdot}4$$

Next,

Area of motor cylinder $A = \frac{\pi}{4} \times 12^2 = 113$ sq. inches (according to equation 4A, p. 18).

From equation 5, p. 39, radius of centre of gravity of rim section, R = 0·45 × 7 feet 3 inches = 3·26 feet.

From equation 2, p. 39, $v = \frac{3{\cdot}26 \times \pi \times 210}{30} = 71{\cdot}6$, or say, 72 feet per second.

The weight of the rim, according to equation 4, will thus be—

$$W = 92 \times 29{\cdot}4 \times \frac{113 \times 1{\cdot}33}{72^2} \times \frac{1}{\delta_0} = 78{\cdot}5 \times \frac{1}{\delta_0}$$

For $\delta_0 = \frac{1}{80}$, W = 78·5 × 80 = 6280 lbs.

It should be noted that this degree of irregularity δ can only occur when the engine is running at maximum load (that is, with an ignition at every two revolutions). At less load the irregularity fluctuates, being greater or less, according to the method of governing. We shall return to this point later.

As regards the permissible amount of cyclic irregularity δ_0, see chapter on "Flywheels."

II. Double-acting Four-cycle Engine, D 4, One Cylinder.

To obtain accurate tangential pressure or turning moment diagrams in *double-acting, multi-cylinder four-cycle* engines, special attention must be paid to the sequence of ignitions.

The moment of ignition with reference to the stroke should also be noted, and whether the ignitions take place during the out- or in-stroke, because the inertia of the moving parts is not the same at the front and the back end of the cylinder.

In double-acting four-cycle engines, as already explained at p. 9, the ignitions are shifted 180° and 540° respectively, occurring after the first and third stroke, and the diagrams of pressure must be made to correspond with this arrangement.

A *continuous diagram* is first drawn, as in single-acting engines, and allowance being made for the inertia of the moving parts, the resultant pressure curves are plotted over it (comp. Fig. 10).

With the help of Figs. 5 to 7, the tangential pressure or turning moment is then determined for different positions of the crank, and carried out for the whole circle described by the crank (Fig. 11).

The turning moment curve is obtained by drawing a curve through the points thus found.

If the area enclosed by the neutral line BB′ and the turning moment curve be converted into a rectangle, it gives the mean pressure reduced to the crank circle, represented in Fig. 11 by the area BCC′B′. The line CC′ cuts the turning moment curve at several points. The areas above and below this line are converted in the same way into rectangles, and the largest is used as the basis of calculation.

The weight of the flywheel is calculated in the same way as in the single-acting engine (comp. p. 39), but the mean pressure is twice as great.

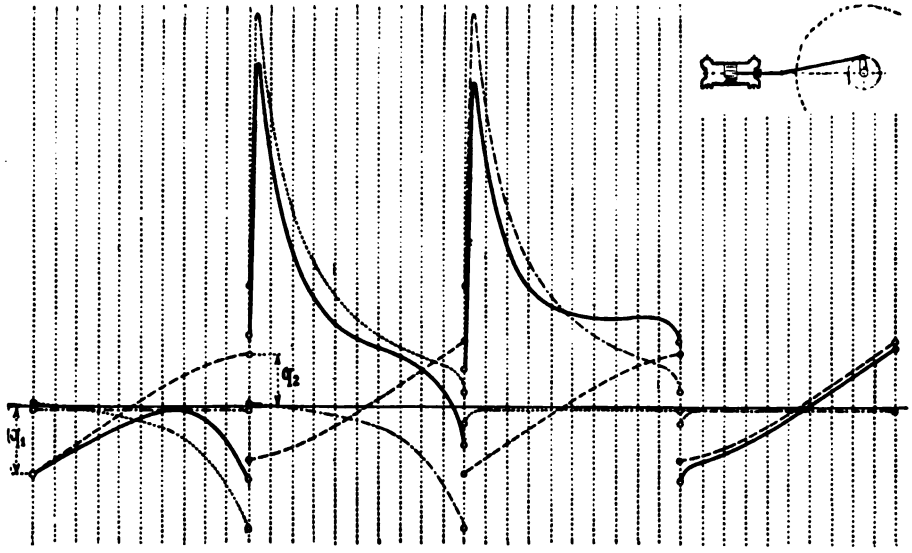


FIG. 10.—Continuous Energy Diagram.

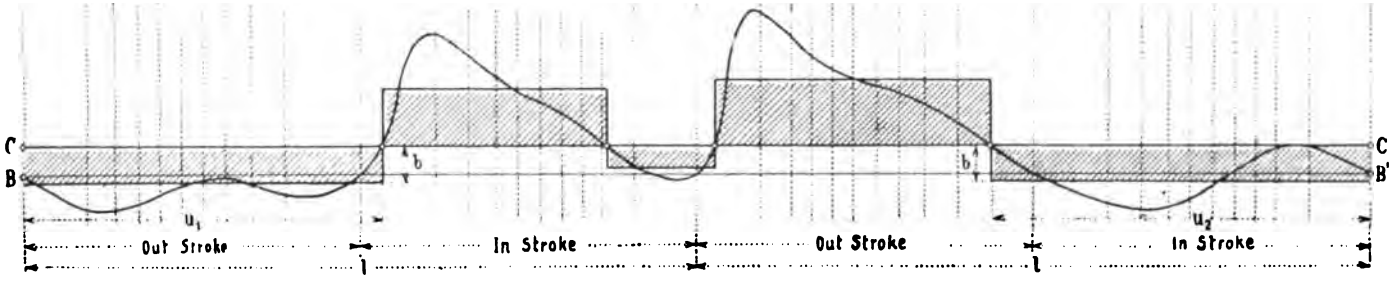


FIG. 11.—Plotting the Turning Moment, and Conversion of the Projecting Areas into Rectangles.

Example.—To show the effect of the double action on the weight of the rotating parts, we will determine the weight of the flywheel rim section for a double-acting engine, having the same dimensions as in the example at p. 40, namely D = 12 inches, S = 16 inches (1·33 feet), $n = 210$, $\delta_0 = \frac{1}{80} = 0{\cdot}0125$; diameter of flywheel 7 feet 3 inches.

From p. 18 we get—

Equation 4*a* : Area of motor cylinder—

$$A = \frac{\pi}{4} \times 12^2 = 113 \text{ sq. inches.}$$

Equation 5, p. 39 : Radius of centre of gravity of rim section—

$$R = 0{\cdot}45 \times 7{\cdot}3 = 3{\cdot}26 \text{ feet.}$$

Equation 2, p. 39 : Circumferential velocity arouud path described by centre of gravity of rim section—

$$v = \frac{3{\cdot}26 \times \pi \times 210}{30} = 72 \text{ feet per second.}$$

Length of greatest area above the mean TE line—

$$u = u_1 + u_2 = 0{\cdot}156 \text{ feet} + 0{\cdot}165 \text{ feet} = 0{\cdot}321 \text{ feet.}$$

Height of greatest area above the mean tangential effort line—

$b = 0{\cdot}196$ inch $\times$ 180 lbs. = 35·3 lbs. per sq. inch (because the scale is 1 inch = 180 lbs.).

Circle described by crank-pin measured on the diagram $l = 0{\cdot}295$ feet—

$$u\frac{b}{l} = 0{\cdot}321\,\frac{35{\cdot}3}{0{\cdot}295} = 38{\cdot}4 \text{ lbs. per sq. inch.}$$

From equation 4, weight of flywheel rim—

$$W = 92 \times 38{\cdot}4\,\frac{113 \times 1{\cdot}33}{72^2} \times \frac{1}{\delta_0} = 101{\cdot}5 \times \frac{1}{\delta_0}$$

For $\delta_0 = \frac{1}{80}$ then $W = 101{\cdot}5 \times 80 = 8120$ lbs.

III. Vis-à-vis Four-cycle Engine, G Z 4, Two Cylinders.

As already explained at p. 10, the sequence of ignitions in this engine is the same as in double-acting four-cycle engines. In both types the weight of the flywheel can be taken as the same. Theoretically this is not quite accurate, but the approximation is sufficiently close for practical purposes.

FIG. 12.—*Vis-à-vis* Four-cycle Engine.

The turning moment diagram, corrected for inertia, Fig. 11, can therefore be used as a basis from which to calculate the flywheel.

IV. Twin Four-cycle Engine, Z 4, Two Cylinders.

From the explanation given at p. 11 of the working method of this engine, it will be seen that the ignitions succeed each other at 360° interval ; that is, an ignition follows at each revolution of the crank, or double stroke to and fro of the piston. Hence the diagrams must be shifted, and follow each other in the same order.

A resultant pressure curve cannot be obtained by adding the pressure diagrams of the two cylinders together, because the motor strokes of the two pistons never take place at the same time. Separate turning moment diagrams must be drawn for each cylinder, and combined to form a resultant curve of tangential pressures.

The turning moment diagram is first drawn for one cylinder, according to the method described at pp. 35 *et seq.* The shape of the diagram is the same as for the single-acting four-cycle engine, Fig. 8, p. 38. The starting-point of the second diagram is shifted 360°, or two single strokes beyond the first (Fig. 14). Both diagrams being plotted, the different

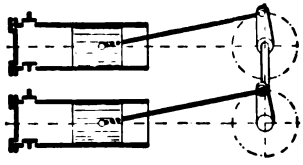


FIG. 13.—Twin Four-cycle Engine.

tangential pressures are added together by the graphic method, as shown at Fig. 14, and a third curve, the so-called *Resultant* is obtained.

If the area enclosed by the neutral line BB′ and the resultant be planimetered, it gives the distance of the line CC′ from the neutral line. This distance BC = B′C′, corresponds to the *mean tangential pressure* reduced to the crank circle.

The areas above the line of mean tangential pressure are then converted into rectangles, and the *largest* (Fig. 14) is taken as the basis of calculation.

Example.—To facilitate comparison with a single-acting engine, we will as before take an engine having the same dimensions as the one at p. 40, and determine the weight of the rim section of the flywheel. Thus D = 12 inches; S = 1·33 feet; n = 210; δ_0 = 0·0125 = $\frac{1}{80}$. Diameter of flywheel, 7 feet 3 inches.

From p. 18 we have—

Equation 4A: Area of each motor cylinder—

$$A = 113 \text{ sq. inches.}$$

Equation 5, p. 39: Circle described by centre of gravity of rim section—

$$R = 0{\cdot}45 \times 7 \text{ feet } 3 \text{ inches} = 3{\cdot}26 \text{ feet.}$$

Equation 2, p. 39: Circumferential velocity around path described by centre of gravity of rim section—

$$v = 72 \text{ feet per second.}$$

By measurement from Fig. 14—

Length of the greatest area above mean tangential pressure line—

$$u = 0{\cdot}125 \text{ feet.}$$

Height of the greatest area above mean tangential pressure line—

$$= b = 0{\cdot}323 \times 180 \text{ lbs.} = 58 \text{ lbs. per sq. inch.}$$

Circle described by crank (measured from diagram) l = 0·293 feet.

$$\text{Therefore } u\frac{b}{l} = 0{\cdot}125\,\frac{58}{0{\cdot}293} = 25 \text{ lbs. per sq. inch.}$$

Then by equation 4: Weight of flywheel rim section—

$$W = 92 \times 25 \times \frac{113 \times 1{\cdot}33}{72^2} \times \frac{1}{\delta_0} = 66{\cdot}7 \times \frac{1}{\delta_0}$$

For $\delta_0 = \frac{1}{80}$; then W = 66·7 × 80 = 5340 lbs.

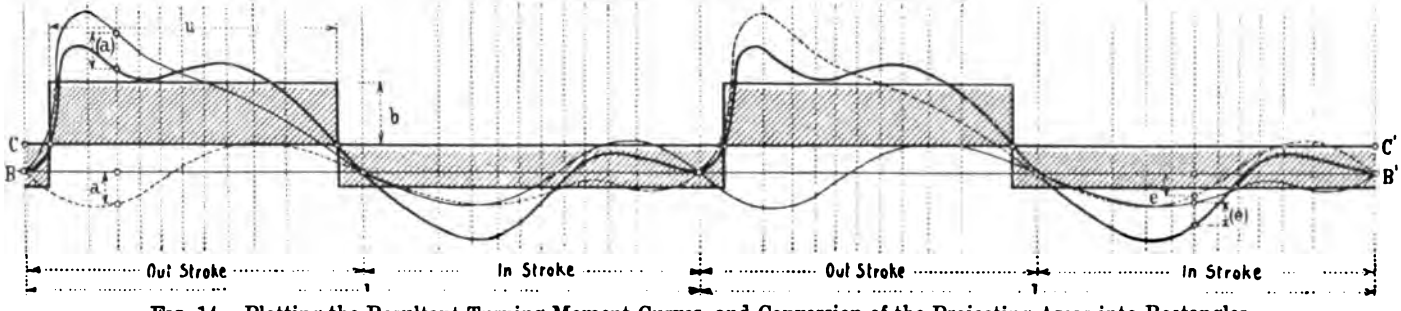


FIG. 14.—Plotting the Resultant Turning Moment Curves, and Conversion of the Projecting Areas into Rectangles.

V. Tandem Four-cycle Engine, R 4, Two Cylinders.

In this type of engine (compare p. 12) the ignitions also occur at intervals of 360°. It should be noted that in these engines the inertia of the moving parts is greater than in single-cylinder engines, because there are two pistons to be allowed for.

In Fig. 15 the two diagrams are continuously developed. The dotted lines ..... denote the back, and the punctuated lines — - — - — the front cylinder. The resultant pressure curves, taking into account the effect of the reciprocating parts, were then determined, and have been plotted as full lines in Fig. 15.

With the help of the diagrams in Figs. 5–7, and the explanation there given, the turning moment diagram corrected for inertia can be plotted for this type of engine (Fig. 16). The areas

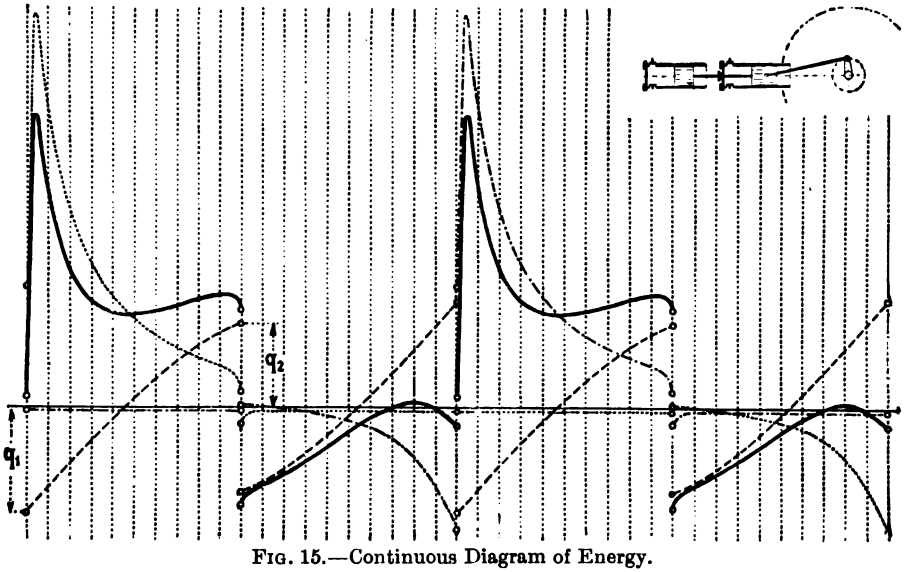


FIG. 15.—Continuous Diagram of Energy.

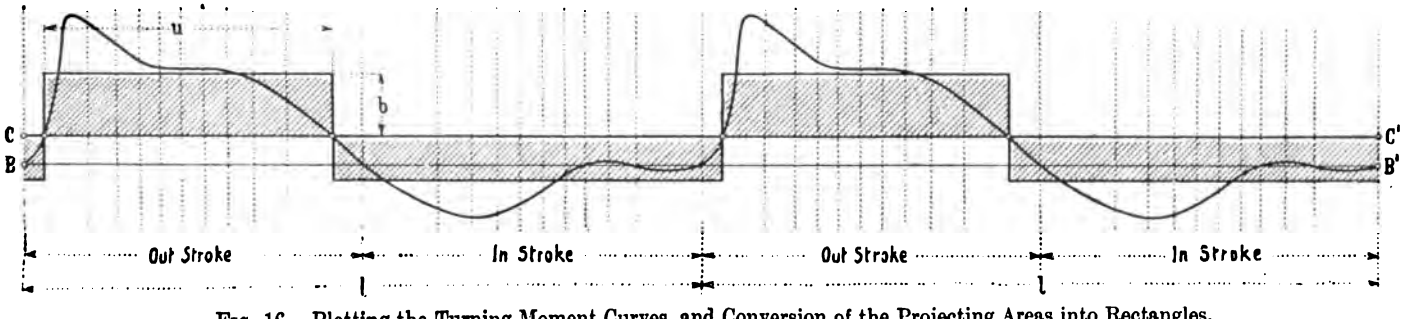


FIG. 16.—Plotting the Turning Moment Curves, and Conversion of the Projecting Areas into Rectangles.

between the turning moment curve and the central line BB' being planimetered, the height of the mean tangential pressure is determined, and the line CC' (Fig. 16) drawn. The areas above the line are converted into rectangles, and their extent determined as before.

Example.—The weight of the rim of the flywheel to be determined for a tandem, single-acting, four-cycle engine of the same dimensions as the example at p. 40, namely, D = 12 inches; S = 1·33 feet (16 inches); $n = 210$, $\delta_0 = 0{\cdot}0125$. Diameter of flywheel, 7 feet 3 inches.

From p. 39—

Equation 4A, p. 18: Sectional area of one motor cylinder—

$$A = \frac{\pi}{4} \times 12^2 = 113 \text{ sq. inches.}$$

Equation 5, p. 39: Radius of centre of gravity of rim section—

$$R = 0{\cdot}45 \times 7 \text{ feet } 3 \text{ inches} = 3{\cdot}26 \text{ feet.}$$

Equation 2, p. 39: Circumferential velocity around path described by centre of gravity of rim section—

$$v = \frac{3{\cdot}26 \times \pi \times 210}{30} = 72 \text{ feet per second.}$$

Length of greatest area above the mean tangential pressure line—

$$u = 0{\cdot}125 \text{ feet.}$$

Height of the greatest area above the mean tangential pressure—

$b = 0{\cdot}323$ inch, therefore $= 0{\cdot}323 \times 180$ lbs. $= 58$ lbs. per sq. inch (since the scale is 1 inch = 180 lbs).

Circle described by crank measured from the diagram $l = 0{\cdot}293$ feet

$$u\frac{b}{l} = 0{\cdot}125\,\frac{58}{0{\cdot}293} = 24{\cdot}8 \text{ lbs. per sq. inch.}$$

Equation 4, p. 39: Weight of rim section of flywheel—

$$W = 92 \times 24{\cdot}8 \times \frac{113 \times 1{\cdot}33}{72^2} \times \frac{1}{\delta_0} = 66{\cdot}3 \times \frac{1}{\delta_0}$$

For $\delta_0 = \frac{1}{80}$; then $W = 66{\cdot}3 \times 80 = 5300$ lbs.

VI. Double Vis-à-vis Four-cycle Engine, Z G Z 4, Four Cylinders.

According to the explanation given at p. 13 of the working method in this engine, there is an ignition at full load at every 180° of the crank circle, in other words, at every stroke to and fro of the piston. Since during each stroke the sequence of operations is the same in one or other of the four cylinders, the weight of the flywheel can be calculated by drawing the turning moment diagram for a single stroke.

The diagram of tangential pressure, or turning moment corrected for inertia, may be plotted from two different points of view, namely—

1. As shown at pp. 35 *et seq.*, the turning moment diagram may be drawn for one cylinder, in other words, for a single-acting four-cycle engine, as at Fig. 17, which is a reproduction of the tangential pressure or turning moment diagram (Fig. 8) to a smaller scale.

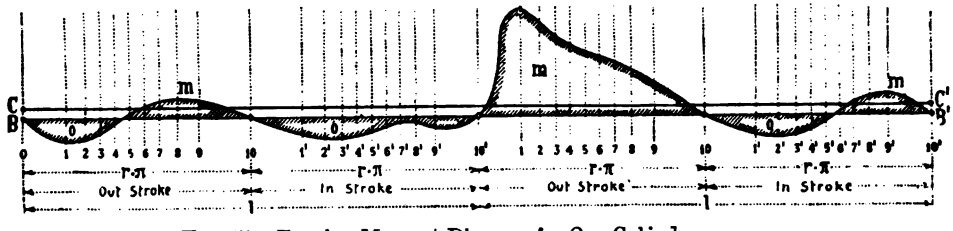


FIG. 17.—Turning Moment Diagram for One Cylinder.

The turning moment curves for the four simultaneous strokes are next combined, and plotted one over the other, as for one piston stroke (Fig. 18). They are then added together, and combined into one resultant curve, which may be used as the basis of calculation. The diagram, Fig. 18, shows the simultaneous action taking place in each cylinder, and in Fig. 20 the curves are numbered, to correspond with the strokes. For the operations carried out during each stroke, see p. 7.

By planimetering the areas enclosed by the neutral line BB′ and the resultant pressure curve, their mean height is determined, and thus the mean pressure which, when reduced to the crank circle, is represented in Fig. 20 by the area BCC′B′.

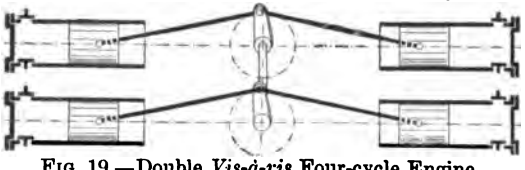


FIG. 19.—Double *Vis-à-vis* Four-cycle Engine.

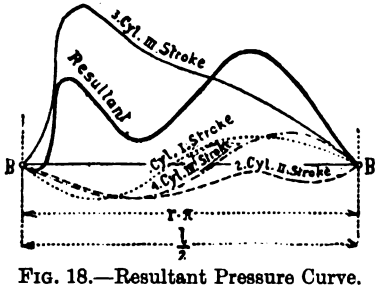


FIG. 18.—Resultant Pressure Curve.

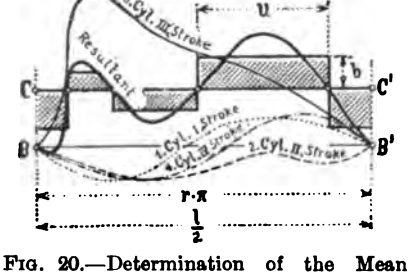


FIG. 20.—Determination of the Mean Pressure, and Conversion of the Projecting Areas into Rectangles.

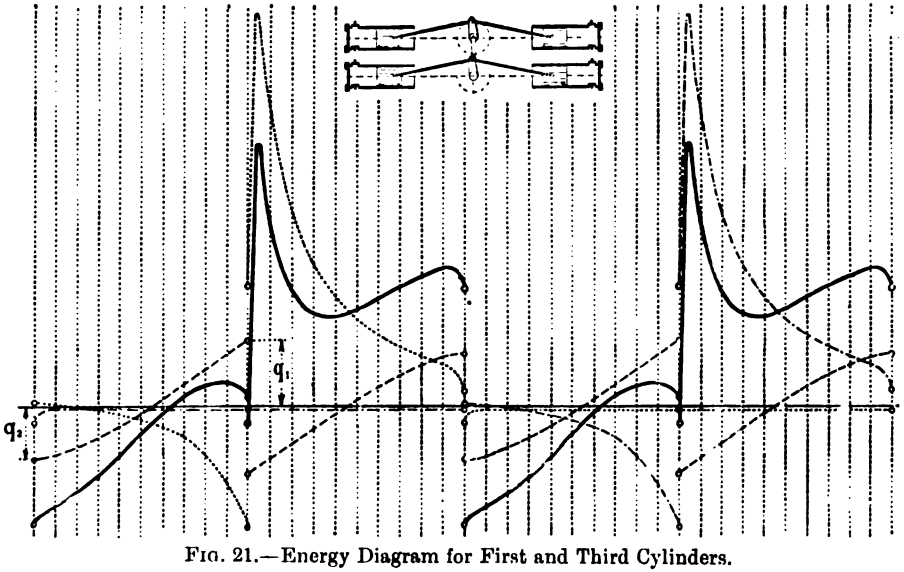


FIG. 21.—Energy Diagram for First and Third Cylinders.

The areas projecting above and below line CC′ are converted into rectangles, and the largest taken as the basis of further calculation (compare p. 39).

2. Draw the pressure curves for each pair of cylinders (1 and 3, and 2 and 4) in Fig. 16, as given at p. 36, and determine the resultant pressure curves. Two diagrams must be plotted, because the movement of the crank and piston are not the same in the two *vis-à-vis* cylinders.

The turning moment curves, as given in Figs. 5 to 7, are next drawn for each pair of cylinders (Fig. 23) and added together to form the resultant pressure curve (Fig. 23).

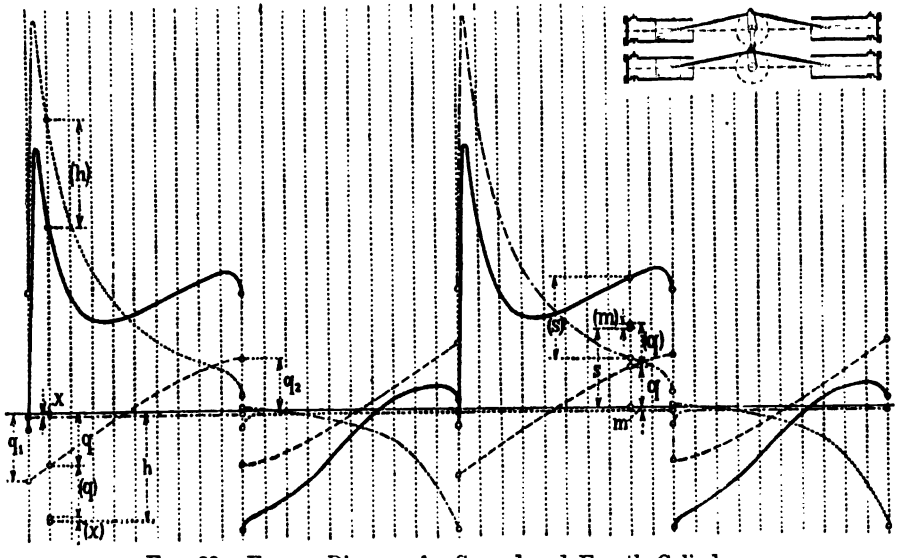


FIG. 22.—Energy Diagram for Second and Fourth Cylinders. (Line x is plotted from (x), and h from (h).)

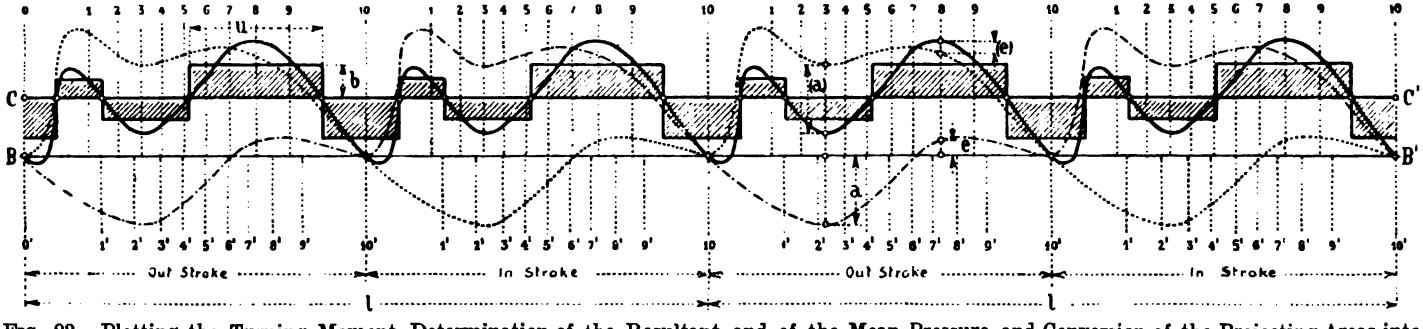


FIG. 23.—Plotting the Turning Moment, Determination of the Resultant, and of the Mean Pressure, and Conversion of the Projecting Areas into Rectangles. (Line a is plotted from (a), and e from (e).)

To determine the mean *height* = *mean pressure* reduced to the crank circle, the areas enclosed by the resultant and the neutral line BB′ are planimetered. In Fig. 23 the line BC is equal to the mean effort.

The areas above and below line CC′ are converted into rectangles, the largest is taken, and the calculation worked out in the same way as at p. 40 for a single-cylinder engine.

In Fig. 23 the rectangles u, b are equal, and are larger than the areas below line CC′.

Example.—For a double *vis-à-vis* four-cycle engine, each of the four cylinders of which is of the same dimensions as the example given at p. 40, viz. D = 12 inches; S = 1·33 feet (16 inches); n = 210; $\delta_0 = \frac{1}{80}$ = 0·0125. Diameter of flywheel 7 feet 3 inches. The weight of the rim section of the flywheel to be determined.

Here (compare example, p. 40) A = 113 sq. inches; v = 72 feet per second (area of one cylinder only to be taken).

From the turning moment diagram (Fig. 23)—

Length of greatest projecting area u = 0·0566 foot.

Height of greatest projecting area b = 0·17 inch.

Therefore = 0·17 × 180 lbs. = 30·6 lbs. per sq. inch (since the scale is 1 inch = 180 lbs.).

Circumference of circle described by crank pin l = 0·295 foot.

$$\text{therefore } u\frac{b}{l} = 0{\cdot}0566\,\frac{30{\cdot}6}{0{\cdot}295} = 5{\cdot}87 \text{ lbs. per sq. inch.}$$

Equation 4, p. 39: Weight of rim of flywheel—

$$W = 92 \times 5{\cdot}87 \times \frac{113 \times 1{\cdot}33}{72^2} \times \frac{1}{\delta_0} = 15{\cdot}7 \times \frac{1}{\delta_0}$$

For $\delta_0 = \frac{1}{80}$; then W = 15·7 × 80 = 1260 lbs.

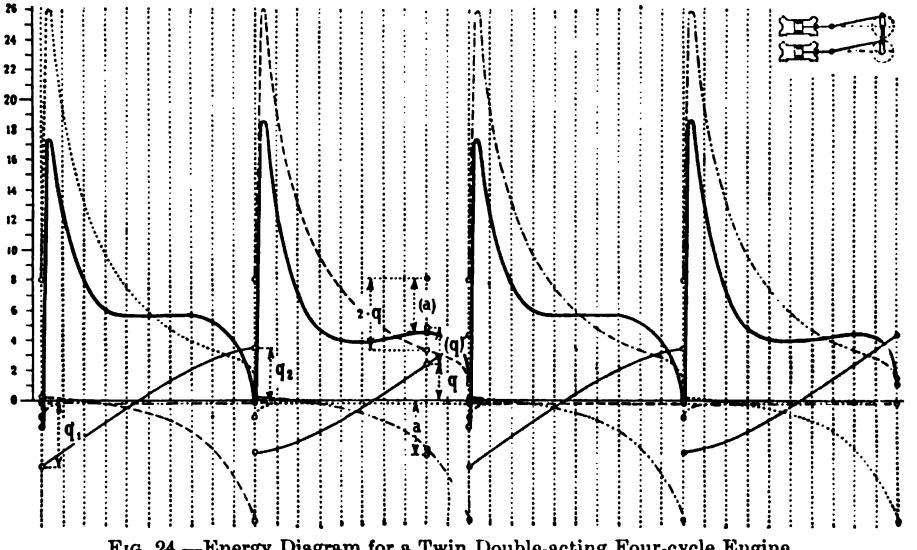


FIG. 24.—Energy Diagram for a Twin Double-acting Four-cycle Engine.

VII. Twin Double-acting Four-cycle Engine, Z D 4, Two Cylinders.

For an explanation of the working method in this engine see p. 14. The sequence of working strokes is shifted 180°, *i.e.* there is an explosion for every half-revolution of the crank shaft. As there are two cylinders, the inertia of the reciprocating parts must be twice allowed for.

The method of drawing the diagram is shown at Fig. 24.

The turning moment diagram (Fig. 25) is constructed as shown at pp. 35 *et seq.* The mean pressure reduced to the crank circle is determined by planimetering the areas enclosed by the neutral line BB′ and the turning moment curve. The line BC = B′C′ corresponds to the mean pressure.

The areas projecting above and below the line CC′ are converted as before into rectangles, and the largest (Fig. 25) is taken as the basis of calculation.

Example.—To make a comparison with the single-acting four-cycle engine, the same dimensions as those given at p. 40 are again taken for determining the weight of the rim of the flywheel, D = 12 inches, S = 1·33 feet (16 inches), $n = 210$, $\delta_0 = \frac{1}{80}$. Diameter of flywheel, 7 feet 3 inches.

Equation 4A, p. 18: Sectional area of one motor cylinder—

$$A = 113 \text{ sq. inches.}$$

Equation 5, p. 39: Radius of centre of gravity of rim section—

$$R = 3{\cdot}26 \text{ feet.}$$

Equation 2, p. 39: Circumferential velocity around path described by centre of gravity of rim section—

$$v = 72 \text{ feet per second.}$$

Measured from Fig 25—

Length of the greatest projecting area $u = 0{\cdot}086$ foot.

Height of the greatest projecting area $b = 0{\cdot}176$ inch.

therefore $= 0{\cdot}176 \times 180$ lbs. $= 31{\cdot}7$ lbs. per sq. inch.

Circumference of circle described by crank-pin measured on diagram (Fig. 25)—

$$l = 0{\cdot}295 \text{ foot}$$

$$\text{therefore } u\frac{b}{l} = 0{\cdot}086\,\frac{31{\cdot}7}{0{\cdot}295} = 9{\cdot}25 \text{ lbs. per sq. inch.}$$

Then by equation 4: Weight of flywheel rim—

$$W = 92 \times 9{\cdot}25\,\frac{113 \times 1{\cdot}33}{72^2} \times \frac{1}{\delta_0} = 24{\cdot}7 \times \frac{1}{\delta_0}$$

For $\delta_0 = \frac{1}{80}$; $W = 24{\cdot}7 \times 80 = 1980$ lbs.

VIII. Tandem Double-acting Four-cycle Engine, R D 4.

As already explained at p. 15, the ignitions in this type of engine occur at intervals of 180°; in other words, there is an ignition (or explosion) at every half-revolution of the crank shaft.

A continuous diagram (as at Fig. 26) is first plotted. Here the lines of the diagrams representing the work done on each face of the piston are denoted by similarly dotted lines. Allowance being made for the inertia of the moving parts, the resultant pressure diagram is drawn in thick lines in the same diagram (Fig. 26).

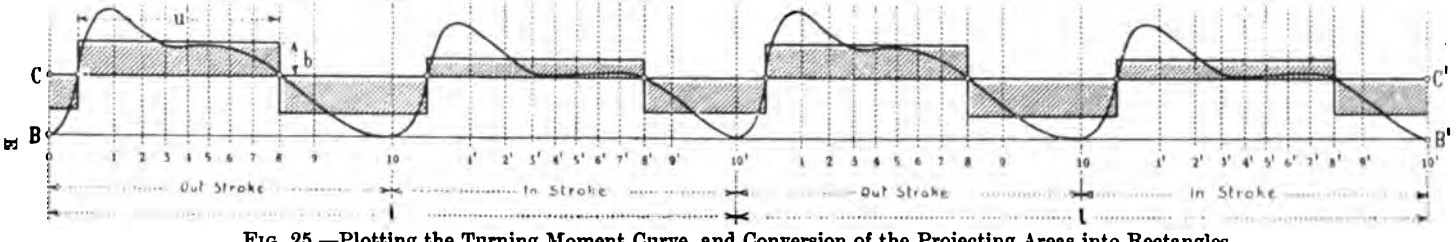


FIG. 25.—Plotting the Turning Moment Curve, and Conversion of the Projecting Areas into Rectangles.

The turning moment diagram (Fig. 27) is then constructed, in the way described at p. 34. The areas enclosed by the neutral line BB′ and the pressure curve are converted by the planimeter into a rectangle of the same area, the height being equal to the mean pressure reduced to the circle described by the crank. In Fig. 27 BC = B′C′ is equal to the mean effort.

The areas projecting above and below CC′ are next converted into rectangles, and the largest of them is taken as the basis of calculation.

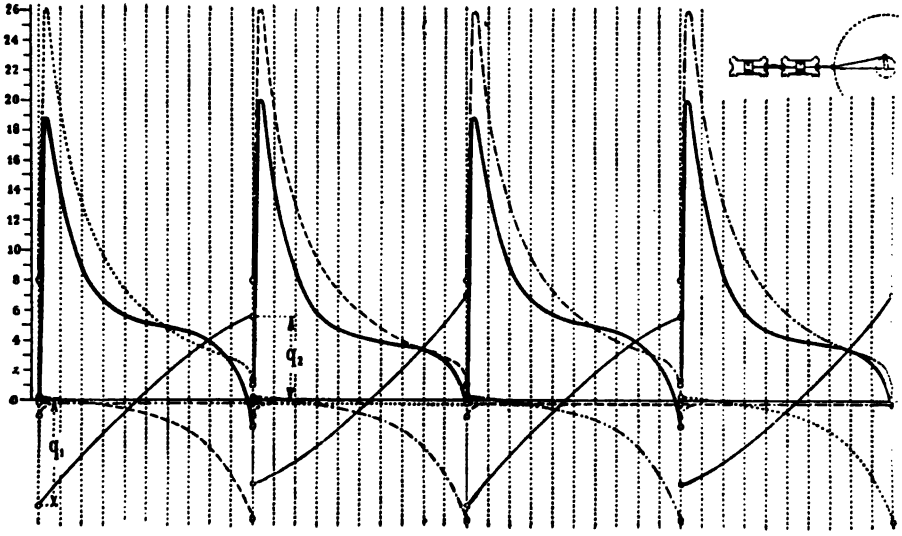


Fig. 26.—Energy Diagram.

Example.—The same dimensions are taken as in the former examples, viz.—

Area of one motor cylinder, A = 113 sq. inches.

Circumferential velocity in circle described by centre of gravity of flywheel, v = 72 ft. per second.

From Fig. 27:

Length of greatest projecting area u = 0·08 foot.

Height of greatest projecting area b = 0·2 inch.

therefore b = 0·2 × 180 lbs. = 36 lbs. per sq. inch.

Circumference of circle described by the crank l = 0·295 foot,

therefore $u\frac{b}{l} = 0{\cdot}08 \times \frac{36}{0{\cdot}295} = 9{\cdot}75$ lbs. per sq. inch.

By equation 4: Weight of flywheel rim—

$$W = 92 \times 9{\cdot}75 \frac{113 \times 1{\cdot}33}{72^2} \times \frac{1}{\delta_0} = 26 \times \frac{1}{\delta_0}$$

For $\delta_0 = \frac{1}{80}$; W = 26 × 80 = 2080 lbs.

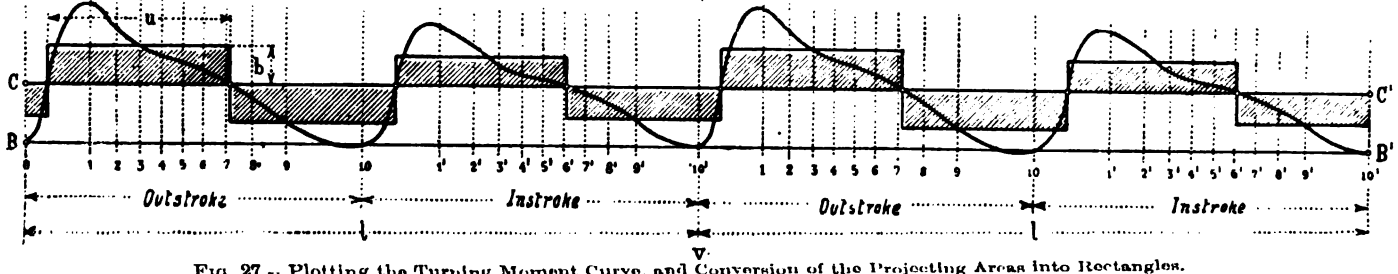


Fig. 27.—Plotting the Turning Moment Curve, and Conversion of the Projecting Areas into Rectangles.

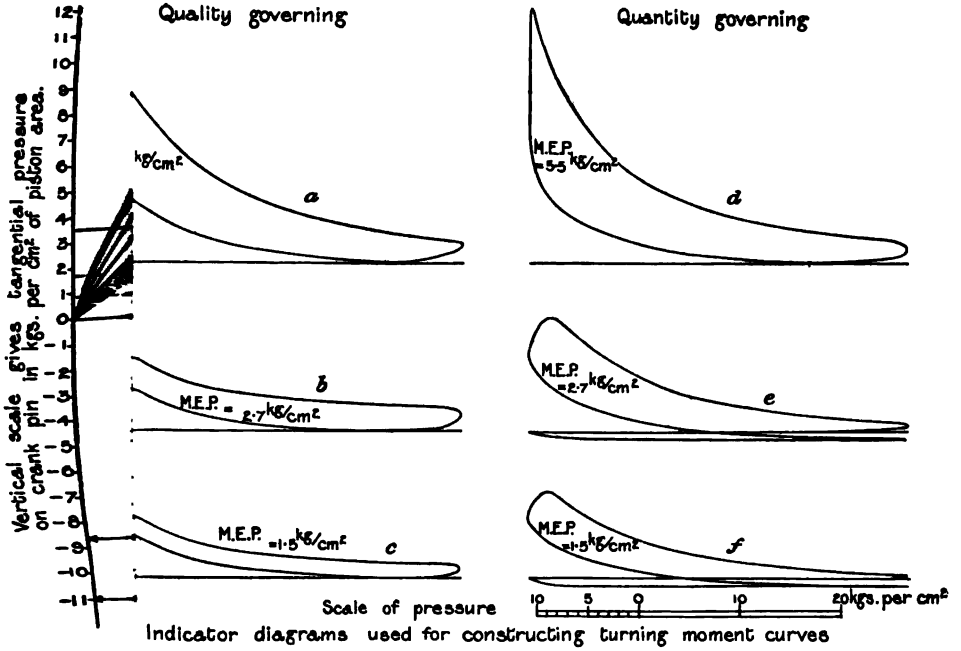
APPENDIX TO CHAPTER V.

NOTE.—The tables and curves given in this Appendix were worked out by Mr. H. A. Humphrey, M.I.C.E., M.I.E.E., etc., and are reproduced by his kind permission. See his contribution to the discussion on Messrs. Allen and Porter's paper, entitled "Use of Gas Engines for Generating Electric Power," read before the Institution of Electrical Engineers, February 25, 1909. Minutes of the Proceeding of the Institution of Electrical Engineers, No. 196, Vol. 43.

TABLE A.

WEIGHT OF RECIPROCATING PARTS OF LARGE GAS ENGINES AND ACCELERATION PRESSURES DUE TO THE RECIPROCATING MASSES (THE ACCELERATION PRESSURES ARE THE MEAN OF THE VALUES AT THE TWO DEAD CENTRES), BASED ON PARTICULARS FURNISHED BY THE RESPECTIVE MAKERS.

Firm.	Size of engine.	Total weight of reciprocating parts.	Weight per unit of piston area. Kg. per cm.²	Revolutions per minute.	$\frac{mr^2}{r}$ Kg. per cm.² of piston.
		Kilogrammes.			
Gasmotoren Fabrik Deutz (Otto cycle)	2,000 B.H.P. (1,100 mm. × 1,300 mm. (double-acting tandem)	23,700	2·66	94	17·10
Erhardt & Sehmer (Otto cycle)	2,000 B.H.P. (1,150 mm. × 1,300 mm. double-acting tandem)	20,000	2·06	94	13·25
Nurnberg (Otto cycle)	1,000 B.H.P.	7,200	1·64	125	14·30
	1,200 B.H.P.	8,600	1·57	107	10·80
	500 B.H.P.	3,500	1·21	128	8·31
Premier (Otto cycle)	1,200 B.H.P.	7,220 (two single-acting tandem sets)	1·585	120	10·70
Mather & Platt (2-cycle)	1,000 B.H.P. (2-cycle Korting)	2,687	0·95	110	6·44
Richardsons, Westgarth & Co. (Otto cycle)	1,000 B.H.P. (double-acting tandem)	7,250	1·24	107	7·92
	500 B.H.P. (double-acting tandem)	3,600	1·40	125	9·80
Bechem & Keetman (Otto cycle)	1,000 B.H.P. (double-acting tandem)	3,940	0·78	107	5·00
Snow Steam Pump Works (Otto cycle)	5,400 B.H.P. (double-acting twin-tandem)	20,500 (total mass per line)	2·63	88	17·35



Indicator diagrams used for constructing turning moment curves

ne.

g rod to crank, 5 : 1. Weight of reciprocating parts, 23,700 kg.
piston area, 17·1 kg. $\left(= \frac{Mv^2}{r}\right)$.

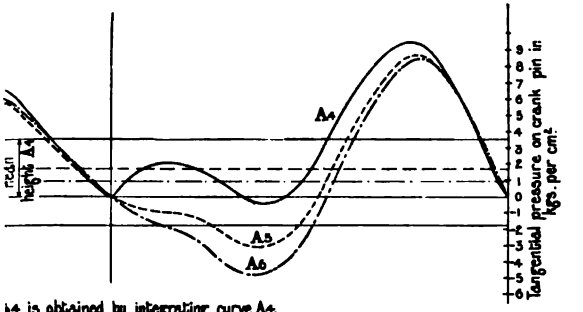
REN FABRIK DEUTZ.

FOR QUANTITY GOVERNING.

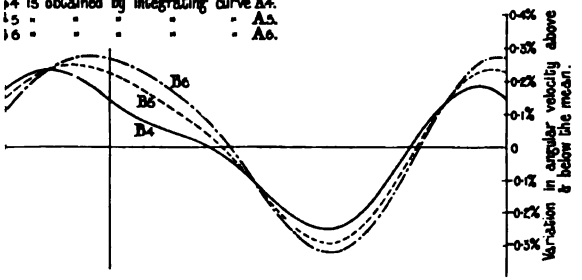
n crank pin due to indicator diagram pressures per cm.².
5 kg. per cm.² mean effective pressure of diagram.
am 2·7 kg. per cm.².
am 1·5 kg. per cm.².
kg. per cm.² (actual value).

Quantity Governing.

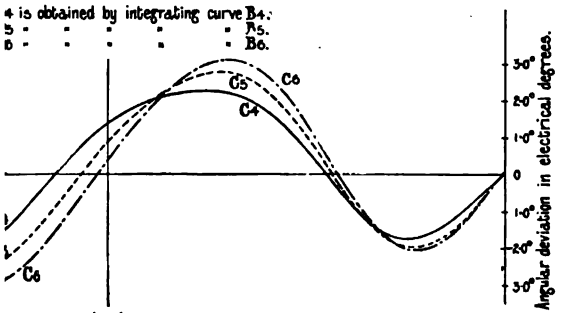
on of curves d and J of Fig. A (M.E.P. = 5·5 kgs. per cm.²)
" " e " J " A { " = 2·7 " " }
" " f " J " A { " = 1·5 " " }



4 is obtained by integrating curve A4.
5 " " " " " A5.
6 " " " " " A6.



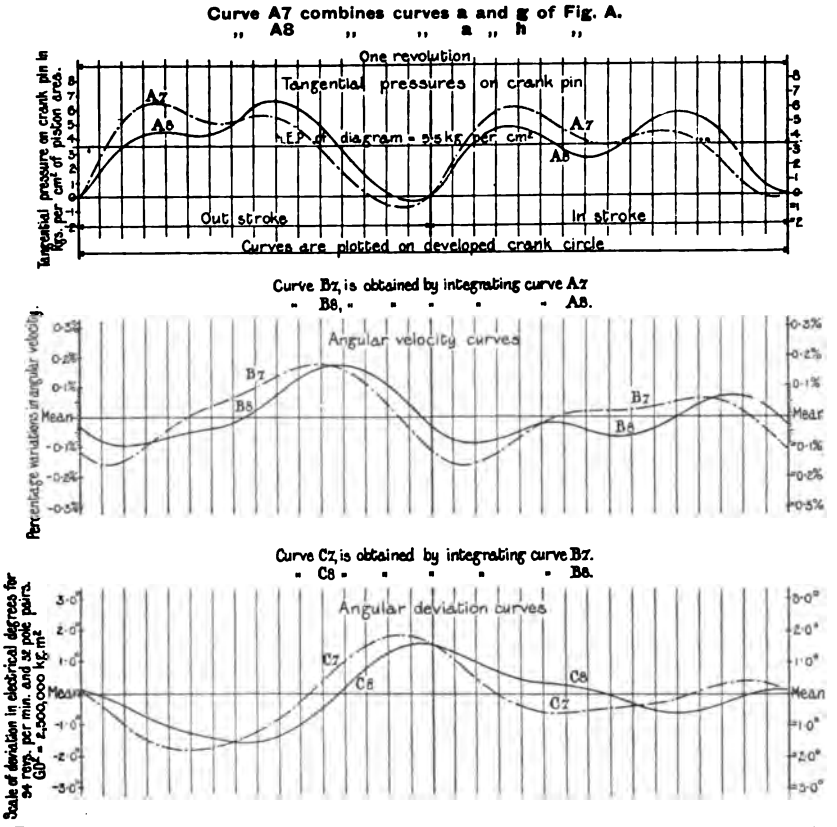
4 is obtained by integrating curve B4.
5 " " " " " B5.
6 " " " " " B6.



e not exactly the true means.

).

: diagram and the forces due to the inertia are algebraically added.
m the mean velocity.
ator with 32 poles. Weight of flywheel to give total angular deviation of

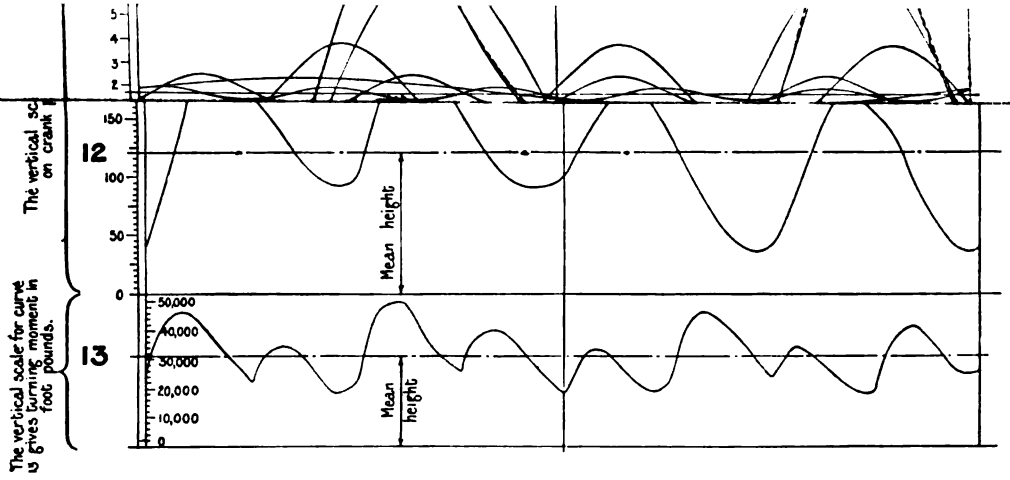


For further particulars, diagrams and curves, see Figs. A and B and curves 1 to 8 of Fig. D.

FIG. C.—2000-B.H.P. Double-acting Tandem Gas Engine (Deutz).

Curves A7 and A8 in this Fig. are directly comparable with curve A1 in Fig. B, and show the effect of combining the full-load tangential pressure of curve A (quality governing) with the reduced inertia forces obtained from curves *g* and *h* in Fig. A.

(For further particulars and curves, see Figs. A to C.)



(*For further particulars and curves, see Figs.* A *to* C.)

Curve 12 is for two Cockrill gas engines as above (Curve 11), coupled with their cranks at 90°, so as to form one twin double-acting tandem engine. Weight of flywheel, 45 tons.

Curve 13 is for 1000-B.H.P. British Westinghouse gas engine. Vertical tandem type, four pairs = 8 cylinders. Cylinders, top, 22 in. diameter; bottom, 21 in. diameter. Stroke, 24 in. Revs., 200 per minute. Weight of reciprocating parts (per line), 2750 lbs. (includes two-thirds of connecting-rod weight). Weight of flywheel, 18,000 lbs. (generator).

FIG. D.—Turning Moment Curves for Large Gas Engines.

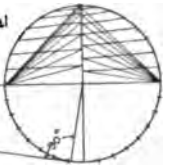
Curves from 1 to 8 are for 2000-B.H.P. double-acting tandem gas engine (Deutz), and are taken from Figs. A to D, and Curves 1 and 2 show in each case the first 5 harmonics in Fourier series.

'N ON FIG. D.

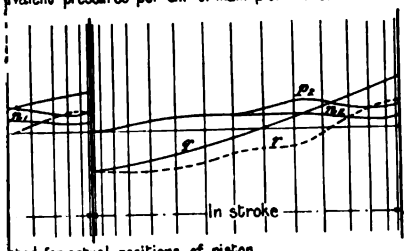
Harmonics.			
	3rd.	4th.	5th.
·655	cos (3θ + 93°) + 0·473	1·425 cos (4θ + 223°) + 0·407	0·370 cos (5θ + 233°) 0·106
14	cos (3θ + 94°) + 1·26	0·778 cos (4θ + 249°) + 0·458	0·180 cos (5θ + 291°) 0·106
05	cos (3θ + 176°) + 2·28	0·678 cos (4θ + 8°) + 0·753	0·303 cos (5θ + 17°) 0·337
85	cos (3θ + 175°) + 0·529	1·3 cos (4θ + 41°) + 0·372	0·315 cos (5θ + 7°) 0·09
02	cos (3θ + 175°) + 1·19	0·690 cos (4θ + 60°) + 0·406	0·089 cos (5θ + 40°) 0·052
·065	cos (3θ + 180°) + 2·29	0·602 cos (4θ + 45°) + 0·669	0·19 cos (5θ + 27°) 0·211
527	cos (3θ + 168°) + 0·151	1·487 cos (4θ + 45°) + 0·425	0·342 cos (5θ + 42°) 0·098
005	cos (3θ + 138°) + 0·288	1·43 cos (4θ + 46°) + 0·409	0·276 cos (5θ + 52°) 0·079
789	cos (3θ + 126°) + 0·147	1·952 cos (4θ + 54°) + 0·365	0·239 cos (5θ + 54°) 0·045
86	cos (3θ + 135°) + 0·313	37·8 cos (4θ + 62°) + 0·298	6·4965 cos (5θ + 76°) 0·051
91	cos (3θ + 180°) + 0·114	30·51 cos (4θ + 41°) + 0·504	6·772 cos (5θ + 15°) 0·112
2	cos (3θ + 132°) + 0·092	59·85 cos (4θ + 41°) + 0·494	11·69 cos (5θ + 72°) 0·096
4·5	cos (3θ + 289°) + 0·205	354·35 cos (4θ + 227°) + 0·0114	3,378·0 cos (5θ + 86°) 0·109

engine cranks

obtaining crank pin tangential
sures for pump parts.

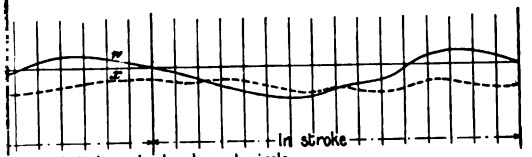


p from actual indicator diagrams.
es in air & gas pump from actual indicator diagrams
ssures to ordinates of curves m_1 & m_2.
es due to inertia of reciprocating parts
" " both indicated pressures & forces due to inertia
ined by combining curves p_1 & q & p_2 & q.
ivalent pressures per cm² of main piston area.



tted for actual positions of piston

pressure curves for pump cylinder.
rank pin for one set of pumps due to indicator diagram pressures & inertia forces
" " " both sets " " " " " " " " " "



Curve plotted on developed crank circle

uation of curves see Fig. F and curve 9, Fig. D.

nders with Cranks at 90°.

000 mm. Revs. 110 per minute. Air and gas pumps, 650 mm.
ight of one line of pump parts, 623 kg. $\left(\frac{Mv^2}{r} = 1{\cdot}27\right.$ kg. per cm.²

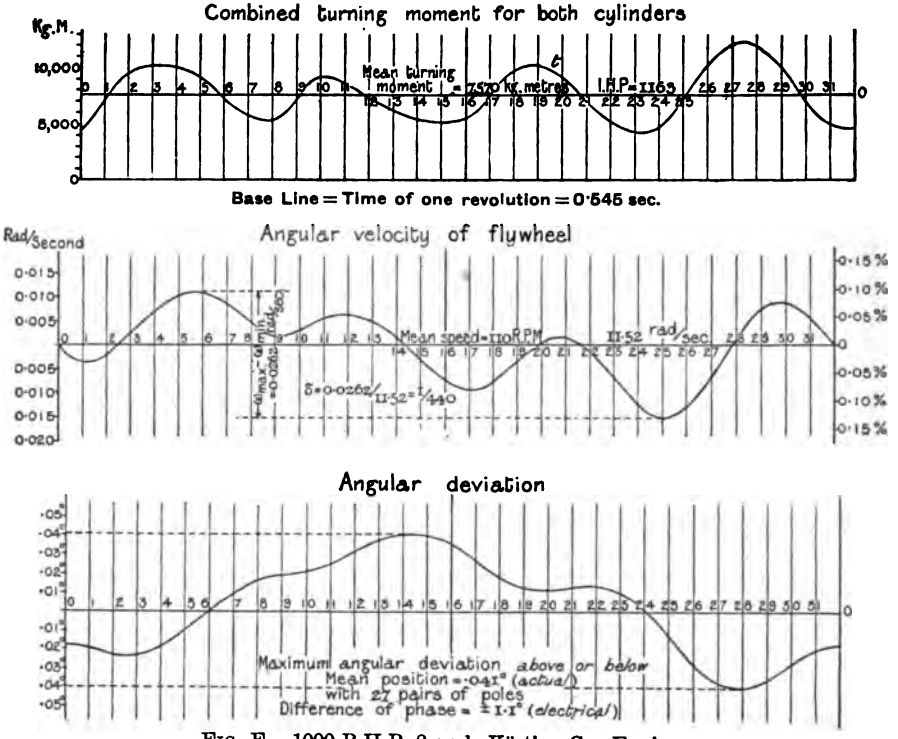


FIG. F.—1000-B.H.P. 2-cycle Körting Gas Engine.
(Messrs. Mather & Platt, Ltd., Engineers.)

DIMENSIONS OF ENGINE—
Twin double-acting cylinders. Cranks at 90°.
Main cylinders: Diameter, 600 mm. Stroke, 1000 mm.
Pump cylinders: Diameter, 650 mm. Stroke, 850 mm.
Revs. per minute, 110.

FLYWHEEL—
Alternator rotor equivalent to 28 tons at 11·9 ft. diameter.
G.D.² = weight × diameter² = 3957 ton-feet² = 374,000 kg.-metres².

CALCULATION OF DEGREE OF IRREGULARITY—
Notation—
M = Difference between turning moment on crank and mean resisting torque in kg.m.
I = Moment of inertia of flywheel (mass × rad. of gyration², in kg.-metre/sec.².
g = Acceleration due to gravity = 9·81 metres/sec.².
G.D.² = Moment of flywheel in kg.-metre².
ω = Angular velocity in rad./sec.
t = Time in seconds.

$$\delta = \text{Degree of irregularity} = \frac{\omega_{max.} - \omega_{min.}}{\omega_0}$$

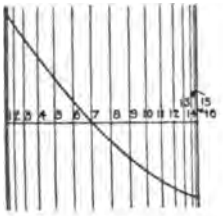
$$M\,dt = I\,d\omega = \frac{G.D.^2}{4g}\,d\omega \qquad \omega - \omega_0 = \frac{4g}{G.D.^2}\int M\,dt.$$

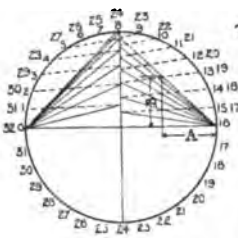
$$\textit{Angular Deviation} = \int(\omega - \omega_0)\,dt.$$

TABLE OF FLYWHEEL WEIGHTS.

Mean Rim Speed.	75 Feet/Sec.			90 Feet/Sec.		
δ	$\frac{1}{250}$	$\frac{1}{300}$	$\frac{1}{350}$	$\frac{1}{250}$	$\frac{1}{300}$	$\frac{1}{350}$
G.D.² ft.² tons ...	2,250·0	2,700·0	3,145·0	2,250·0	2,700·0	3,145·0
D ft.	13·0	13·0	13·0	15·6	15·6	15·6
D²	169·0	169·0	169·0	244·0	244·0	244·0
Rim weight G, tons	13·3	16·0	18·6	9·2	11·9	12·9
Total weight ...	18·0	21·6	25·1	12·4	14·9	17·4

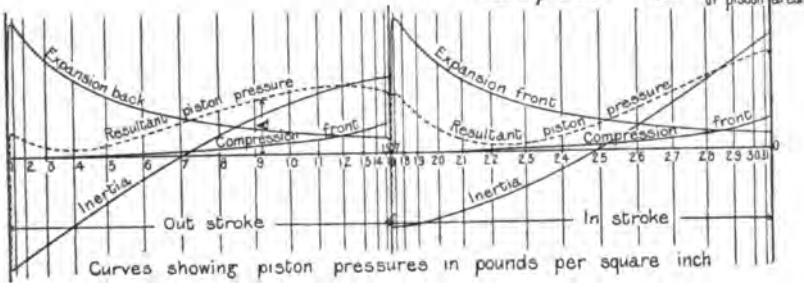
(For other Diagrams and Curves, see Fig. E.)



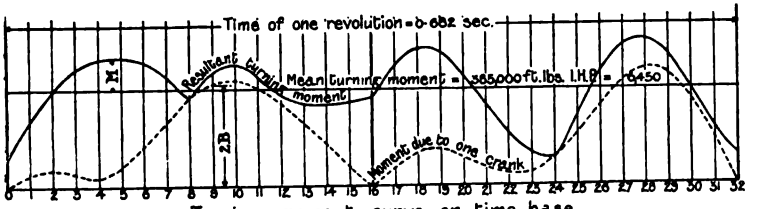


5,400 B.H.P. Gas Engine
four-cylinder double acting twin tandem
The Snow Steam Pump Works Buffalo N.Y.

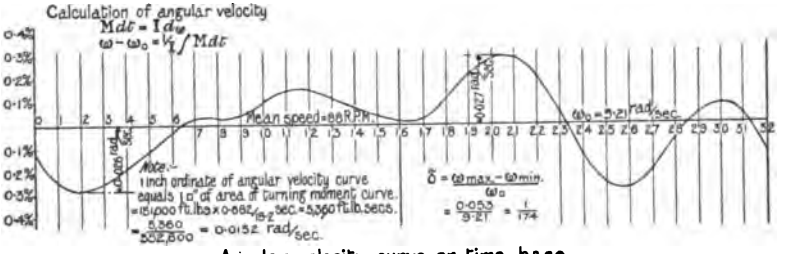
Dimensions of engine
Cylinders Stroke = 60"
Bore = 42"
R.P.M. = 88
Ratio of length of connecting rod to crank 5:1
Flywheel alternator
Moment of inertia = $\frac{W}{g} \cdot R^2$ = 352,800 ft. lbs. sec²
Number of pairs of poles = 34
~ per sec 25
Volts 13,200
M.E.P. of indicator diagram = 99 lbs/□"
Weight of reciprocating parts for one line of cylinders & including 2/3 of weight of connecting rod = 45,300 lbs.
Value of $\frac{W}{g}R^2$ for each line of cylinders = 247 lbs. per sq. in. of piston area.



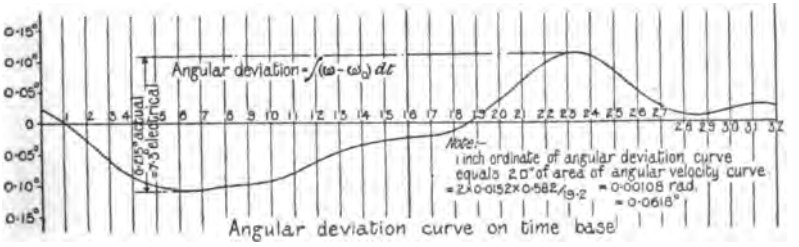
Curves showing piston pressures in pounds per square inch



Turning moment curve on time base



Angular velocity curve on time base



Angular deviation curve on time base

FIG. G.

Before proceeding to treat of the *different parts of the engine*, it is necessary clearly to understand the forces acting upon them, which have to be taken into account.

CHAPTER VI.

Forces acting on the Cross Head, Piston, Connecting Rod, and Main Bearings.

(a) Forces acting on the Cross Head.

THE pressures developed in gas engines are of many kinds. They depend not only on the dimensions of the engine and the explosion pressures, but also on the clearance or slack in the brasses, or any slackness of the flywheel on the shaft, or of the frame on the foundation, or to any sudden rise in pressure due to pre-ignition, and so forth.

It is impossible to estimate these *accidental pressures* accurately, and we are obliged, therefore, to determine the strength of the different parts of the engine by calculating the *theoretical pressures only*. Of these we have specially to consider—

1. *Forces acting on the cross head.*
2. *Pressure on the crank-shaft bearings.*

The strength of many parts of the engine—for instance, of the frame—can only be tested in a few places. Thus it is hardly possible to determine the shearing stress on the holding-down or foundation bolts, because the strains on them are of a very complex nature. In this case it is usual to adopt dimensions that have been arrived at by practical experience. Any calculations the results of which deviate considerably from those obtaining in practice should be rejected.

1. Forces acting on the Piston Rod, Connecting Rod, Crank Pin, and Crank Journals.

The maximum pressure developed in the neighbourhood of the dead point is—

$$P' = A \times p \text{ in lbs. per sq. inch} \quad . \quad . \quad . \quad (1)$$

A being the area of the cylinder in sq. inches, and P being pressure of explosion in lbs. per sq. inch (Fig. 1).

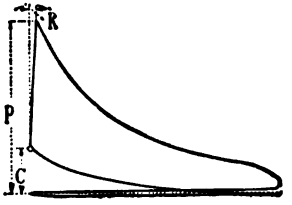


FIG. 1.

We will take a value within working limits, making due allowance for the continually increasing pressures of compression and explosion. In order to have the working parts strong enough to bear powers above the normal rated horse power, let—

$$p = 26 \textit{ atmospheres} \text{ above atmospheric pressure * } (= 370 \text{ lbs. per sq. inch}) \quad (2)$$

With hit-and-miss governing, the full pressure of explosion must be taken into account, even if, as at less than full load, it is not often developed. If the engine is governed by

* As much as 30 atmospheres (430 lbs. per sq. inch) are sometimes allowed for.

varying the proportions of the charge, that is, an ignition is obtained at each working stroke (and therefore, in four-cycle engines, for every two revolutions of the crank shaft), the pressure of explosion varies. We must, however, take the *maximum* pressure developed as a basis of calculation.

2. Pressure on the Crank-shaft Bearings.

These bearings form one of the most important parts of the engine, and it is necessary for the engineer clearly to understand the pressures acting on them.

This he can only do by working out an example for himself—

We take the case of a single-cylinder, four-cycle engine with two bearings, and therefore having the flywheel overhung at the end of the crank shaft.

What are the pressures on the bearings at the different crank positions? in which position is the greatest pressure developed? and in what direction is it exerted?

Further, can the engine continue running if the caps are taken off the bearings?

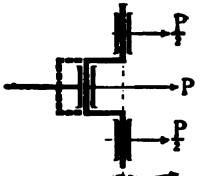


FIG. 2.

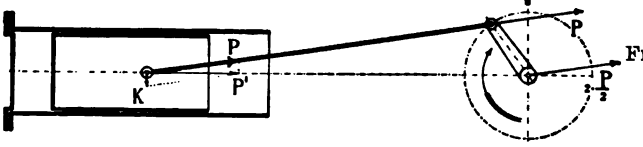


FIG. 3.—Diagram of Forces.

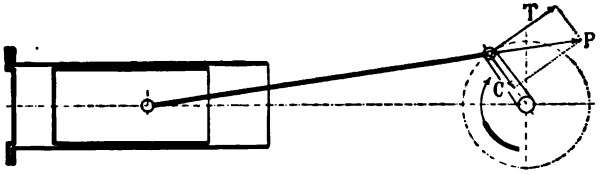


FIG. 4.—Plotting the Thrust.

First draw the diagram of forces without taking the length of the crank shaft or weight of the flywheel into account (Fig. 3).

We are dealing here with *a cranked shaft* (*with two crank arms*), and therefore two bearings (Fig. 2), but for the sake of simplicity the diagram of effort is shown with one crank only (Figs. 3 and 4).

Let P′ be the pressure on the *piston* in lbs.
P be the thrust on the connecting rod in lbs.
K be the *vertical* pressure exerted on the cylinder liner in lbs. (Fig. 3).
β be the angle of inclination of the connecting rod in degrees (Fig. 8, p. 27).

Then the—

$$\text{Thrust } P = \frac{P'}{\cos \beta} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (3)$$

With ordinary lengths of the connecting rod the maximum is $\beta = 10°$, corresponding to $\cos \beta = 0{\cdot}985$, where—

$$P = \frac{P'}{0{\cdot}985} = \text{about } 1{\cdot}015 + P'.$$

This difference is so small that it may be neglected, and we may make—

$$P = P' \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (4)$$

The pressure P is resolved into T, tangential pressure or turning moment, and C radial pressure, Fig. 4 (compare p. 34).

The energy expended in these two directions can easily be calculated, if the pressure on the piston at the corresponding crank position is known. If we suppose the bearings to be in the same plane as the crank pin, then the pressure on the two bearings will be equal in extent and direction to that on the crank pin.

As both the main bearings are at the same distance from the crank pin, and are thus symmetrically arranged (Fig. 2), then the—

$$\text{Pressure on the bearings} = \frac{P}{2} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (5)$$

P being the pressure exerted by the piston corresponding to the simultaneous pressure on the connecting rod.

The pressure on the crank-shaft bearings, and the direction in which it is developed, vary for each position of the piston. It is greatest during explosion, and decreases during the expansion stroke, and the following exhaust, admission, and compression strokes.

Effect of the Flywheel on the Pressure on the Bearings.

The weight W of the flywheel acts downwards at right angles, and endeavours on the flywheel side to hold the shaft down on the lower brass of the main bearing A_2 and against the cap of the bearing A_1 on the other side.

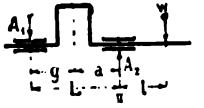


FIG. 5.

Outer Bearing. Main Bearing (flywheel side).

From Fig. 5 we get—

Bearing on outer side, A_1—

Left-hand turning moment: $M_l = A_1(g + a)$

Right-hand ,, ,, $M_r = W \times l$

$$\text{Hence pressure upward on the bearing } A_1 = \frac{W \times l}{g + a} = W \times \frac{l}{L} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (6)$$

Main bearing at A_2—

Left-hand turning moment: $M_l = A_2 \times (g + a)$

Right-hand ,, ,, $M_r = W \times (L + l)$

$$\text{Therefore downward pressure on the bearing } A_2 = W\frac{L + l}{g + a} = W \times \frac{L + l}{L} \quad . \quad . \quad . \quad . \quad . \quad (7)$$

As there are two bearings, and the weight of the flywheel has a different effect on each, they must be treated separately.

I. Bearing on Outer Side (commonly called the Side-shaft Side), A_1.

After the above explanation, we can draw the diagram for this bearing, Fig. 6.

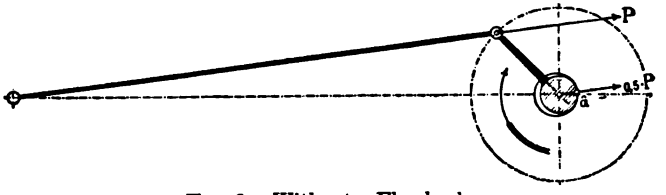
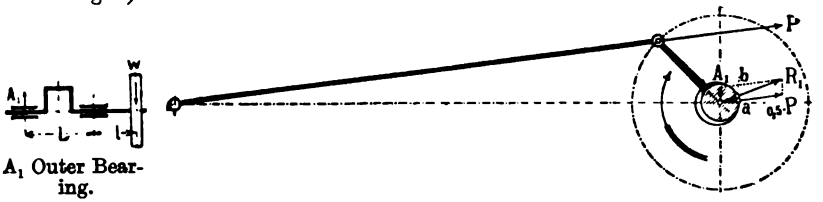


FIG. 6.—Without a Flywheel.

P is the magnitude and direction of the thrust on the connecting rod.

0·5 P ,, ,, ,, ,, this pressure on the main bearing (neglecting the flywheel, Fig. 6).

a would be the theoretical point of contact of the journal with the brass of the bearing (neglecting the flywheel).
R_1 is the direction and magnitude of the resultant pressure.
b ,, theoretical point of contact of the journal and brass under this pressure (with a flywheel, Fig. 7).



A_1 Outer Bearing.

FIG. 7.—With a Flywheel.

Thus the effect of the flywheel is always to shift the point of contact in this outer bearing slightly upwards.

II. Main Bearing (Flywheel Side), A_2.

Here

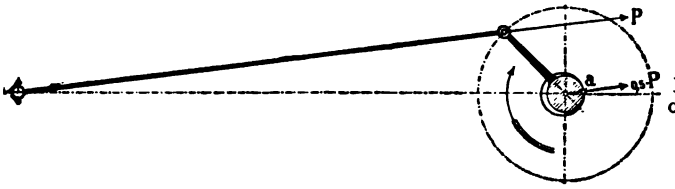
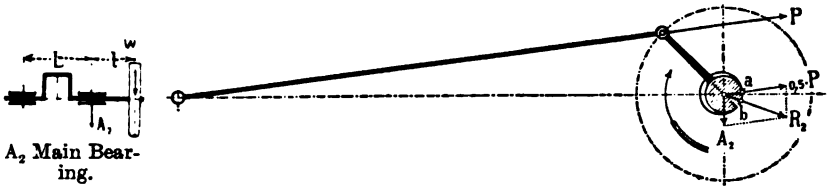


FIG. 8.—Without a Flywheel.



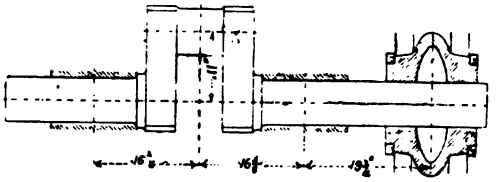
A_2 Main Bearing.

FIG. 9.—With a Flywheel.

The letters have the same meaning as before, when dealing with the outer bearing A_1, but Fig. 9 shows that the resultant pressure R_2 is directed downwards, because the weight of the flywheel is exerted downwards.

We will now calculate a numerical *example* for a single-acting four-cycle engine, having cylinder diameter D = 16 inches; effective cylinder area A = 201 sq. inches; stroke S = 22 inches; explosion pressure = 400 lbs. per sq. inch, weight of flywheel W = 11,600 lbs.

FIG. 10.



Outer Bearing. Main Bearing.

The dimensions of that part of the crank shaft which must be taken into account are given at Fig. 10.

We will base our calculations on the diagram Fig. 3A (p. 36), showing the inertia of the moving parts, but will confine ourselves to the third stroke (explosion and expansion). Compare Fig. 11.

If the actual pressures be calculated for crank positions 1, 5, and 10, they work out as follows :—

Table I.—Pressure on the Bearings.

For Crank Positions.	Measured from Fig. 11.	On the Crank Pin.	On the Bearings.
1	$p_1 = 1\frac{1}{5}$ in. = 230 lbs. per sq. in.	$P = A \times p_1 = 201 \times 230 = 46{,}230$ lbs.	$0{\cdot}5 \times P =$ **23,115** lbs.
5	$p_5 = {\cdot}5$,, = 90 ,, ,, ,,	$P = A \times p_5 = 201 \times 90 = 18{,}090$,,	$0{\cdot}5 \times P =$ **9,045** ,,
10	$p_{10} = {\cdot}35$,, = 64 ,, ,, ,,	$P = A \times p_{10} = 201 \times 64 = 12{,}864$,,	$0{\cdot}5 \times P =$ **6,432** ,,

A = area of the cylinder in sq. inches.

This result is the same for both bearings, but the influence of the flywheel is exerted in different directions.

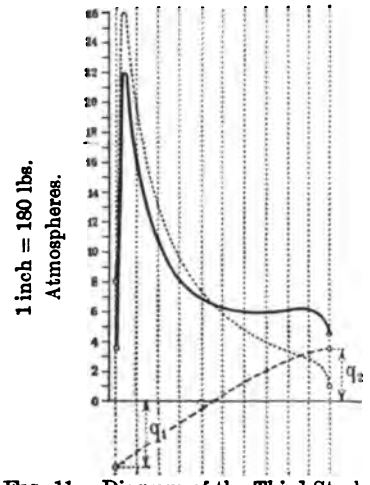


FIG. 11.—Diagram of the Third Stroke.

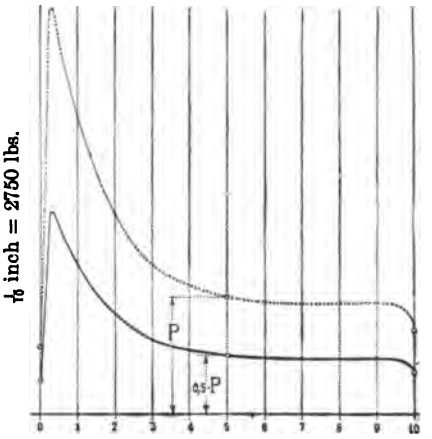


FIG. 12.—Diagram of Actual Pressure on the Bearings.

To make the subject clearer, a diagram of pressures on the bearings should be drawn, the figures printed in black type in Table I. being plotted as in Fig. 12. The scale chosen is $\frac{1}{10}$ inch = 2750 lbs. (For instance, for position 2 of the piston, measure from Fig. 12 a height of = 0·54 inch, which gives a pressure on the bearings of 0·54 × 2750 = 14,850 lbs.)

I. Outer Bearing (Side-shaft Side), A_1.

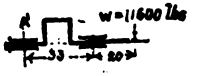


FIG. 13.

The effect of the weight of the flywheel is the same for every position of the crank. In this bearing A_1, according to equation 6 and Fig. 13, the pressure against the cap of the bearing A_1 will be—

$$\frac{11{,}600 \times 20}{33} = 7000$$

If these numbers be transferred to the scale of Fig. 12, *i.e.* $\frac{1}{10}$ inch = 2750 lbs., the ordinate gives—

$$\frac{7000}{2750 \times 10} = \text{about } 0{\cdot}25 \text{ inch.}$$

Determination of the Resultant Thrust, R_1.

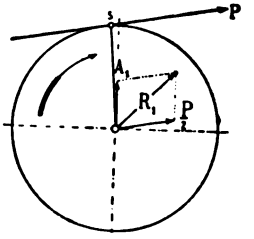


FIG. 14.—Determination of the Resultant Thrust, R_1.

From the pressure on the bearing 0·5 P (parallel to the direction of the connecting rod), and the vertical upward pressure A_1 upon the cap of the bearing, a resultant can be determined. The method adopted for position 5 of the piston is shown at Fig. 14. The pressure on the

bearing, 0·5 P (from Table I. or Fig. 12) is 8700 lbs. With the scale of $\frac{1}{10}$ inch = 2750 lbs., this gives a length of 0·316 inch, and these values are plotted parallel to the direction of the thrust. The pressure against the cap of the bearing A_1 amounts to 7000 lbs. = 0·25 inch for every position of the piston, and is carried out vertically. By plotting the parallelogram of forces, as at Fig. 14, the resultant R_1 is obtained.

The resultants in Fig. 15 are determined in the same way, and are plotted for positions 0, 5, and 10 of the piston. The curve is drawn by joining the different points of the resultants.

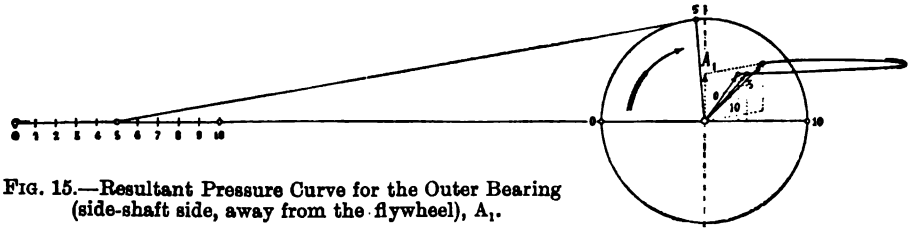
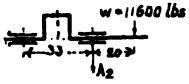


FIG. 15.—Resultant Pressure Curve for the Outer Bearing (side-shaft side, away from the flywheel), A_1.

Thus, in ordinary gas engines under normal conditions, all the pressures are seen to be directed upwards, towards the upper half of the circle described by the crank.

There is therefore a *pressure* on the *cap* of the *bearing*, and the engine could not work *if the cap were taken off.*

II. Main Bearing (Flywheel Side), A_2.



The effect of the weight of the flywheel is a constant, and for this bearing, according to equation 7, it amounts to—

$$A_2 = 11{,}600 \times \frac{33 + 20}{33} = 18{,}650 \text{ lbs.}$$

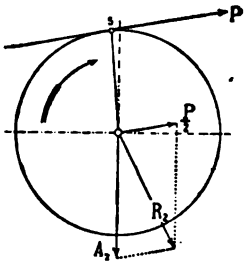


FIG. 16.—Determination of the Resultant Thrust, R_2.

Taking as before the scale of $\frac{1}{10}$ inch = 2750 lbs., the ordinate gives—

$$\frac{18{,}650}{2750 \times 10} = 0{\cdot}68 \text{ inch.}$$

From the pressure on the bearing (0·5 P) in the direction of the connecting rod, and the share borne by the weight of the flywheel, A_2, which acts downwards, the resultant R_2 is obtained for each position of the crank, after completing the parallelogram of forces (compare Fig. 16).

In Fig. 17 all the resultant pressures are plotted for one stroke, and show clearly that they are all exerted downwards, in the lower half of the circle described by the crank.

Hence there is *no pressure* on the *cap of the bearing.*

In *theory*, therefore, the engine would work if the cap of the main bearing on the flywheel side were taken off.

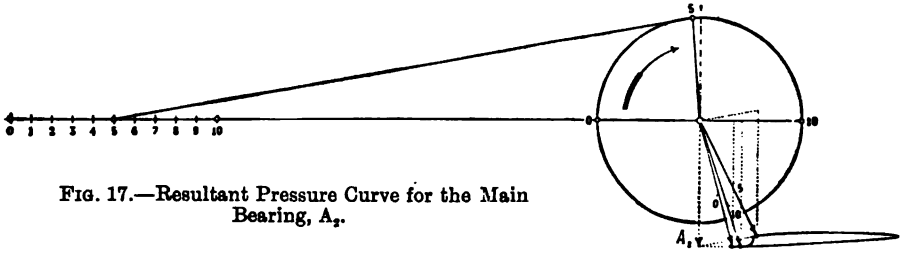


FIG. 17.—Resultant Pressure Curve for the Main Bearing, A_2.

From the above calculations the pressures on the bearings can be determined for other types of engine.

(b) Forces acting on the Cross Head, Crank Pin, etc.

Under this heading we will deal with the loads on the piston transmitting tension or compression loads on the cross head, crank pin, etc., through the agency of the piston rod, connecting rod, etc. Since no engine has yet been built with a piston load exceeding 660,000 lbs.—

we will take a maximum piston load $\overline{\overline{<}}$ 660,000 lbs. . . . . (8)

We must again make use of the steam engine for the sake of comparison, because sufficiently numerous tests have been made upon it to enable us to determine the loads of the moving parts.

The numerical examples given in Tables II. to V. are based upon the following values :—

The friction of the piston is neglected.

Admission pressure $p_e = 0{\cdot}9$ atm. absolute $(1 - p_e) = 0{\cdot}1$ atm. (= 1·5 lbs. per sq. inch).

Exhaust ,, $p_a = 1{\cdot}1$,, ,, $(p_a + 1) = 0{\cdot}1$,, (= 1·5 lbs. ,, ,,).

$(1 - p_e) \times A \sim 1{\cdot}5 \times A$. Energy required in lbs. per sq. inch to draw in the charge of gas and air.

$(p_a - 1) \times A \sim 1{\cdot}5 \times A$. Energy required in lbs. per sq. inch to expel the charge of gas and air.

$1{\cdot}5 \times C \times A$. Energy required in lbs. to overcome the compression pressure, when the piston is halfway through its stroke.

$0{\cdot}25\, p \times A = 81 \times A$. Pressure on the piston when halfway through the stroke, in lbs. per sq. inch.

Compression pressure C = 120 lbs. per sq. inch *above atmosphere* (gas engines).

Back pressure C = 2·9 lbs. ,, ,, ,, (condensing steam engines).

Explosion pressure $p \sim 330$ lbs. ,, ,, ,, (23 atms.).

w = exhaust pressure in lbs. per sq. inch above atmosphere.

r = radius of the crank in feet.

$A = \frac{\pi}{4} D^2$ = sectional area of cylinder in sq. inches.

p_a = back pressure of exhaust steam in lbs. per sq. inch (*absolute*) $(1 - p_a) = 10{\cdot}3$ lbs. per sq. inch. Table V.

W = weight of reciprocating parts, in lbs.

$\frac{W}{g} = \frac{W}{32}$ = mass of the reciprocating parts.

$\frac{n^2 \times r}{78}$ acceleration on *back* centre, $\frac{n^2 \times r}{110}$ acceleration on *front* centre in ft. per second^2.

⊙ = *change of pressure*; in this crank position the pressure always falls upon a different side of the brass.

○ = release of the rods (pressure taken off). Here the brass remains on the same side as before, but has no pressure on it, because there is no change in the tensile and compressive stress upon the rods.

By *change of pressure* in the rods we mean the reversal of the direction in which the pressure of the load is exerted in the bearings (compare p. 63).

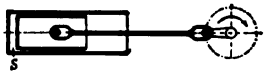
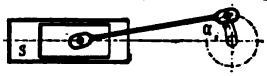
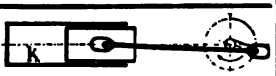
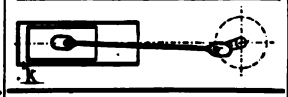
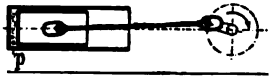
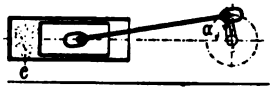
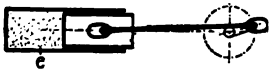
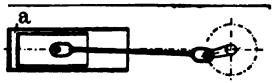
In the *motor strokes* (explosion and expansion), the part of the cylinder in which expansion takes place is indicated by dots.

The formulas for calculating the forces acting on the crank pin in the last column but one of the tables is very simple. For instance, Table IV., line 1. Pressure on the piston at the dead point, in lbs. per sq. inch. The whole of this pressure is not transferred to the piston rod; part of it, viz. acceleration × weight of the moving parts, or $\frac{W}{32} \times \frac{n^2 \times r}{78}$ in lbs., commonly called the inertia of the reciprocating parts, is employed to set the mass of the piston and rod in motion. The actual pressure on the piston rod is only the difference between the two values, viz. $p \times A - \frac{W}{32} \times \frac{n^2 \times r}{78}$.

Variation and Magnitude of Forces acting on the Crank Pin.

Table II.—Single-cylinder Four-cycle Engine, E 4.

(For meaning of the signs and letters, see pp. 59 and 63.)

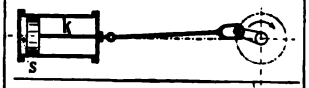
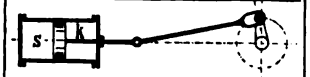
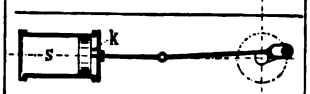
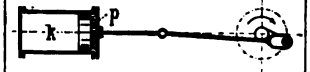
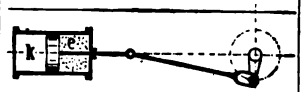
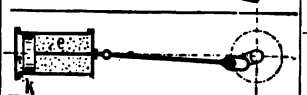
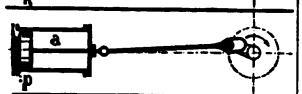
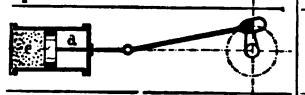
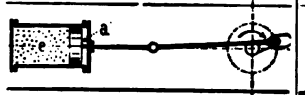
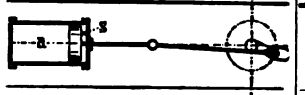
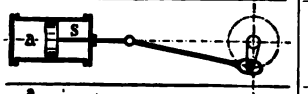
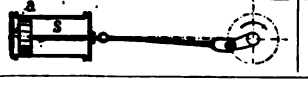
Strokes.	Successive positions of the piston during one working cycle, two revolutions.	Tension or compression in the rods.	Load (minus the inertia of the reciprocating parts) in lbs.	Examples.* 25 HP D = 12 ins. S = 16 ,, n = 210 ,,	600 HP. D = 47 ins. S = 51 ,, n = 90 ,,
s = admission		Tension	$=\frac{W}{32}\times\frac{n^2r}{78}+(1-p_s)\times A$	6,600 lbs. (58·3 lbs.)	78,500 lbs. (45 lbs.)
		⊙	If $a \cong 80°$. Change of pressure in the rods. See diagram, Table VI.		
		Compression	$=\frac{W}{32}\times\frac{n^2r}{110}-(1-p_s)\times A$	4,420 lbs. (39 lbs.)	51,500 lbs. (30 lbs.)
k = compression		Compression	$=\frac{W}{32}\times\frac{n^2r}{110}-(1-p_s)\times A$	4,420 lbs. (39 lbs.)	51,500 lbs. (30 lbs.)
		Compression	$0{\cdot}1C\times A$	1,230 lbs. (10·9 lbs.)	19,900 lbs. (11·5 lbs.)
		Compression	$C\times A-\frac{W}{32}\times\frac{n^2r}{78}$	5,900 lbs. (52·2 lbs.)	123,000 lbs. (71 lbs.)
p = explosion e = expansion		Compression	$p\times A-\frac{W}{32}\times\frac{n^2r}{78}$	29,000 lbs. (256 lbs.)	495,000 lbs. (286 lbs.)
		Compression	$0{\cdot}25p\times A$	8,800 lbs. (78 lbs.)	143,000 lbs. (82·5 lbs.)
		Compression	$w\times A-\frac{W}{32}\times\frac{n^2r}{110}$	6,230 lbs. (55 lbs.)	79,000 lbs. (45·5 lbs.)
a = exhaust		Compression	$(p_a-1)\times A+\frac{W}{32}\times\frac{n^2r}{110}$	4,730 lbs. (42 lbs.)	56,300 lbs. (32·5 lbs.)
		⊙	If $a = \sim 100°$. Change of pressure in the rods. See diagram, Table VI.		
		Tension	$\frac{W}{32}\times\frac{n^2r}{78}-(p_a-1)\times A$	6,300 lbs. (55·7 lbs.)	73,500 lbs. (42·4 lbs.)

* The figures within brackets give the pressure in lbs. per square inch of piston area.

Variation and Magnitude of Forces acting on the Crank Pin.

Table III.—Double-acting Four-cycle Engine, D 4.

(*For meaning of the signs and letters, see pp.* 59 *and* 63.)

Strokes.	Successive positions of the piston during one working cycle, two revolutions.	Tension or compression in the rods.	Load (minus the inertia of the reciprocating parts) exerted on both faces of the piston in lbs.	EXAMPLE.* 600 HP. D = 33¼ inches S = 51 ,, n = 90 ,,
s = admission k = compression		Tension	$\frac{W}{32} \times \frac{n^2 r}{78}$	50,600 lbs. (57·5 lbs.)
		Tension	$0{\cdot}1C \times A + (1 - p_s)A$	11,200 lbs. (12·7 lbs.)
		○	Release of pressure on rods. Table VI.	See diagram,
		Tension	$C \times A - \frac{W}{32} \times \frac{n^2 r}{110}$	63,800 lbs. (72·5 lbs.)
k = compression p = explosion e = expansion		Tension	$p \times A - \frac{W}{32} \times \frac{n^2 r}{110} + (1 - p_s)A$	**254,000** lbs. (288 lbs.)
		Tension	$25p \times A - 0{\cdot}1C \times A$	61,600 lbs. (70 lbs.)
		⊙	Change of pressure. See diagram, Table VI.	
		Compression	$C \times A - \frac{W}{32} \times \frac{n^2 r}{78} - w \times A$	36,000 lbs. (40·8 lbs.)
p = explosion a = exhaust e = expansion		Compression	$p \times A - \frac{W}{32} \times \frac{n^2 r}{79} - (p_a - 1) \times A$	236,000 lbs. (268 lbs.)
		Compression	$0{\cdot}25p \times A - (p_a - 1) \times A$	70,500 lbs. (80 lbs.)
		Compression	$w \times A + \frac{W}{32} \times \frac{n^2 r}{110} - (p_a - 1) \times A$	47,000 lbs. (53·4 lbs.)
a = exhaust s = admission		Compression	$(p_a - 1) \times A + \frac{W}{32} \times \frac{n^2 r}{110} - (1 - p_s) \times A$	36,200 lbs. (41 lbs.)
		⊙	Change of pressure. See diagram, Table VI.	
		Tension	$\frac{W}{32} \times \frac{n^2 r}{78} - (p_a - p_s) \times A$	48,200 lbs. (54·7 lbs.)

* See footnote, p. 60.

Variation and Magnitude of Forces acting on the Crank Pin.

Table IV.—Double-acting Two-cycle Engine, D 2.

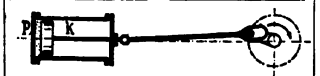
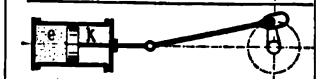
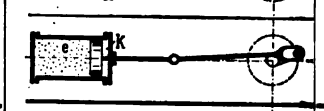
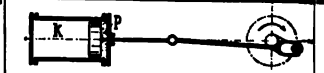
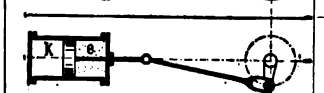
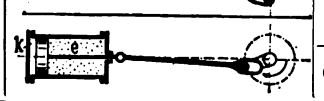
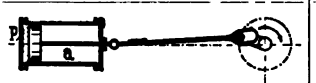
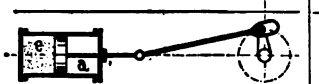
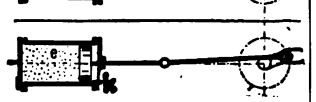
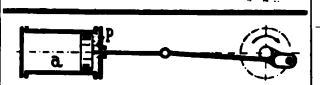
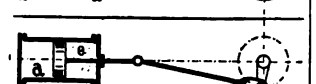
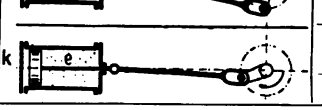
Strokes.	Successive positions of the piston during one working cycle, one revolution.	Tension or compression in the rods.	Load (minus the inertia of the reciprocating parts) exerted on both faces of the piston in lbs.	EXAMPLE.* 600 HP. D = 23¾ ins. S = 51 ,, n = 90 ,,
p = explosion e = expansion k = compression		Compression	$p \times A - \frac{W}{32} \times \frac{n^2 r}{78}$	**118,000** lbs. (266 lbs.)
		Compression	$0{\cdot}25p \times A - 0{\cdot}1C \times A$	30,800 lbs. (69·6 lbs.)
		⊙	Change of pressure. See diagram, Table VI.	
		Tension	$C \times A - \left(w \times A + \frac{W}{32} \times \frac{n^2 r}{110}\right)$	22,040 lbs. (49·7 lbs.)
p = explosion e = expansion k = compression		Tension	$p \times A - \frac{W}{32} \times \frac{n^2 r}{110}$	**125,000** lbs. (282 lbs.)
		Tension	$0{\cdot}25p \times A - 0{\cdot}1C \times A$	30,800 lbs. (69·6 lbs.)
		⊙	Change of pressure. See diagram, Table VI.	
		Compression	$C \times A - \left(w \times A + \frac{W}{32} \times \frac{n^2 r}{78}\right)$	13,200 lbs. (29·8 lbs.)

Table V.—Single-cylinder Condensing Steam Engine.

Strokes.	Successive positions of the piston during one working cycle, one revolution.	Tension or compression in the rods.	Load (minus the inertia of the reciprocating parts) exerted on both faces of the piston in lbs.	EXAMPLE.* 600 HP. D = 29¼ ins. S = 51 ,, n = 90 ,,
p = full steam pressure e = expansion a = exhaust k = compression		Compression	$p \times A + (1 - p_o) \times A - \frac{W}{32} \times \frac{n^2 r}{78}$	**66,200** lbs. (96·8 lbs.)
		Compression	$0{\cdot}3p \times A + (1 - p_o) \times A$	26,500 lbs. (38·7 lbs.)
		Compression	$(w \times A - C \times A) + \frac{W}{32} \times \frac{n^2 r}{110}$	16,700 lbs. (24·5 lbs.)
		⊙	Change of pressure. See diagram, Table VI.	
p = full steam pressure e = expansion a = exhaust k = compression		Tension	$p \times A + (1 - p_o) \times A - \frac{W}{32} \times \frac{n^2 r}{110}$	**70,500** lbs. (103 lbs.)
		Tension	$0{\cdot}3p \times A + (1 - p_o) \times A$	26,400 lbs. (38·6 lbs.)
		Tension	$(w \times A - C \times A) + \frac{W}{32} \times \frac{n^2 r}{78}$	23,200 lbs. (34 lbs.)
		⊙	Change of pressure. See diagram, Table VI.	

* During two single strokes there are two changes of pressure. The figures within brackets give the pressure in lbs. per sq. inch of piston area.

To explain the expression "*change of pressure,*" the diagrams in Tables II. to V. must be studied, to see which side of the bearings comes under pressure. In these diagrams

when the pressure falls on the back brass of the bearings, it is called compression in the rods;

when there is no pressure on either side of the bearings, it is called change of pressure, or release of pressure on the rods;

when the pressure falls on the front brass, it is called tension in the rods.

In Tables II. to V. the load on the piston (minus the inertia of the reciprocating parts) is calculated for given dimensions of engine. If these pressures are plotted vertically for the different types, we get the diagrams shown at Table VI.

All the engines are assumed to have the same length of stroke, to develop the same power, and to run at the same number of revolutions, and therefore at the same piston speeds.

The four engines, diagrams of which are given, are—

1. Single-acting four-cycle.
2. Double-acting „
3. „ „ two-cycle.
4. Steam engine.

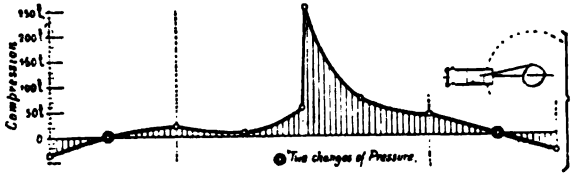
All with one cylinder only.

Table VI.—Comparison of the Load on the Piston (less Inertia) of a Gas Engine and a Steam Engine, each having one Motor Cylinder.

HP about 600. Stroke = 51 inches. $n = 90$. c = piston speed = 765 feet per minute.

W = weight of reciprocating parts in lbs.

$\frac{W}{HP}$ = weight of reciprocating parts per HP in lbs.



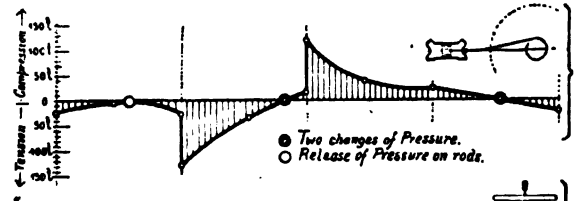
Single-acting Four-cycle Engine (E 4, one cyl.).

$D = 47\frac{1}{4}$ inches; $\frac{p_m}{4} = \frac{82{\cdot}5}{4} = 20{\cdot}6$ lbs.

$W = 11{,}200$ lbs.; $\frac{W}{HP} = \frac{11{,}200}{600} = 18{\cdot}7$ lbs.

inertia (out stroke) $q_2 = 30$ „

„ (in „) $q_1 = 43$ „



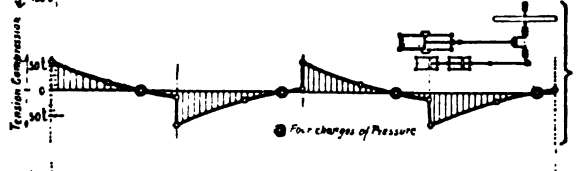
Double-acting Four-cycle Engine (D 4, one cyl.).

$D = 33\frac{1}{2}$ inches; $\frac{p_m}{2} = \frac{82{\cdot}5}{2} = 41{\cdot}2$ lbs.

$W = 7500$ lbs.; $\frac{W}{HP} = \frac{7500}{600} = 12{\cdot}5$ lbs.

inertia (out stroke) $= q_2 = 41{\cdot}2$ „

„ (in „) $= q_1 = 58$ „



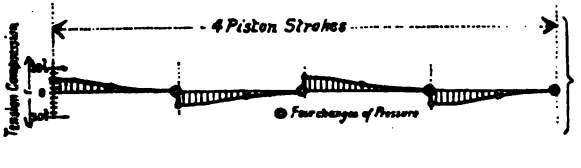
Double-acting Two-cycle Engine (*Koerting*) (D 2, one cyl.)

$D = 23\frac{3}{4}$ inches; $p_1 = p_m$ 82·5 lbs.

$W = 3750$ lbs.; $\frac{W}{HP} = \frac{3750}{600} = 6{\cdot}25$ lbs.

inertia (out stroke) $q_2 = 41{\cdot}2$ „

„ (in „) $q_1 = 58$ „



Steam Engine (*Condensing*).

$D = 29\frac{1}{2}$ inches; $p_1 = p_m = 47$ lbs.

$W = 3300$ lbs.; $\frac{W}{HP} = \frac{3300}{600} = 5{\cdot}5$ lbs.

inertia (out stroke) $q_2 = 23$ lbs.

„ (in „) $q_1 = 33$ „

Although the above values have been determined for an engine developing 600 HP in one cylinder, they may also be used for smaller powers, as the inertia of the reciprocating parts (with a piston of the usual dimensions per sectional area) does not vary much, whatever the power developed. The diagram in Table VI. for a 600 HP single-acting engine can also be plotted for the 25 HP engine in Table II. Table VII., giving the comparative values, will not, as we have said, reveal any marked difference between them.

Example from Table VI. In a double-acting four-cycle engine (Deutz), the load on the piston, minus the inertia of the reciprocating parts, is three times, and in a double-acting, two-cycle engine one and a half times as great as in a steam engine, although the latter and a two-cycle gas engine have more or less the same working cycle. The apparent superiority of engine D 2 over engine D 4 is lost, because of the necessity for separate charging pumps in the former.

Table VII.—Comparison of the Load on the Piston (minus the Inertia of the Reciprocating Parts) in a Four-cycle Gas Engine and a Steam Engine.

(*The figures show how many times greater the piston loads (minus the inertia of the reciprocating parts) are in gas engines than in steam engines of the same power.*)

Type.			Steam Engine.			
			Single cylinder.	Tandem.	Compound.	Triple.
Four-cycle.	Single-acting.	(a)	6	6	12	24
		(b)	3	3	6	12
		(c)				
		(d)				
		(e)	1·5	1·5	3	6
Gas engine.	Double-acting.	(f)	3	3	6	12
		(g)	1·5	1·5	3	6
		(h)				
		(i)	0·75	0·75	1·5	3

TABLE VII.—*Example* 1.—The first line in this table shows that, for the same power, the load on the piston (minus the inertia of the reciprocating parts) in a single-acting, four-cycle gas engine (type *a*) is ***six times*** greater than in a single-cylinder steam engine, and twenty-four times greater than in a triple engine.

Example 2.—In a four-cylinder, single-acting, four-cycle engine (type *e*) the piston load (minus the inertia of the reciprocating parts) is three times greater than in a compound two-cylinder steam engine.

Example 3.—In double-acting, four-cycle gas engines the pressure is rather better distributed, but even here the steam engine is superior. In the double-acting, four-cycle, four-cylinder engine (type *i*), the piston load is still three times greater than in the three-cylinder steam engine.

According to equation 8 (p. 59) the maximum allowable pressure on the rods is taken at 660,000 lbs., and according to the above table this pressure is developed much sooner in gas than in steam engines.

With a pressure on the rods (piston load minus inertia of reciprocating parts) of **600,000** lbs. we get (from Table VII.)—

Table VIII.—Maximum Horse-Power at Normal Load.

In a four-cylinder, single-acting, four-cycle gas engine (type *e*):	About 600 HP per cylinder.	Total	2,400 HP.
,, ,, double-acting, four-cycle gas engine (type *i*):	About 1000 HP per cylinder.	,,	4,000 HP.
,, three-cylinder steam engine (triple expansion):	About 3600 HP per cylinder.	,,	10,800 HP.

NOTE.—The various parts of a gas engine have to be designed strong enough to resist the ignition pressure, which of course influences the cost of the engine, but the power of the engine is determined by the mean pressure.

CHAPTER VII.

The Parts of the Engine.

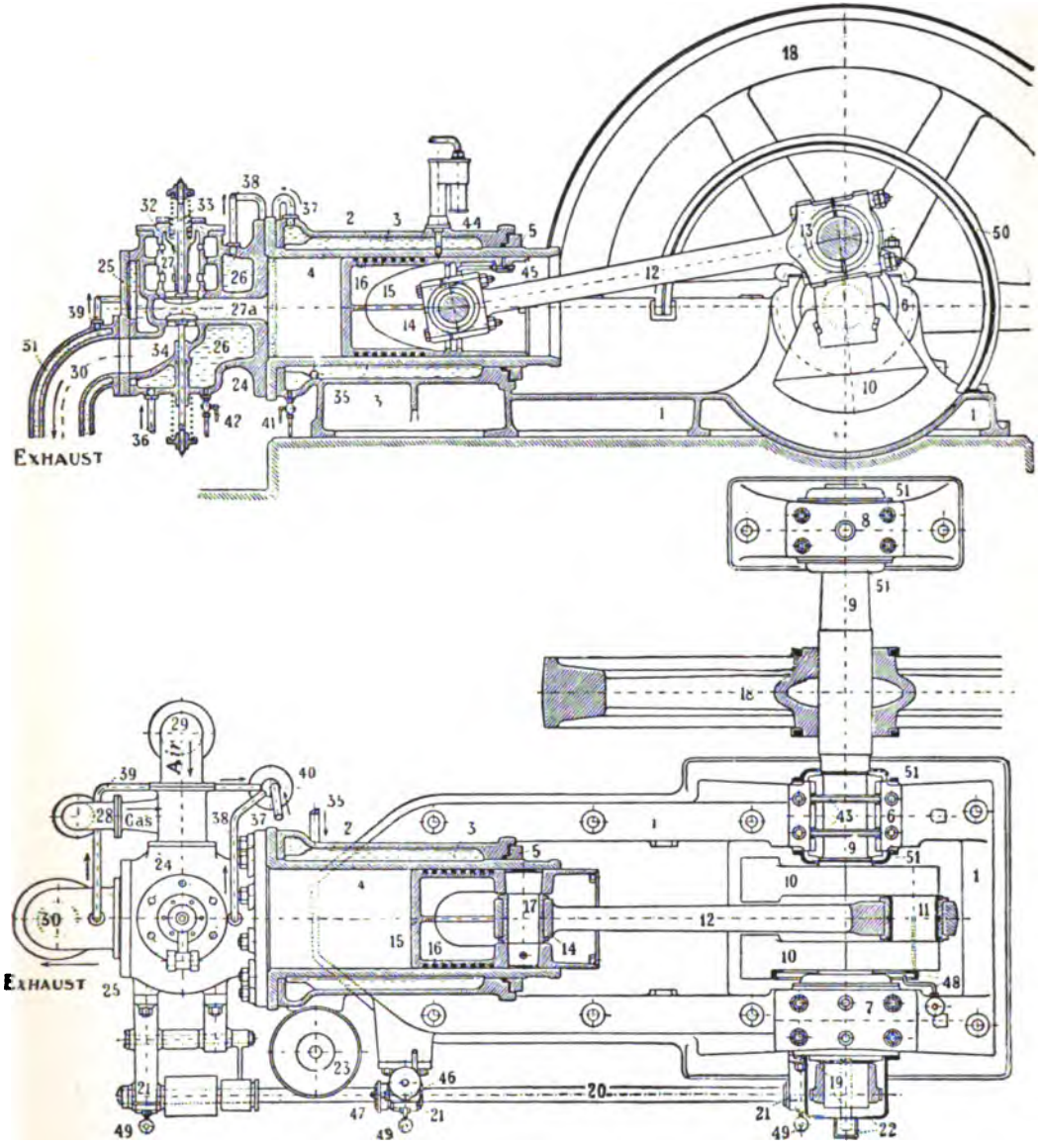
THE beginner should make himself familiar with the various parts of a gas engine, and with their names. The following definitions will be found useful to begin with:—

Names of the Various Parts of an Ordinary Four-cycle Gas Engine.

Figs. 1, 2 (next page).

No.	
	Frame.
1	Frame or bed.
2	Cylinder water jacket.
3	Water space.
4	Cylinder liner.
5	Liner joint ring (to water jacket).
	Bearings.
6	Main bearing. Flywheel side.
7	,, ,, Side-shaft (outer) side.
8	Out-end bearing.
	Shaft.
9	Crank shaft.
10	Balance weights.
11	Crank pin.
	Connecting Rod.
12	Connecting rod.
13	Big end of connecting rod (crank end).
14	Small ,, ,, (piston end).
	Piston.
15	Piston.
16	Piston ring.
17	Piston pin (cross-head).
	Flywheel.
18	Flywheel.
	Gearing.
19	Crank worm (driving the valve gear).
20	Side shaft.
21	Side-shaft bearing.
22	Guard for gearing.
	Governor.
23	Governor.

No.	
	Valve Gear.
24	Breech end.
25	Breech-end cover.
26	Water jacket to valves.
27	Mixing chamber for gas and air.
27*a*	Combustion chamber (dotted).
28	Gas pipe.
29	Air pipe.
30	Exhaust pipe.
31	Water jacket to exhaust.
	Valves.
32	Admission valve block.
33	,, valve.
34	Exhaust valve.
	Cooling Water Pipes.
35	Cooling water inlet pipe to cylinder.
36	,, ,, ,, breech end.
37	,, outlet from cylinder.
38	,, ,, ,, breech end.
39	,, ,, ,, exhaust.
40	Main outlet pipe for cooling water.
	Drain Cocks.
41	Drain cock from cylinder water jacket.
42	,, ,, breech end jacket.
	Lubrication.
43	Oiling rings for main bearings.
44	Piston lubricator.
45	Piston-pin lubricator.
46	Oil pump for above.
47	,, driving gear.
48	Centrifugal lubricator for crank pin.
49	Lubricator for side-shaft bearing.
50	Crank splasher guard.
51	Oil guard on main bearings.



FIGS. 1, 2.—Diameter of cylinder, 400 mm. (15¾″); Stroke, 550 mm. (21⅝″); Scale, 1 : 30.
See Table I., p. 24.

Calculation and Construction of the Parts of an Engine.

In studying the construction of the different parts of a gas engine, we assume that the reader has an accurate knowledge of steam-engine construction. To retain, when constructing a large gas engine, the same design of the parts as that used for a small engine, and merely to increase the scale of dimensions, is a serious error. We must utilize the experience accumulated during two hundred years in building steam engines, especially with regard to the strength of the working parts, and the effects of variations in the temperatures. These are shown in Table I. for steam engines and for internal combustion engines.

Table I.—Pressures and Temperatures (Figs. 3–5).

	Pressures in lbs. per sq. inch above atmosphere.					Temperatures in degrees Centigrade.				
	Initial ignition or explosion pressure p	Terminal pressure w	Back pressure p_0 for steam engines p_s for gas engines absolute.	Compression pressure C	Mean pressure p_m	At ignition or explosion t_p	After expansion (termnl.) t_w	At exhaust t_a	At admission t_s	During compression t_c
1. Steam engine, 1 cyl. condensing	100–200	8·5*–21·5	4·3–15·6	0–171	30–71·2	160–300	90–120	65–105	—	100–200
2. *Lighting gas*	175–320	21·5–42·7	11·1–15·6	42·7–114	50–93	1250–1750	500–800	300–500	50–100	300–500
3. *Press and suction producer* gas	242–430	,,	,,	114–214	43–78	,,	,,	,,	,,	,,
4. *Blast furnace* gas	215–355	,,	,,	171–228	43–78	,,	,,	,,	,,	,,
5. Benzine	170–285	,,	,,	43–71	57–85	,,	,,	,,	,,	,,
6. Oil	142–256	,,	,,	43–71	43–57	,,	,,	250–400	,,	,,
7. Diesel (oil)	455–500	,,	,,	455–500	85–114	1200–1600	,,	,,	,,	500–600
8. Banki benzine	570	,,	,,	171–228	100	,,	,,	100–200	,,	200–300

* Lbs. per sq. inch absolute.

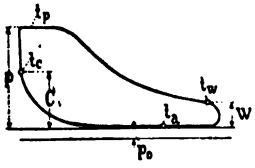


Fig. 3.—Steam Engine, exhausting to atmosphere.

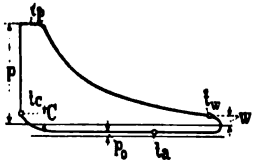


FIG. 4.—Steam Engine, condensing.

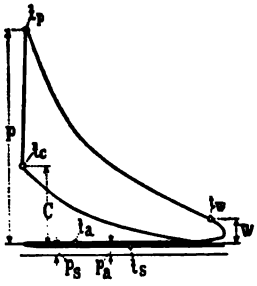


FIG. 5.—Four-cycle Gas Engine.

Section I. The Crank Shaft.

It is the universal practice of engineers to build solid forged cranked shafts (not over hung crank pins).

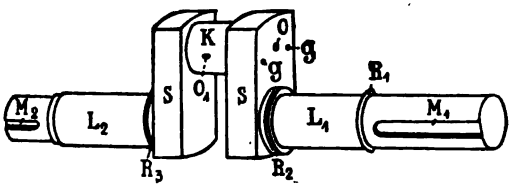


FIG. 6. — Design for a Small Engine with Light Flywheel (2 Bearings, 1 Flywheel).

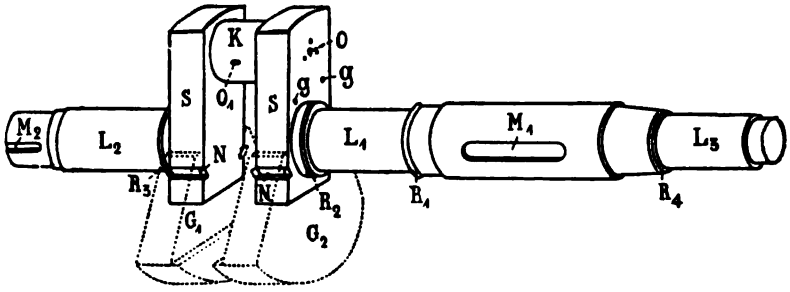


FIG. 7.—Crank Shaft (3 Bearings, 1 Flywheel).

L_1 Journal for main bearing, flywheel side. L_2 Journal for main bearing, side-shaft side.
L_3 ,, ,, out-end bearing.
SS Crank webs or cheeks.
R_1 to R_4 Oil fender rings, to prevent the oil splashing, when the bearings are lubricated by rings (see p. 75).
OO_1 Oil holes in crank pin K; *gg*, tapped holes for set screws, to screw the centrifugal lubricator ring to the crank web.
M_1 Sunk or feather key for flywheel.
M_2 ,, ,, ,, crank worm.
NN Keys for securing balance weights to crank webs (see p. 79).
G_1G_2 Balance weights (to balance the rotating parts) (see p. 79).

In *designing the crank shaft* the following points must be considered:—

1. Are there to be two or three bearings, one or two flywheels?
2. In choosing the dimensions, careful note must be taken of engines already built.
3. Provision must be made that the engine when built can be run with a cylinder of larger diameter.
4. Oil must not be thrown out from the main bearings.
5. Oil feed to the crank pin must be automatic and certain in action.
6. Driving the valve gear. If the valves are not worked from the side shaft, some arrangement for driving the governor will be necessary.
7. Balancing the reciprocating and rotating parts.
8. Choice of suitable materials for the crank shaft, etc.
9. The strength of the crank pin, journals, and crank webs should be tested. The pressure on the bearings, as well as the friction, and the factor of safety, should be determined. Occasions may arise when the dimensions of these parts have to be altered.
10. For hand-turning gear, or a pulley and belt for compressing air, see chapter on "Starting."

We will now consider these different points in detail.

1. Numbers of Bearings and Flywheels.

As a general rule, the following are the six most usual methods of arranging a given number of bearings, flywheels, and pulleys (Figs. 8 to 13).

FIG. 8.—2 Bearings, 1 Driving Pulley Flywheel.

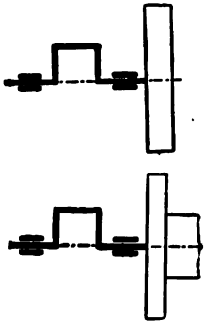


FIG. 9.—2 Bearings, 2 Driving Pulley Flywheels.

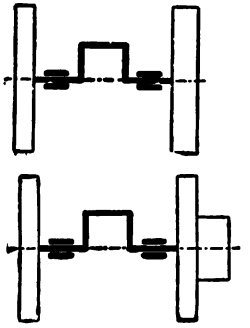


FIG. 10.—3 Bearings, 1 Driving Pulley Flywheel.

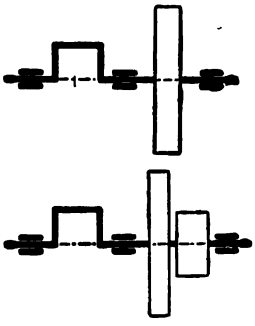


FIG. 11.—2 Bearings, 1 Flywheel, 1 Driving Pulley.

FIG. 12.—2 Bearings, 2 Flywheels, 1 Driving Pulley.

FIG. 13.—3 Bearings, 1 Flywheel, 1 Driving Pulley.

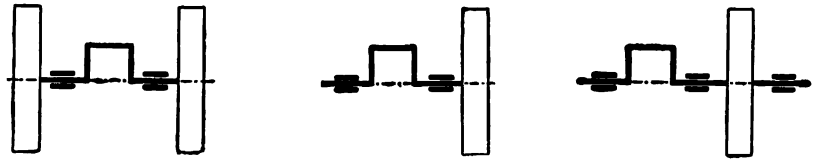
From these we will select as *most suitable* for study :—

FIG. 14.—2 Bearings, 1 Flywheel, 1 Driving Pulley Flywheel.

FIG. 15.—2 Bearings, 1 Driving Pulley Flywheel.

FIG. 16.—3 Bearings, 1 Driving Pulley Flywheel.

Connecting rods, etc., difficult of access.



Up to 20 P for power in factories and workshops, but with a heavy flywheel a third bearing is necessary.

Above 20 P for industrial purposes, and *in general for electric lighting*, and therefore a heavy flywheel is required.

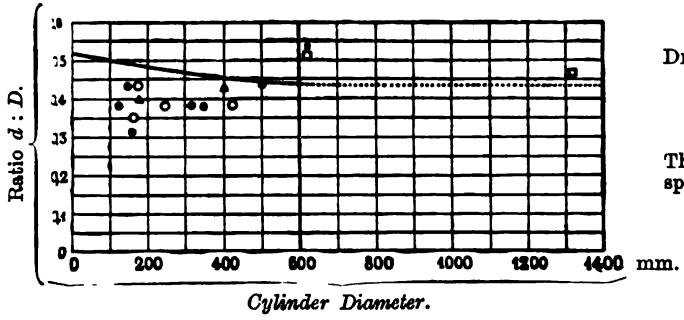
Engines in factories, workshops, mines, etc., generally have a separate pulley, as shown at Figs. 11 and 13, otherwise the speed of the drive would be too great. In engines driving dynamos the flywheel always forms the driving pulley (compare "Flywheels," pp. 164 *et seq.*).

2. Dimensions of the Crank Shaft.

The most important of these, and first to be considered, are—

Diameter and length of the crank pin.
" " " of the main bearing journals.
Distance L between the crank-shaft bearings.

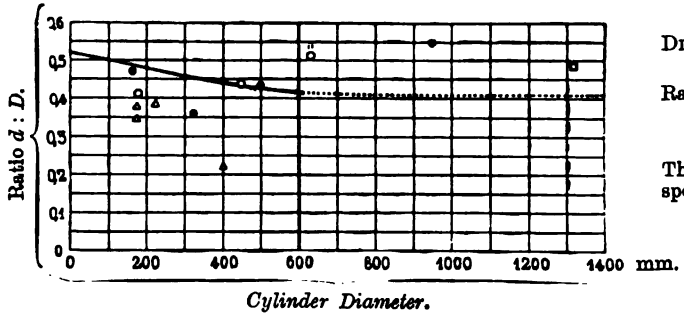
These dimensions are generally directly proportioned to the diameter of the cylinder. In the following diagrams the ratios for some engines are plotted.



DIAGRAM, FIG. 17.—Horizontal Engines.

Ratio $\frac{\text{crank-pin diameter}}{\text{cylinder diameter}}$

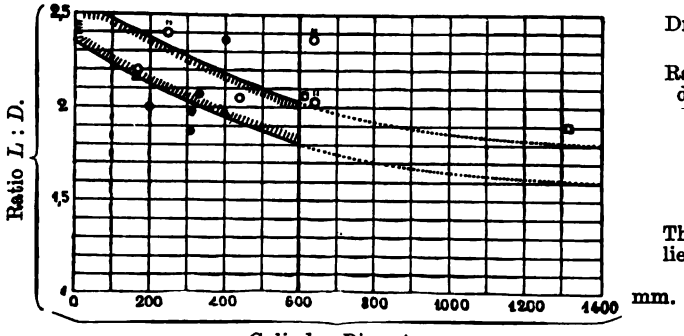
The thick line ——— corresponds to the dimensions of engines at p. 73.



DIAGRAM, FIG. 18.—Horizontal Engines.

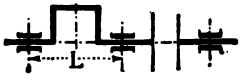
Ratio $\frac{\text{diameters of main bearings}}{\text{cylinder diameter}}$

The thick line ——— corresponds to the dimensions of engines at p. 73.



DIAGRAM, FIG. 19.—Horizontal Engines.

Ratio $\frac{\text{distance between the bearings}}{\text{cylinder diameter}}$

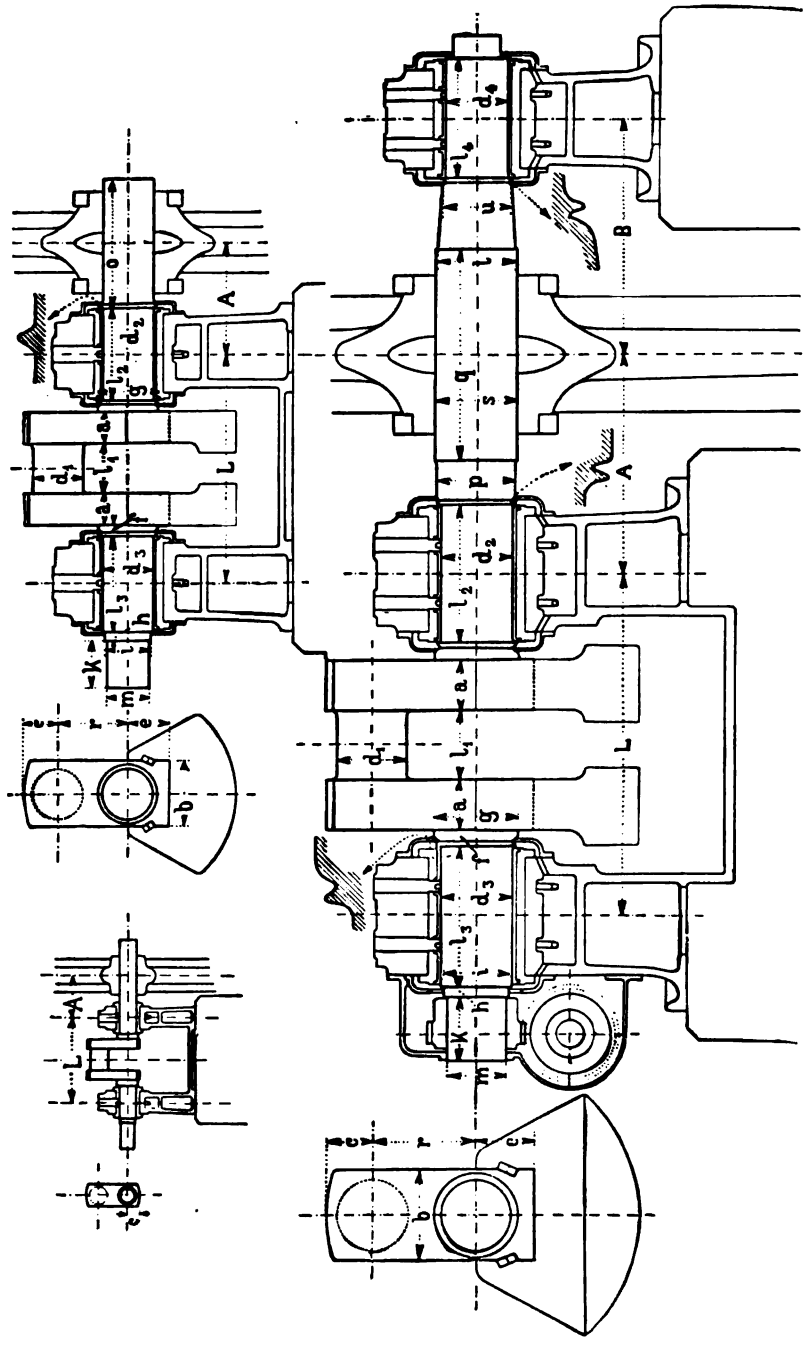


The dimensions of engines in p. 73 lie within the shaded areas ///////

Engines by the same makers are, in the above diagrams, denoted by the same sign. Thus O signifies Deutz, □ Nürnberg E_4, △ Swiderski, Ö Koerting D_2 (two-cycle).

In engines of over 14 inches cylinder diameter the crank pin is made larger in diameter than the main bearing journals. In engines up to 20 inches the diameter of the crank pin may be = diameter of the main bearing journal.

The *standard types of crank shafts* given at pp. 72, 73, represent actual engines, and are based on the calculations at pp. 83–94.



FIGS. 20, 21.—Crank Shaft for Engine. Diam. = 100 mm. (4"). Stroke = 160 mm. (6¼"). Scale 1 in 20.

FIGS. 22, 23.—Crank Shaft for Engine with Light Flywheel. Diam. = 300 mm. (11¾"). Stroke = 400 mm. (15¾"). Scale 1 in 20.

FIGS. 24, 25.—Crank Shaft for Engine. Diam. = 480 mm. (18⅞"). Stroke = 600 mm. (25⅝"). Scale 1 in 20.

Table II.—Crank Shaft.

(Dimensions in millimetres.)

							Main	Main	Out-end						Extension for worm										
ins.	ins.	ins.	ins.	ins.	ins.	ins.	ins.	ins.		ins.	ins.	ins.	ins.	ins.	ins.	ins.	ins.	ins.	ins.		ins.	ins.	ins.	ins.	ins.	ins.
5	3 1/4	7 7/8	4 1/2	5 3/8	1 5/8	2	1 5/8	2 7/8	Same as main bearing on flywheel side.	1 9/16	2 3/4	1	2 3/16	1 1/8	—	9/16	2	3/8	1 1/2	See dimensions under side-shaft wheels.	1 1/4	6	2	5	2 3/16	2	2
6 1/4	4	9 1/2	4 3/4	5 1/2	2	2 1/4	2	3 1/2		1 3/4	3 1/4	1 3/16	2 9/16	1 1/4	—	5/8	2 3/8	7/16	1 3/4		1 9/16	6 1/8	2 3/8	5 1/2	2 9/16	2 3/8	2 3/16
7 1/2	5	10 7/8	5 1/2	6	2 3/8	2 9/16	2 3/8	4 1/4		2	3 1/2	1 7/16	3 1/16	1 9/16	—	9/16	2 7/8	1/2	2 3/16		2	6 1/4	2 1/4	6 1/2	3	2 1/4	2 3/8
8 3/4	5 3/4	12 5/8	6 1/4	6 1/2	2 3/4	3 3/16	2 3/4	5		2 3/8	4 1/4	1 5/8	3 9/16	1 3/4	—	5/8	3 3/8	1/2	2 9/16		2 3/8	7 1/4	3 1/8	7 7/8	3 9/16	3 3/16	2 3/4
10 5/8	7 1/2	17	8 1/2	8 1/4	3 1/4	4 1/8	3 1/4	6 5/8		2 3/4	5	2 1/4	4 7/8	2 7/16	—	3/4	4 1/2	9/16	3 1/2		3 1/8	9	4 3/8	9 7/8	4 1/4	4 3/8	3 9/16
12 1/4	8 3/4	19 5/8	9 7/8	10 1/4	4 5/16	4 3/4	4 3/8	7 7/8		3 1/4	6 5/8	2 5/8	5 3/4	2 7/8	4	1	5 3/16	5/8	4		3 1/2	11 1/2	5	11 7/8	5 3/8	5	4 3/8
13 3/4	10 1/4	22	11	12 1/4	5	5	5	9		4 3/8	7 7/8	3 1/16	6 3/8	3 1/4	4 3/8	1	6	11/16	4 1/2		4 1/8	12 5/8	5 1/2	13 3/4	6	5 1/2	5
15 3/4	11 7/8	25 5/8	13	13 3/4	5 3/4	5 3/4	5 3/4	10 1/2		5	9	3 9/16	7 7/16	3 3/4	4 3/4	1 3/16	6 7/8	3/4	5 5/16		4 3/4	15	6 1/4	15 3/4	6 7/8	6 1/4	5 3/4
17 3/4	13	27 3/4	17 3/4	19 1/4	6 1/8	6 1/8	6 1/8	11 1/2		5 3/4	10 1/2	4	7 7/8	4	5 3/16	1 3/16	7 1/4	7/8	5 3/4		5 1/8	—	6 7/8	17 3/4	7 1/2	6 7/8	6 1/2
19 3/4	14 1/4	29 1/2	19 5/8	31 5/8 21 3/8	6 3/4	6 3/4	6 1/2	12		6 1/8	11 1/2	4 3/16	8 1/2	4 1/4	5 5/8	1 5/16	7 5/8	1	6 1/8		5 1/2	—	7 1/4	21 7/8 19 3/4	7 7/8	7 1/2	6 7/8
21 5/8	15 3/4	32 5/8	21 5/8	35 1/8 23 5/8	7 1/4	7 1/4	7 1/16	13 3/8		6 1/2	12	4 3/4	9 1/4	4 5/8	6 1/8	1 5/16	8 1/2	1 3/16	6 11/16		6 1/8	—	7 7/8	35 1/2 21 5/8	8 1/2	7 7/8	7 1/4
23 5/8	19	38	24 5/8	29 26 5/8	8 1/2	8 1/2	7 7/8	15 1/2		7 1/8	13 3/8	5 3/4	10 7/16	5 1/8	6 9/16	1 5/16	9 7/16	1 3/8	7 1/4		6 5/8	—	8 5/8	39 23 5/8	9 1/2	8 5/8	7 7/8
27 1/2	23 5/8	46 1/4	29 1/2	42 1/2 32 7/8	10 1/4	10 1/4	9 5/8	18 7/8		7 7/8	15 1/2	7 1/8	12 9/16	6 1/4	7 1/2	1 1/2	11 5/8	1 9/16	8 7/8		8 1/4	—	10 1/2	42 1/2 25 3/8	11 1/2	10 1/2	9 1/16
				49 5/8																				47 1/4			

The dimensions of *h*, *i*, and *m* can be more accurately determined when we come to the side-shaft wheels.

The higher of the two values given for B and *q* in engines of more than 16 inches stroke developing power in factories, mines, etc., should be used, if a driving pulley can be fitted on the shaft next the flywheel (Fig. 13, p. 70). The lower values apply in engines in which the flywheel acts as the driving pulley, that is, the belt is on the rim of the flywheel.

* If there is a separate driving pulley (in power engines), *o* is either made correspondingly longer, or the driving pulley is bolted to the arms of the flywheel.

3. Dimensions of Shoulder between the Crank Web and Main Bearings.

In the standard types of engines given in the preceding table, the main brasses are not, as in Fig. 26, fitted close up to the crank web, but a shoulder is interposed, the diameter of which is not less than 1·2 *d*, and should be somewhat smaller than the width of the crank web (see Fig. 27). The dangerous point is where movement is transmitted from the crank web to the journals. If the same pattern of frame is used for different diameters of cylinder, the dimensions of *a* must be correspondingly increased or diminished. In certain cases the shoulder may be made very narrow (compare Fig. 30).

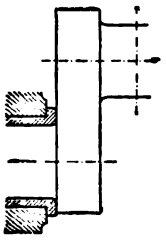


FIG. 26.

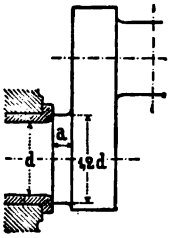


FIG. 27.

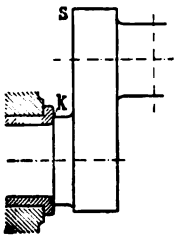


FIG. 28.

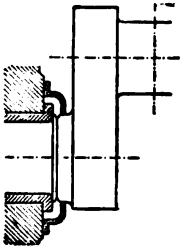


FIG. 29.

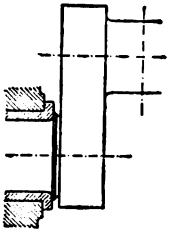


FIG. 30.

4. Oil Fender Ring.

The oil escaping from the bearings at *k* is thrown outwards and upwards towards *s* (Fig. 28) by centrifugal force, and might then be flung out from the crank web. This is a defect common to many gas engines, and is avoided by having the oil fender ring turned on the shaft (Fig. 29). The oil cannot then creep along to the crank web, but only to the outer periphery of the ring. If the circumferential velocity is small, it drops from here into the oil-guard ; if the speed is great, it is flung off.

When the shoulder is very narrow, the fender ring may be shaped as shown at Fig. 30.

The bottom of the groove *z* should have no sharp angles, as at Fig. 31, but should be rounded off, and be at least 10 per cent. larger in diameter than the crank shaft journal, as in Fig. 32. All sharp corners are liable to start a fracture.

Table III.—Dimensions of the Oil Fender Ring.
(Fig. 33.)

$d = 3\frac{1}{4}''$	$3\frac{1}{2}''-4\frac{3}{8}''$	$4\frac{3}{4}''-6''$	$6\frac{1}{4}''-7\frac{1}{2}''$	$8''-12''$
$a = \frac{5}{16}''$	$\frac{1}{4}''$	$\frac{5}{16}''$	$\frac{3}{8}''$	$\frac{1}{2}''$
$b = \frac{3}{8}''$	$\frac{1}{2}''$	$\frac{5}{8}''$	$\frac{3}{4}''$	$1''$

FIG. 31.

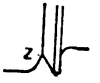


FIG. 32.

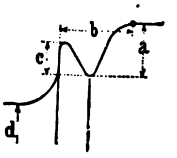


FIG. 33.

The oil which drips from the fender ring is caught in the guard shown at Fig. 29, and from thence escapes into the reservoir of the oil-ring bearing, as seen in the drawings at p. 105.

5. Crank Pin Lubrication.

For oiling the crank pin the so-called *centrifugal ring lubricator*, which has proved very satisfactory in high-speed steam engines with cranked shafts, is exclusively used.

The oiling ring R (Fig. 34) is secured to the crank web by set screws.

The *action* of the centrifugal-ring lubricator is as follows :—

The oil is fed from the sight-feed lubricator T through the small pipe *r* into the ring R, is driven by centrifugal force into the deep channel turned in the inside of this ring, and passes to the crank pin through the hole *z*.

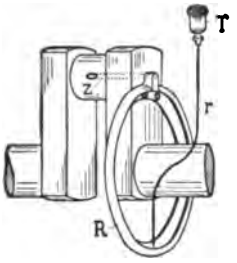


FIG. 34.—Oiling Ring.

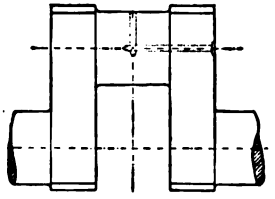
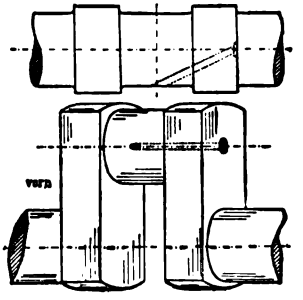


FIG. 35.—Wrong.



FIGS. 36, 37.—Right.

To allow the oil to pass freely out of the hole, the latter should not discharge parallel to the direction of the crank web, as at Fig. 35, because after compression and during explosion the big end brasses are pressed hard against the pin at this point. It is better to drill the oil hole at the side, towards the front when the crank is vertical, as shown in Figs. 36, 37, and preferably in an oblique direction.

Construction of the Centrifugal Ring Lubricator.

FIGS. 38, 39.

Smaller rings are made of gun metal, larger of cast iron. They may be either semicircular or *square* in section (Figs. 38, 39).

The lubricator ring is preferably made in two parts, as otherwise it is impossible to take it off after the engine has been erected.

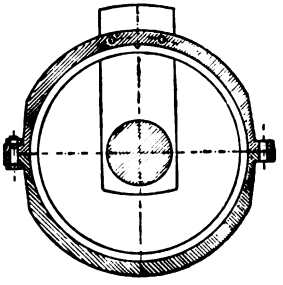


FIG. 40.—Lubricator Ring.

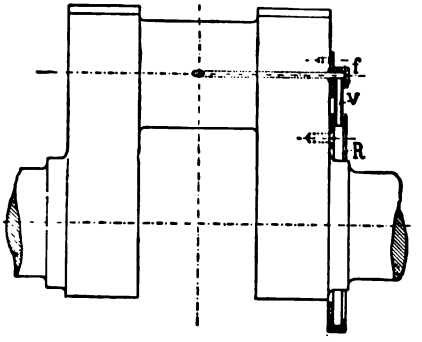
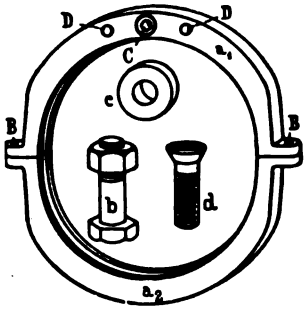


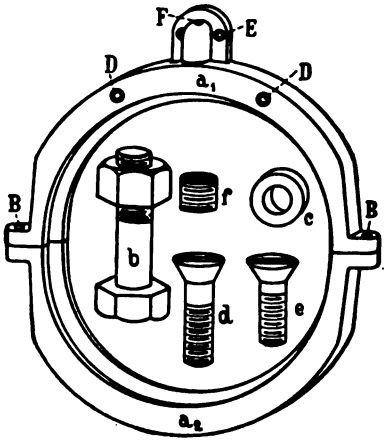
FIG. 41.—Lubricator Ring for Engines above 32″ Stroke.

To avoid too large a diameter of the lubricator ring R, making it liable to "wobble," a small pipe *v* with flange *f* is added; for engines above 10 inches throw it is cast on (Fig. 46), and above 16 inches throw screwed on to the ring R (Fig. 41). The ring must always, however, be large enough to allow ample clearance between the bearing, oil fender guard and oiling ring, that oil may be poured direct into the latter, if necessary (Fig. 41).

Parts of a Centrifugal Oiling for the Crank Pin.



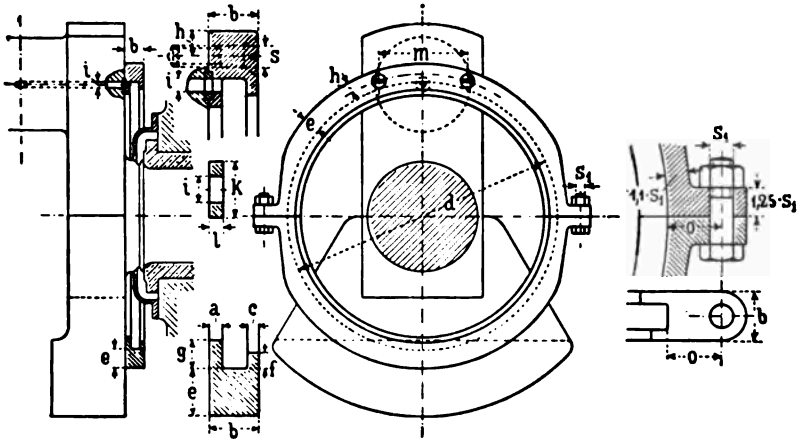
FIGS. 42–45.—For Engines up to 20″ Stroke.



FIGS. 46–51.—For Engines from 22″ to 32″ Stroke.

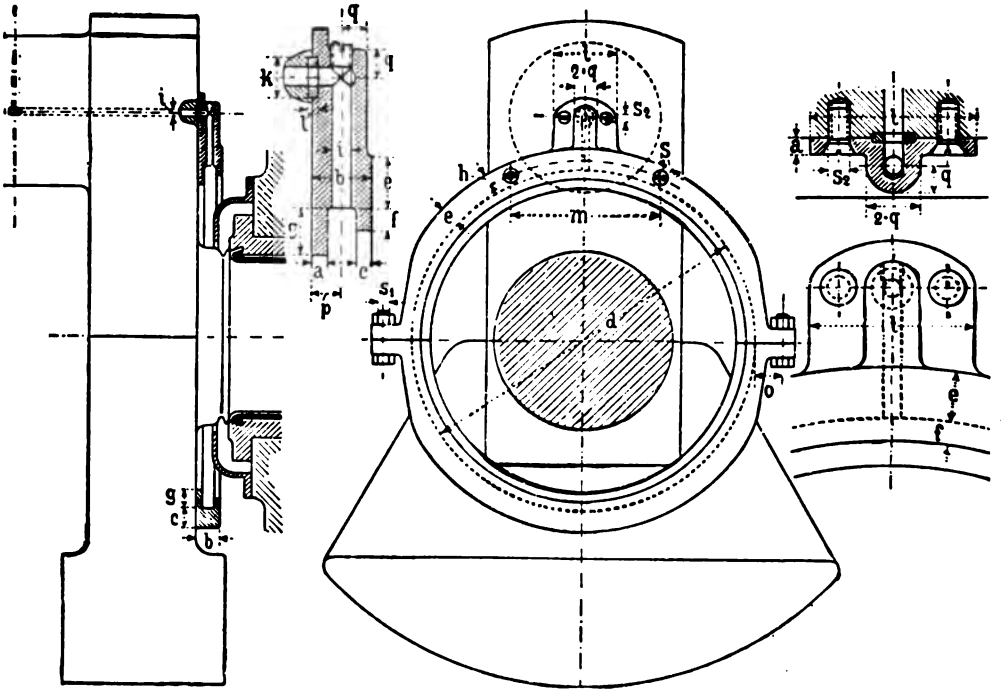
a_1, a_2 Top and bottom halves of ring (gun metal or cast iron).
b Bolts for holes B, to bolt the two halves of the ring together.
c Leather packing washer for oil channel C.
d Screws for holes D, to secure the oiling ring to the cheek of the crank.
e " " E " branch to the crank cheek.
f Nipple to close the oiling hole F in the branch E.

Standard Dimensions of the Centrifugal Oiling Ring.



FIGS. 52–57.—For Engines up to 20″ Stroke (see Table IV.).
(The drawing shows a centrifugal oiling ring for an engine 14″ stroke. Scale $\frac{1}{10}$.)

The following measurements are merely to serve as a guide in designing these parts. Deviations from them are naturally allowable.



FIGS. 58–62.—For Engines above 22″ Stroke.
(This drawing shows a centrifugal lubricator for an engine of 24″ stroke. Scale $\frac{1}{10}$.)

Table IV.—Centrifugal Lubricator for Crank Pin.
(Figs. 52–57 and 58–62.)

	Piston stroke.	*a*	*b*	*c*	*d*	*e*	*f*	*g*	*h*	*i*	*k*	*l*	*o*	*p*	*q*	*s*	s_1	s_2	*t*
Fig. 52	130 to 190	4	14	3	*d* = stroke + *i*	17	6	9	9	6	10	3	20	—	—	8	8	—	—
	220 ,, 310	5	18	4		21	8	12	11	6	14	4	24	—	—	10	10	—	—
	350 ,, 400	6	22	5		25	10	16	13	8	18	4	30	—	—	12	13	—	—
	450 ,, 500	8	28	6		27	12	20	14	8	24	4	33	—	—	13	16	—	—
Fig. 58	550	8	32	7	430	27	12	20	14	10	24	4	33	16	16	13	16	12	84
	600	9	35	7	460	27	13	20	14	10	26	5	33	18	17	13	16	12	86
	700	9	35	8	540	27	13	20	14	10	28	5	33	18	17	13	16	13	94

The dimension *m*, *i.e.* the distance between the centre of the screws fastening the ring to the crank web, must be determined when designing the parts.

Table IVa.—Centrifugal Lubricator for Crank Pin.

(Figs. 52–57 and 58–62.)

	Stroke.	*a*	*b*	*c*	*d*	*e*	*f*	*g*	*h*	*i*	*k*	*l*	*o*	*p*	*q*	*s*	s_1	s_2	*t*
	ins.	ins.	ins.	ins.	*d* = stroke + *i*.	ins.	ins.	ins.	ins.	ins.	ins.	ins.	ins.	ins.	ins.	ins.	ins.	ins.	ins.
Fig. 52	$5\frac{1}{2}$–$7\frac{1}{2}$	$\frac{3}{16}$	$\frac{9}{16}$	$\frac{1}{8}$		$1\frac{1}{16}$	$\frac{1}{4}$	$\frac{3}{8}$	$\frac{3}{8}$	$\frac{1}{4}$	$\frac{3}{8}$	$\frac{1}{8}$	$\frac{3}{4}$	—	—	$\frac{5}{16}$	$\frac{5}{16}$	—	—
	$8\frac{5}{8}$–$12\frac{1}{4}$	$\frac{3}{16}$	$\frac{11}{16}$	$\frac{3}{8}$		$\frac{7}{8}$	$\frac{5}{16}$	$\frac{5}{8}$	$\frac{5}{16}$	$\frac{1}{4}$	$\frac{9}{16}$	$\frac{5}{8}$	1	—	—	$\frac{3}{8}$	$\frac{3}{8}$	—	—
	$13\frac{3}{4}$–$15\frac{3}{4}$	$\frac{1}{4}$	$\frac{7}{8}$	$\frac{3}{16}$		1	$\frac{3}{8}$	$\frac{5}{8}$	$\frac{1}{4}$	$\frac{5}{16}$	$\frac{11}{16}$	$\frac{5}{32}$	$1\frac{3}{16}$	—	—	$\frac{7}{16}$	$\frac{1}{2}$	—	—
	$17\frac{3}{4}$–$19\frac{3}{4}$	$\frac{5}{16}$	$1\frac{1}{8}$	$\frac{1}{4}$		$1\frac{1}{8}$	$\frac{7}{16}$	$\frac{3}{4}$	$\frac{3}{16}$	$\frac{5}{16}$	1	$\frac{3}{32}$	$1\frac{5}{16}$	—	—	$\frac{1}{2}$	$\frac{3}{8}$	—	—
Fig. 58	$21\frac{3}{4}$	$\frac{5}{16}$	$1\frac{1}{4}$	$\frac{1}{4}$	17	$1\frac{1}{16}$	$\frac{7}{16}$	$\frac{3}{4}$	$\frac{9}{16}$	$\frac{3}{8}$	1	$\frac{3}{16}$	$1\frac{5}{16}$	$\frac{5}{8}$	$\frac{5}{8}$	$\frac{1}{2}$	$\frac{5}{8}$	$\frac{7}{16}$	$3\frac{3}{8}$
	$23\frac{3}{4}$	$\frac{3}{8}$	$1\frac{3}{8}$	$\frac{1}{4}$	$18\frac{1}{8}$	$1\frac{1}{16}$	$\frac{7}{16}$	$\frac{3}{4}$	$\frac{9}{16}$	$\frac{3}{8}$	1	$\frac{3}{16}$	$1\frac{5}{16}$	$\frac{11}{16}$	$\frac{11}{16}$	$\frac{1}{2}$	$\frac{5}{8}$	$\frac{7}{16}$	$3\frac{3}{8}$
	$27\frac{1}{2}$	$\frac{3}{8}$	$1\frac{3}{8}$	$\frac{5}{16}$	$21\frac{1}{4}$	$1\frac{1}{16}$	$\frac{7}{16}$	$\frac{3}{4}$	$\frac{9}{16}$	$\frac{3}{8}$	$1\frac{1}{8}$	$\frac{3}{16}$	$1\frac{5}{16}$	$\frac{11}{16}$	$\frac{11}{16}$	$\frac{1}{2}$	$\frac{5}{8}$	$\frac{1}{2}$	$3\frac{3}{4}$

6. The valve gear is usually worked from a side shaft driven by worm wheels keyed on to the extension of crank shaft (Figs. 1, 2, p. 67); the dimensions of these wheels depend upon the perpendicular distance between the crank shaft and the side shaft. The diameter of the extension of the crank shaft must be determined by the bore of the worm.

7. Balancing the Reciprocating and Rotating Parts.

This is intended to lessen the shock caused by the engine rocking to and fro on its foundation. Let—

W_1 = weight of the balance weight in lbs.

R = radius of the centre of gravity of the balance weight in feet.

W_2 = weight of the crank pin and the big end of the connecting rod + half the weight of the body of the connecting rod, in lbs. (see p. 28).

r = throw or radius of the crank in feet.

W_3 = weight of the piston and piston pin in lbs. + half the body of the rod and small end (p. 28).

Taking count of the inertia we get—

$$\text{For } \textit{horizontal} \text{ gas engines } W_1 = 0{\cdot}7\ (W_2 + W_3)\frac{r}{R} \text{ lbs.} \quad . \quad . \quad . \quad (1)$$

$$\text{,, } \textit{vertical} \text{ ,, ,, } W_1 = W_2\frac{r}{R} \text{ lbs.} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (2)$$

Balance weights are also very desirable to facilitate starting the engine. Equation 1 would apply for vertical, and equation 2 for horizontal engines.

Assuming $y = y_1$ and $b = 2{\cdot}2\ r$ (Fig. 64), the radius of the centre of gravity of the balance weight is found thus: Divide the area of the weight into sections $f_1 f_2$ (Fig. 63), and determine the distances e_1 e_2 of the centre of gravity of each section from the axis of the crank shaft AA. Then the distance of the centre of gravity of the whole balance weight from the axis of the shaft—

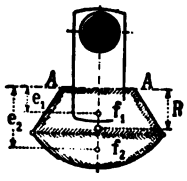


FIG. 63.

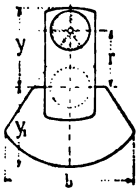


FIG. 64.

$$R = \frac{f_1\ e_1 + f_2\ e_2}{f_1 + f_3} \text{ in feet} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (3)$$

at which point the whole weight is supposed to be concentrated.

If the balance weight is thicker at the bottom than the top, and $b_2 > b_1$ (compare Fig. 79), R is increased, and is expressed by the equation $\frac{Rb_2}{b_1} = R$ . . . . . (4)

To calculate the stress upon the *balance weight bolts* and *side keys*, centrifugal force exerted when the balance weight is revolving at the speed of the engine must be known, namely—

$$CF = \frac{W_1}{g} \times \frac{V_2}{R} = \frac{WRn^2}{2935} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (5)$$

in which

W_1 = weight of balance weight in lbs. (equations 1 and 2).
R = radius of centre of gravity of balance weight in feet (Fig. 63).
n = revolutions per minute.

$$V = \frac{R \times 3{\cdot}14 \times n}{30} = \text{circumferential velocity of centre of gravity in feet per second} \quad . \quad (6)$$

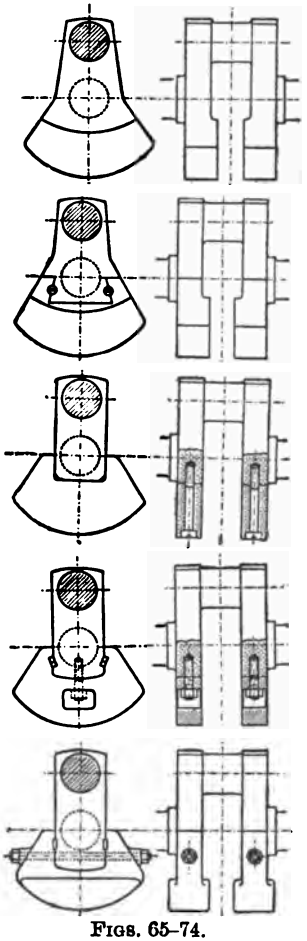
Equation 5 gives the centrifugal force exerted by the whole of the balance weight, which must be halved to get the weight on each crank web.

Rules for Constructing the Balance Weights.

The weights should be placed *as nearly as possible in the same place* as the reciprocating parts. With forged cranked shafts in gas engines this arrangement is not difficult, as each crank web carries a balance weight equal to half the total weight.

Balance weights should not be placed on the flywheel, as dangerous lateral thrusts may be set up (compare "Steam Engines," 8th Edition).

In any case it is often difficult, if the frame is to be kept low, to place the balance weights in the crank pit and allow them sufficient room to slew round, without having the crank pit projecting below the frame. It should also be possible to withdraw the piston without taking off the balance weights (see "Kranker Gas Motor," par. 72*b*).



FIGS. 65–74.

The balance weight should either be *forged* solid with the crank web, or cast in a separate piece, and firmly secured to it, as shown in various ways in Figs. 65 to 74.

Figs. 65, 66. Balance weight *forged* solid with the crank web. This method is rarely adopted, because it makes the crank shaft much more difficult to manufacture and erect.

Figs. 67, 68. Cast-iron balance weights *dovetailed on* to the crank web, and locked with set screws.

Figs. 69, 70. Cast-iron balance weights secured with bolts. Side keys or bolts should be provided as an additional safeguard, as otherwise the centrifugal force falls only on the bolts.

Figs. 71, 72. Balance weights fixed on by studs, and secured by *side keys*. The hole for tightening up the nut or stud is closed by a plate, painted and varnished.

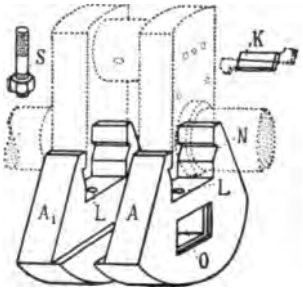
Figs. 73, 74. Balance weights secured by *long bolts* and side keys (Deutz 60 H.P.). The nuts on the bolts can be let into recesses in the balance weights, that they may not project beyond them.

Parts of the Balance Weights.

In Figs. 75–77—

AA denote balance weight (cast iron).
K „ key for keyways N.
S „ studs to fit holes L.
O „ hole to tighten nut on stud S.

To improve the appearance of the balance weights the openings O are covered with a plate.



FIGS. 75–77.

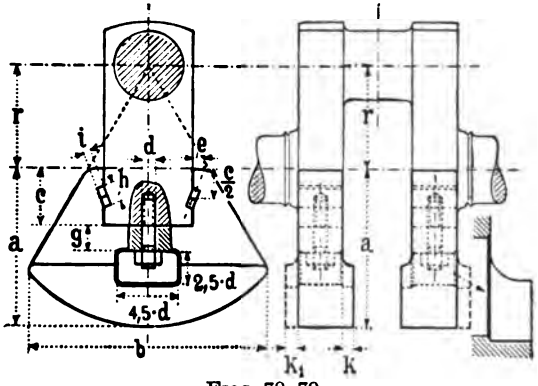
Table V.—Balance Weights.

(Figs. 78, 79.)

(The drawings show the balance weights for an engine of 22″ stroke. Scale 1 : 20.)

The studs are intended to facilitate the putting together of the balance weights, and the side keys to strengthen them.

It should be possible (after disconnecting the connecting rod) to withdraw the piston without removing the balance weights, and this must be allowed for when calculating the dimensions.



FIGS. 78, 79.

Dimensions in inches.

Diameter of cylinder	Stroke.	*a*	*b*	*c*	*d*	*e*	*g*	*h*	*i*	*k*	k_1
ins.	ins.	ins.	ins.	ins.	ins.	ins.	ins.	ins.	ins.	ins.	ins.
$8\frac{3}{4}$	$12\frac{1}{4}$	$9\frac{1}{2}$	14	4	$\frac{3}{4}$	$\frac{3}{4}$	$1\frac{9}{16}$	1	$\frac{3}{8}$	$\frac{7}{8}$	—
$10\frac{1}{4}$	$13\frac{3}{4}$	$10\frac{5}{8}$	$15\frac{3}{4}$	$4\frac{3}{8}$	1	$\frac{13}{16}$	2	$1\frac{3}{16}$	$\frac{7}{16}$	$\frac{7}{8}$	—
$11\frac{1}{8}$	$15\frac{1}{4}$	$12\frac{1}{4}$	$13\frac{1}{8}$	$4\frac{3}{4}$	1	$\frac{15}{16}$	$2\frac{3}{16}$	$1\frac{7}{16}$	$\frac{9}{16}$	1	—
13	$17\frac{3}{4}$	$13\frac{3}{4}$	$20\frac{1}{2}$	$5\frac{3}{16}$	$1\frac{1}{8}$	1	$2\frac{3}{8}$	$1\frac{9}{16}$	$\frac{3}{8}$	$1\frac{1}{16}$	—
$14\frac{1}{4}$	$19\frac{3}{4}$	$15\frac{3}{8}$	$22\frac{5}{8}$	$5\frac{5}{8}$	$1\frac{1}{4}$	1	$2\frac{9}{16}$	$1\frac{13}{16}$	$\frac{11}{16}$	$1\frac{1}{8}$	—
$15\frac{3}{4}$	$21\frac{5}{8}$	$16\frac{3}{4}$	25	$6\frac{1}{8}$	$1\frac{3}{8}$	1	$2\frac{11}{16}$	2	$\frac{13}{16}$	$1\frac{3}{16}$	$1\frac{3}{8}$
19	$23\frac{5}{8}$	$18\frac{3}{8}$	$27\frac{1}{4}$	$6\frac{9}{16}$	$1\frac{1}{2}$	$1\frac{3}{32}$	$2\frac{7}{8}$	$2\frac{3}{16}$	$\frac{7}{8}$	$1\frac{1}{4}$	$1\frac{9}{16}$
$23\frac{5}{8}$	$27\frac{1}{2}$	$21\frac{1}{2}$	$31\frac{1}{2}$	$7\frac{1}{2}$	$1\frac{3}{4}$	$1\frac{3}{16}$	$3\frac{3}{16}$	$2\frac{9}{16}$	1	$1\frac{3}{8}$	$1\frac{3}{4}$

Dimensions in millimetres.

Cylinder diameter.	Piston stroke.	*a*	*b*	*c*	*d*	*e*	*g*	*h*	*i*	*k*	k_1
220	**310**	240	355	100	20	18	40	25	10	22·5	
260	**350**	270	400	110	23	21	50	30	12	22·5	
300	**400**	310	460	120	26	23	55	36	14	26	
330	**450**	350	520	132	29	24	60	40	16	27	
360	**500**	390	575	144	32	25	65	46	18	28	
400	**550**	425	635	155	35	26	68	50	20	30	35
480	**600**	465	690	167	39	28	72	56	22	32	40
600	**700**	545	800	190	45	30	80	65	24	35	45

These measurements amount to about 75 per cent. of the weight W_1 given by equation 1, and in most cases this approximation is sufficient.

8. Material of the Crank Shaft.

For this the best quality of forged steel is almost exclusively used.

9. Calculating the Strength of the Crank Shaft.*

It is desirable, as already stated, to determine approximately, *at the outset, the strength of the journals*, and other dimensions of the crank shaft. For these the diagrams (p. 71) and Table of Standard Measurements of the various parts of the crank shaft (p. 73) may be used, but other types of engine must also, as far as possible, be studied.

The *diameter of the crank shaft* should not be *worked out* from a given formula; the student is too much inclined to place reliance on results thus obtained. To calculate a crank shaft, the main dimensions must first be determined either graphically, or by analysing the stresses. The graphic method is preferable, but it can only be used to advantage by those who are specially practised in the art of graphic representation, and have thoroughly mastered the subject. We will therefore base our calculations on an *analysis* of the stresses, because it is more easily understood, and forms a check on the results graphically obtained.

The following calculations apply to *single* as well as *double* acting engines, because, in both, the sequence of explosion strokes is so arranged that there is no increase in the pressure of explosion (compare the developed diagrams, pp. 16, 17). Therefore the bending stresses upon the crank shaft are more often repeated in the same unit of time in double-acting and multi-cylinder than in single-acting engines, while the intensity of the stresses remains the same.

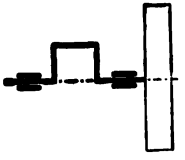


FIG. 80.

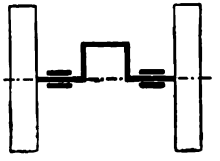


FIG. 81.

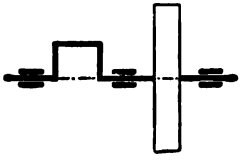


FIG. 82.

We will now proceed to consider the three methods of arranging the crank, crank-shaft bearings, and flywheel shown at Figs. 80–82, which according to p. 70 are those in most general use. For clearness' sake the calculations for the three types are tabulated side by side, although this has necessitated more elaborate equations than would otherwise have been required. In vertical engines, the maximum stresses in the flywheel bearings are developed when the crank shaft is below (that is, when the crank is on the top centre), because the weight of the flywheel and pressure of the piston are both exerted in the same direction.

In engines in factories and workshops, where there is a separate driving pulley at the side of the flywheel, the weight of the pulley ought to be taken into account. Its effect upon the stresses is, however, only small, and the tension of the belt may also be neglected.

We have to consider and test the—

1. *Strength* of the crank shaft.
2. *Surface pressure* on the journals (to ensure *cool running*).

---

* The moving parts must always be calculated for the maximum pressure, as has been done here; the mistake should never be made of calculating these parts for the actual loads coming on them during working (p. 59), and not allowing for the greater load thrown on them when starting or running slow. The inertia of the moving parts in small engines does not amount to much; in large engines, especially tandem engines, the inertia is very great, but it would not be advisable to allow for it in our calculation, for the reasons mentioned above; also it must be borne in mind that an engine which has been designed for a maximum ignition pressure of 350 lbs. per sq. inch may sometimes have to stand a pressure of 420 lbs., owing to a very complete mixture of the charge and consequent rapid ignition. Again, we must consider the mischief produced by pre-ignitions; however careful the designer may be to avoid projections or pockets in the combustion chamber, pre-ignitions occasionally occur in large gas engines driven at their maximum load with gases rich in hydrogen. Again, a careless driver may start pre-ignitions whilst shifting the ignition gear from the starting to the working position, therefore *the moving parts must always be calculated for the maximum pressure.*

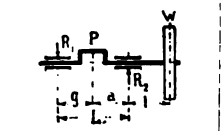
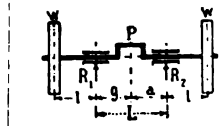
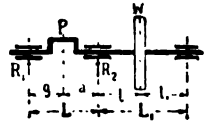
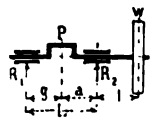
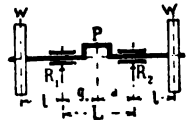
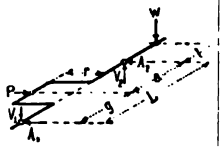
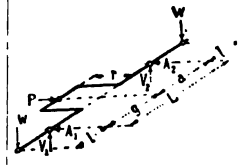
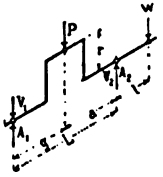
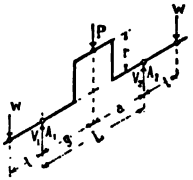
It is also necessary to study the stresses coming on the crank shaft in two positions, namely—

1. At ignition; the full initial pressure comes on the crank when on the dead centre.

2. When the crank is at an angle of 34°, because, according to the turning moment diagram (p. 38), it is in this position that the maximum turning moment occurs.

Let A = sectional area of piston in sq. inches (equations 3–5, p. 18).
p = pressure of explosion in lbs. per sq. inch.
P = $p \times A$ = maximum pressure on piston at explosion at dead point = load on piston, lbs. . . . . (7)
P′ = pressure on piston when the crank is in the position where greatest turning moment occurs, lbs.
W = weight of flywheel in lbs.

(a) Crank on Dead Point at Explosion.

	Horizontal Engines.			Vertical Engines.	
	2 Bearings, 1 Flywheel.	2 Bearings, 2 Flywheels.	3 Bearings, 1 Flywheel.	2 Bearings, 1 Flywheel.	2 Bearings, 2 Flywheels.
					
					
Pressure on bearing from load on piston P.	$A_1 = P \times \frac{a}{L}$	$A_1 = P \times \frac{a}{L}$	$A_1 = P \times \frac{a}{L}$	$A_1 = P \times \frac{a}{L}$	$A_1 = P \times \frac{a}{L}$ . . (8)
	$A_2 = P \times \frac{g}{L}$	$A_2 = P \times \frac{g}{L}$	$A_2 = P \times \frac{g}{L}$	$A_2 = P \times \frac{g}{L}$	$A_4 = P \times \frac{g}{L}$ . . (9)
Pressure on bearing from weight of flywheel W.	$V_1 = W \times \frac{l}{L}$	$V_1 = W\frac{L+l}{L} - W\frac{l}{L} = W$	$V_1 = \text{zero}$ *	$V_1 = -W \times \frac{l}{L}$	$V_1 = W\frac{L+l}{L} - W\frac{l}{L} = W$ (10)
	$V_2 = W \times \frac{L+l}{L}$	$V_2 = W\frac{L+l}{L} - W\frac{l}{L} = W$	$V_2 = W\frac{l_1}{L_1}$	$V_2 = W\frac{l+L}{L}$	$V_2 = W\frac{L+l}{L} - W\frac{l}{L} = W$ (11)
			$V_3 = W\frac{l}{L_1}$ . . . . . (12)		
Resultant pressure from the combined pressures A and V.	$R_1 = \sqrt{A_1^2 + V_1^2}$	$R_1 = \sqrt{A_1^2 + V_1^2}$	$R_1 = A_1$	$R_1 = A_1 + V_1$	$R_1 = A_1 + V_1$. (13)
	$R_2 = \sqrt{A_2^2 + V_2^2}$	$R_2 = \sqrt{A_2^2 + V_2^2}$	$R_2 = \sqrt{A_2^2 + V_2^2}$	$R_2 = A_2 + V_2$	$R_2 = A_2 + V_2$. (14)

This letter R must not be confounded with the $\mathfrak{R}$ in equations 35 and 36.

* Compare footnote, p. 94.

(b) Crank in the Position where the Greatest Turning Moment is Developed (Fig. 93).

We must next determine in which position of the crank the greatest turning moment is developed. For this the diagram of tangential pressure or turning moment diagram may be used.

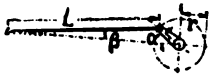


FIG. 93.—Position of Piston at Greatest Turning Moment.

Fig. 94 (taken from Fig. 9, p. 38) represents the tangential pressure diagram of the third or working stroke of a single-acting four-cycle engine. It shows that the greatest rise in the pressure curve is at distance x from the dead point. If we take—

$$\frac{r}{L} = \frac{\text{throw of the crank}}{\text{length of connecting rod}} = \frac{1}{5{\cdot}8}$$

we get from the indicator diagram (Fig. 95) an angle of 34° for the crank in this position. Figs. 94 and 95 also show that by plotting this distance x, the corresponding angle of the crank can be found at once.

Fig. 95 gives the indicator diagram for a pressure of explosion p = **370 lbs. per sq. inch,** the scale of pressure being 1 inch = 180 lbs. In crank position x, where the greatest turning moment occurs, the height of the diagram = $1\frac{9}{16}$ inch approximately = 280 lbs. In this position of the piston, therefore, the pressure is—

$$p_{0{\cdot}1} = \frac{280}{370} \times p = 0{\cdot}75 \times p \quad . \quad . \quad . \quad . \quad . \quad (15)$$

or $\frac{3}{4}$ of the explosion pressure.

Thus for this position we get a pressure on the piston * of

$$\mathbf{P' = A \times 0{\cdot}75}\, p \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (16)$$

where A = sectional area of the piston in sq. inches, p = ignition or explosion pressure of the mixture in lbs. per sq. inch.

If a = 34° and $\frac{r}{L}$ = 5·8, the angle of inclination β of the connecting rod is given by equation 13, p. 27, as $\sin \beta = \frac{1}{5{\cdot}8} \times \sin 34° =$ 0·0965, whence $\beta \sim 5°\ 30'$.

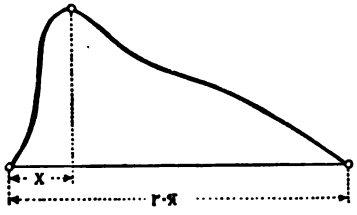


FIG. 94.—Turning Moment Diagram, III. Stroke.

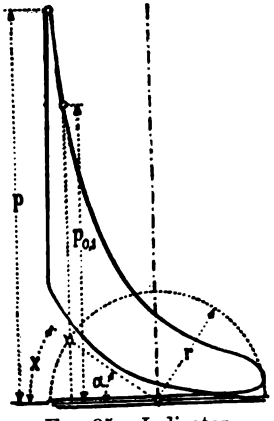
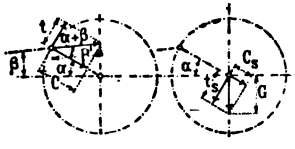
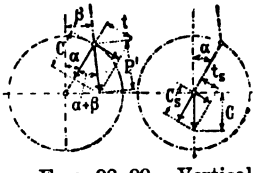


FIG. 95.—Indicator Diagram.

The pressures due to the thrust of the connecting rod may now be resolved into two components, viz. radial pressure C in the direction of the crank webs, and tangential pressure t at right angles to it (Figs. 96, 97).



FIGS. 96, 97.—Horizontal Engines.



FIGS. 98, 99.—Vertical Engines.

The weight of the flywheel is similarly resolved into two components, C_s and t_s, which follow the same direction as C and t (Figs. 97 and 99).

* Here, as at p. 55, load on the piston rod = load on the connecting rod.

For equations 17–20, we get, from p. 83—

$\alpha = 34°$

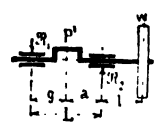
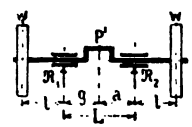
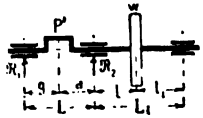
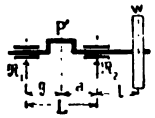
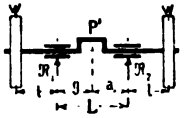
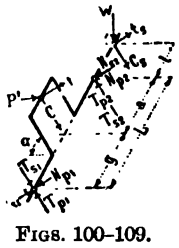
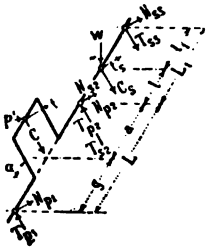
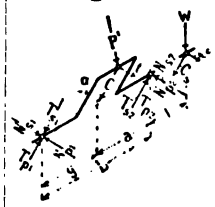
$\beta = 5° \; 30'$

$\sin \alpha = \mathbf{0{\cdot}56}$

$\cos \alpha = \mathbf{0{\cdot}83}$

$\sin (\alpha+\beta) = \mathbf{0{\cdot}64}$

$\cos (\alpha+\beta) = \mathbf{0{\cdot}77}$

	Horizontal Engines.			Vertical Engines.		
	2 Bearings, 1 Fly-wheel.	2 Bearings, 2 Fly-wheels.	3 Bearings, 1 Fly-wheel.	2 Bearings, 1 Fly-wheel.	2 Bearings, 2 Fly-wheels.	
						
						
	FIGS. 100–109.					
Radial component due to pressure of piston	$C = P' . \cos (\alpha+\beta)$	$C = P' . \cos (\alpha+\beta)$	$C = P' . \cos (\alpha+\beta)$	$C = P' . \cos (\alpha+\beta)$	$C = P' . \cos (\alpha+\beta)$.	(17)
Tangential component due to pressure of piston	$t = P' . \sin (\alpha+\beta)$	$t = P' . \sin (\alpha+\beta)$	$t = P' . \sin (\alpha+\beta)$	$t = P' . \sin (\alpha+\beta)$	$t = P' . \sin (\alpha+\beta)$.	(18)
Radial component due to weight of fly-wheel	$C_s = W \times \sin \alpha$	$C_s = W \times \sin \alpha$	$C_s = W \times \sin \alpha$	$C_s = W \times \cos \alpha$	$C_s = W \times \cos \alpha$.	(19)
Tangential component due to weight of flywheel	$t_s = W \times \cos \alpha$	$t_s = W \times \cos \alpha$	$t_s = W \times \cos \alpha$	$t_s = W \times \sin \alpha$	$t_s = W \times \sin \alpha$.	(20)
Pressure on bearings due to radial component C	$T_{p_1} = C \times \frac{a}{L}$	$T_{p_1} = C \times \frac{a}{L}$	$T_{p_1} = C \times \frac{a}{L}$	$T_{p_1} = C \times \frac{a}{L}$	$T_{p_1} = C \times \frac{a}{L}$ . .	(21)
	$T_{p_2} = C \times \frac{g}{L}$	$T_{p_2} = C \times \frac{g}{L}$	$T_{p_2} = C \times \frac{g}{L}$	$T_{p_2} = C \times \frac{g}{L}$	$T_{p_2} = C \times \frac{g}{L}$ . .	(22)
Pressure on bearings due to radial component C_s	$T_{s_1} = -C_s \times \frac{l}{L}$	$T_{s_1} = C_s \times \frac{L+l}{L} - C_s \times \frac{l}{L} = C_s$	$T_{s_1} =$ zero *	$T_{s_1} = -C_s \frac{l}{L}$	$T_{s_1} = C_s \frac{L+l}{L} - C_s \frac{l}{L} = C_s$	(23)
	$T_{s_2} = C_s \times \frac{L+l}{L}$	$T_{s_2} = C_s \times \frac{L+l}{L} - C_s \times \frac{l}{L} = C_s$	$T_{s_2} = C_s \frac{l_1}{L_1}$	$T_{s_2} = C_s \frac{L+l}{L}$	$T_{s_2} = C_s \frac{L+l}{L} - C_s \frac{l}{L} = C_s$	(24)
			$T_{s_3} = C_s \frac{l}{L_1}$			(25)

* Compare footnote, p. 94.

	Horizontal Engines.			Vertical Engines.		
Total pressure on bearings	$T_1 = T_{p_1} + T_{s_1}$ $T_2 = T_{p_2} + T_{s_2}$	$T_1 = T_{p_1} + T_{s_1}$ $T_2 = T_{p_2} + T_{s_2}$	$T_1 = T_{p_1}$ $T_2 = T_{p_2} + T_{s_2}$	$T_1 = T_{p_1} + T_{s_1}$ $T_2 = T_{p_2} + T_{s_2}$	$T_1 = T_{p_1} + T_{s_1}$. (26) $T_2 = T_{p_2} + T_{s_2}$ . . (27)
Pressure on bearings due to tangential Component t	$N_{p_1} = t \times \frac{a}{L}$ $N_{p_2} = t \times \frac{g}{L}$	$N_{p_1} = t \times \frac{a}{L}$ $N_{p_2} = t \times \frac{g}{L}$	$N_{p_1} = t \times \frac{a}{L}$ $N_{p_2} = t \times \frac{g}{L}$	$N_{p_2} = t \times \frac{g}{L}$	$N_{p_1} = t \times \frac{a}{L}$ . . . (28) $N_{p_1} = t \times \frac{g}{L}$ . . . (29)
Pressure on bearings due to tangential component t_s	$N_{s_1} = -t_s \times \frac{l}{L}$ $N_{s_2} = t_s \times \frac{L+l}{L}$	$N_{s_1} = t_s \times \frac{L+l}{L} - t_s \times \frac{l}{L} = t_s$ $N_{s_2} = t_s \times \frac{L+l}{L} - t_s \times \frac{l}{L} = t_s$	$N_{s_1} = \text{zero}$ * $N_{s_2} = t_s \times \frac{l_1}{L_1}$	$N_{s_1} = -t_s \times \frac{l}{L}$ $N_{s_1} = t_s \times \frac{L+l}{L}$	$N_{s_1} = t_s \times \frac{L+l}{L} - t_s \times \frac{l}{L} = t_s$ (30) $N_{s_2} = t_s \times \frac{L+l}{L} - t_s \times \frac{L}{l} = t_s$ (31)
			$N_{s_3} = t_s \times \frac{l}{L_1}$		. . . . . (32)
Total pressure on bearings	$N_1 = N_{p_1} + N_{s_1}$ $N_2 = N_{p_2} + N_{s_2}$	$N_1 = N_{p_1} + N_{s_1}$ $N_2 = N_{p_2} + N_{s_2}$	$N_1 = N_{p_1} + N_{s_1}$ $N_2 = N_{p_2} + N_{s_2}$	$N_1 = N_{p_1} + N_{s_1}$ $N_2 = N_{p_2} + N_{s_2}$	$N_1 = N_{p_1} + N_{s_1}$ . . (33) $N_2 = N_{p_2} + N_{s_2}$ . . (34)

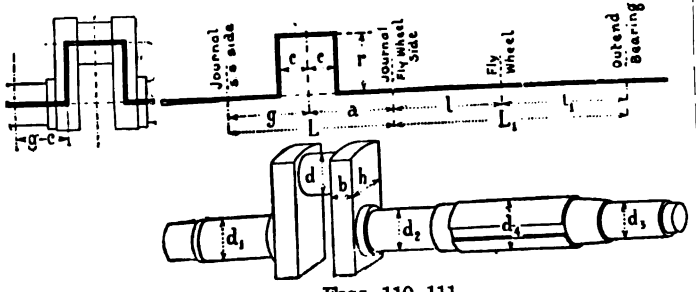
These pressures on the bearings are combined into a resultant pressure, namely—

Resultant pressure on bearings	$\mathfrak{R}_1 = \sqrt{T_1^2 + N_1^2}$ $\mathfrak{R}_2 = \sqrt{T_2^2 + N_2^2}$	$\mathfrak{R}_1 = \sqrt{T_1^2 + N_1^2}$ $\mathfrak{R}_2 = \sqrt{T_2^2 + N_2^2}$	$\mathfrak{R}_1 = \sqrt{T_1^2 + N_1^2}$ $\mathfrak{R}_2 = \sqrt{T_2^2 + N_2^2}$	$\mathfrak{R}_1 = \sqrt{T_1^2 + N_1^2}$ $\mathfrak{R}_2 = \sqrt{T_2^2 + N_2^2}$	$\mathfrak{R}_1 = \sqrt{T_1^2 + N_1^2}$ . . (35) $\mathfrak{R}_2 = \sqrt{T_2^2 + N_2^2}$ . . (36)

This letter $\mathfrak{R}$ must not be confused with the printed letter R in equations 13 and 14. The resulting pressures on the bearings obtained from equations 13, 14, and 35, 36 may now be used to determine the—

Bending and Twisting Moments and Stresses.

In the following calculations for the five types of crank shaft here considered, the pressures are given in lbs. per sq. inch, and are taken from equations 8 to 36. All dimensions should be stated in inches.

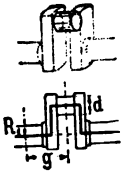


FIGS. 110, 111.

* Compare footnote, p. 94.

(a) Ignition with Crank on Dead Centre.

1. *Crank Pin.*



Resultant load on bearing R_1† (from equation 13).

Bending moment $M_b = R_1 \times g$ in inch-lbs. . (37)

Modulus of resistance $Z_b \sim 0{\cdot}1 \times d^3$ in cubic inches (38)

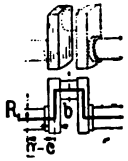
Stress due to bending $= f_b = \frac{M_b}{Z_b}$ in lbs. per sq. inch . . . . . (39)

Allowable stress or safe working load $f_b =$ **14,000** to 17,000 lbs. per sq. inch . . . . . (40)

† R_1 equation 13 must not be confused with $\mathfrak{R}_1$ equation 35.

2. *Crank-shaft webs, side-shaft side* (see Figs. 110, 111).

Resultant load on bearing R_1 (equation 13).



Bending moment $M_b = R_1 (g - c)$ inch-lbs. . (41)

Modulus of resistance $Z_b = \frac{1}{6} b^2 h$ in cubic inches (42)

Stress due to bending $f_b = \frac{M_b}{Z_b}$ lbs. per sq. inch (43)

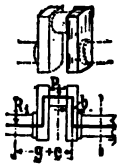
Stress due to compression $f_c = 0{\cdot}5 \frac{P}{bh}$ lbs. per sq. inch . . . . . (44)

Combined stress $f = f_b \times f_c$ lbs. per sq. inch . (45)

Allowable stress or safe working load $f = 11{,}000$ to **12,500** lbs. per square inch . . . . (46)

3. *Crank-shaft webs, flywheel side.*

Resultant load on bearing R_1 from equation 13; maximum load on piston P from equation 7.



Bending moment $M_b = R_1 (g + c) - Pc$ in inch-lbs. . . . . (47)

Modulus of resistance $Z_b = \frac{1}{6} b^2 h$ in cubic inches (48)

Stress due to bending $f_b = \frac{M_b}{Z_b}$ lbs. per sq. inch . . . . . (49)

Stress due to compression $f_c = \frac{0{\cdot}5P}{bh}$ lbs. per sq. inch . . . . . (50)

Combined stress $f = f_b + f_c$ lbs. per sq. inch . (51)

Allowable stress, or safe working load $f = 11{,}000$ to **12,500** lbs. per sq. inch . . . . . (52)

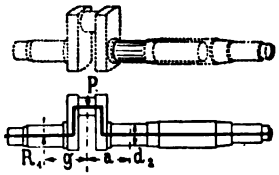
4. *Main bearing journal flywheel side.*

Maximum load on piston P from equation 7; load on bearing A_1 (equation 8); load on bearing V_1 (equation 10).

Bending moment $M_{b1} = Pa - A_1 (g + a)$ in inch-lbs. . . (53)

Bending moment $M_{b2} = V_1 (g + a)$ inch-lbs. . . . . (54)

Bending moment $M_b = \sqrt{M_{b1}^2 + M_{b2}^2}$ inch-lbs. . . . . (55)



Modulus of resistance $Z_b = 0{\cdot}1\, d_2^3$ cubic inches . . . . . (56)

Stress due to bending $f_b = \frac{M_b}{Z_b}$ lbs. per sq. inch . . . . . (57)

Allowable stress or safe working load $f_b = 4200$ to **5600** lbs. per sq. inch . . . . . (58)

In shafts with three bearings (Fig. 82, p. 82) the bending moment M_b = zero.*

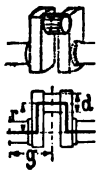
For crank shafts with an overhung flywheel, equation 55 should read as follows:—

Bending moment $M_b = W \times l$ in inch-lbs. . . . . (59)

(b) Crank in Position where Greatest Turning Moment occurs (see Fig. 93).

1. *Crank pin.*

Resultant load on bearing $\mathfrak{R}_1$ from equation 35; resultant load on bearing N_1 from equation 33.



Bending moment $M_b = R_6 \times g$ inch-lbs. . . . (60)

Twisting moment $M_t = N_1 \times r$ inch-lbs. . . (61)

Modulus of resistance to bending $Z_b = 0{\cdot}1\, d^3$ cubic inches . . . . . (62)

Modulus of resistance to twisting $Z_t = 0{\cdot}2 \times d^3$ cubic inches . . . . . (63)

Stress due to bending $f_b = \frac{M_b}{Z_b}$ in lbs. per sq. inch . . . . . (64)

Stress due to twisting $f_t = \frac{M_t}{Z_t}$ in lbs. per sq. inch . . . (65)

Combined stress—

$$f = 0{\cdot}35 f_b + 0{\cdot}65 \sqrt{f_b^2 + 4 f_t^2} \quad . . . . (66)$$

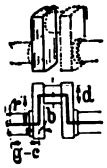
Allowable stress or safe working load

$f = 11{,}000$ to **14,000** lbs. per sq. inch . . . . (67)

* Compare footnote, p. 94.

2. *Crank-shaft webs, side-shaft side.*

Load T_1 on bearing . . . . . equation 26.
Load N_1 on bearing . . . . . „ 33.



In the *middle of the longer sides of the webs.*

Bending moment $M_b = T_1\,(g-c)$ in inch-lbs. (68)
Twisting moment $M_t = N_1\,(g-c)$ inch-lbs. . (69)
Modulus of resistance to bending Z^b $\Big\} = \frac{1}{6}b^2h$ cubic inches . (70)
Modulus of resistance to twisting Z_t $\Big\} = \frac{2}{9}b^2h$ cubic inches . (71)

$$\left.\begin{array}{l}\text{Stress due to bending } f_b = \dfrac{M_b}{Z_b} \text{ lbs. per sq. inch .} \\ \text{„ \quad „ \quad twisting } f_t = \dfrac{M_t}{Z_t} \text{ \quad „ \quad „}\end{array}\right\} (72)$$

Combined stress—

$$f = 0{\cdot}35 f_b + 0{\cdot}65\sqrt{f_b^2 + 4 f_t^2} \text{ lbs. per sq. inch} \quad . \; . \quad (73)$$

Allowable stress or safe working load—

$$f = 8500 \text{ to } \mathbf{10{,}000} \text{ lbs. per sq. inch} \quad . \; . \; . \; . \quad (74)$$

At the point where the crank pin begins, in the middle of the shorter sides of the webs—

$$\text{Bending moment } M_b = N_1\left(r - \frac{d}{2}\right) \text{ inch-lbs.} \quad . \; . \; . \; . \; . \quad (75)$$

Twisting moment $M_t = N_1\,(g-c)$ „ . . . . . (76)
Modulus of resistance to bending $Z_b = \frac{1}{6}bh^2$ cubic inches . (77)
„ „ twisting $Z_t = \frac{2}{9}bh^2$ „ . . (78)

$$\left.\begin{array}{l}\text{Stress due to bending } f_b = \dfrac{M_b}{Z_b} \text{ lbs. per sq. inch} \quad . \; . \; . \; . \\ \text{„ \quad „ \quad twisting } f_t = \dfrac{M_t}{Z_t} \text{ \quad „ \quad „ \quad „ \quad . \; . \; .}\end{array}\right\} (79)$$

Combined stress—

$$f = 0{\cdot}35 f_b + 0{\cdot}65\sqrt{f_b^2 + 4 f_t^2} \text{ lbs. per sq. inch} \quad . \; . \quad (80)$$

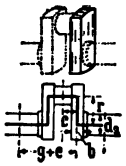
Allowable stress or safe working load—

$$f = 2800 \text{ to } \mathbf{4200} \text{ lbs. per sq. inch} \quad . \; . \; . \; . \quad (81)$$

3. *Crank-shaft webs, flywheel side.*

Load T_1 on bearing . . . . . equation 26.
Load N_1 on bearing . . . . . „ 33.

Tangential component t (eq. 18), radial component C (eq. 17).



In the *middle of the longer sides of the webs.*

Bending moment $M_b = T_1\,(g+c) + C \times c$ inch-lbs. (82)
Twisting moment $M_t = N_1\,(g+c) + t \times c$ inch-lbs. (83)
Modulus of resistance to bending Z_b $\Big\} = \frac{1}{6}b^2h$ cubic inches (84)
Modulus of resistance to twisting Z_t $\Big\} = \frac{2}{9}b^2h$ „ (85)

$$\left.\begin{array}{l}\textit{Stress due to bending } f_b = \dfrac{M_b}{Z_b} \text{ lbs. per sq. inch} \quad . \; . \; . \; . \\ \text{„ \quad „ \quad } \textit{twisting } f_t = \dfrac{M_t}{Z_t} \text{ \quad „ \quad „ \quad . \; . \; . \; .}\end{array}\right\} (86)$$

Combined stress—

$$f = 0{\cdot}35 f_b + 0{\cdot}65\sqrt{f_b^2 + 4 f_t^2} \text{ lbs. per sq. inch} \quad . \; . \; . \quad (87)$$

Allowable stress or safe working load—

$$f = 7000 \text{ to } \mathbf{9000} \text{ lbs. per sq. inch} \quad . \; . \; . \; . \; . \quad (88)$$

At the point where the shaft connects, stresses are also developed in the *middle of the shorter sides of the webs*, namely—

$$\text{Bending moment } M_b = t\left(r - \frac{d_2}{2}\right) + N_1 \times 0{\cdot}5\, d_2 \text{ inch-lbs.} \quad . \; . \quad (89)$$

Twisting moment $M_t = -N_1\,(g+c) + t \times c$ inch-lbs. . . (90)
Modulus of resistance to bending $Z_b = \frac{1}{6}bh^2$ cubic inches . (91)
„ „ twisting $Z_t = \frac{2}{9}bh^2$ „ . (92)

$$\left.\begin{array}{l}\text{Stress due to bending } f_b = \dfrac{M_b}{Z_b} \text{ lbs. per sq. inch} \quad . \; . \; . \; . \\ \text{„ \quad „ \quad twisting } f_t = \dfrac{M_t}{Z_t} \text{ \quad „ \quad „ \quad . \; . \; . \; .}\end{array}\right\} (93)$$

Combined stress—

$$f = 0{\cdot}35 f_b + 0{\cdot}65\sqrt{f_b^2 + 4 f_t^2} \text{ lbs. per sq. inch} \quad . \; . \quad (94)$$

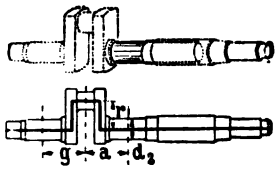
Allowable stress or safe working load—

$$f = 5000 \text{ to } 6250 \text{ lbs. per sq. inch} \quad . \; . \; . \; . \quad (95)$$

4. *Main bearing journal.*

Load T_1 on bearing . . . . . equation 26
Radial component C . . . . . „ 17
Load N_1 on bearing . . . . . „ 33
Tangential component t . . . . . „ 18

Bending moment $M_{b1} = -T_1(g + a) + C \times a$ . . . . (96)
„ „ $M_{b2} = -N_1(g + a) + t \times a$ . . . . (97)
Resultant bending moment $M_b = \sqrt{M_{b1}^2 + M_{b2}^2}$ . . . . (98)
Twisting moment $M_t = tr$ (99)



Modulus of resistance due to bending—

$Z_b = 0{\cdot}1 d_2^3$ cubic inches (100)

Modulus of resistance due to twisting—

$Z_t = 0{\cdot}2 d_2^3$ cubic inches (101)

Stress due to bending $f_b = \frac{M_b}{Z_b}$ lbs. per sq. inch . . . .
„ „ twisting $f_t = \frac{M_t}{Z_t}$ „ „ . . . . (102)

Combined stress—

$$f = 0{\cdot}35 f_b + 0{\cdot}65 \sqrt{f_b^2 + 4 f_t^2} \quad . \quad . \quad . \quad . \quad . \quad (103)$$

Allowable stress, or safe working load—

$$f = 6250 \text{ to } 7750 \text{ lbs. per sq. inch} \quad . \quad . \quad . \quad . \quad . \quad (104)$$

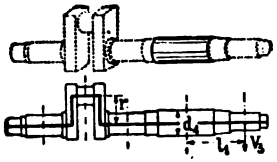
In an engine with two bearings, and overhung flywheel, *i.e.* the flywheel fitting over the end of the shaft, and projecting beyond the bearing, the bending moment is—

$$M = W \times l \text{ in inch-lbs.} \quad . \quad . \quad . \quad . \quad . \quad (105)$$

5. *In the swell for flywheel in an engine with three bearings—*

Load V_3 on bearing . . . . . equation 12
Tangential component t . . . . . „ 18

Bending moment $M_b = V_3 \times l_1$ inch-lbs. . . . . (106)
Twisting „ $M_t = 0{\cdot}5 \times t \times r$ inch-lbs. . . . . (107)



Modulus of resistance to bending—

$Z_b = 0{\cdot}1 \times d_4^3$ cubic inches (108)

Modulus of resistance to twisting—

$Z_t = 0{\cdot}2 \times d_4^3$ cubic inches (109)

Stress due to bending $f_b = \frac{M_b}{Z_b}$ lbs. per sq. inch . . .
„ „ twisting $f_t = \frac{M_t}{Z_t}$ „ „ . . . . (110)

Combined stress—

$$f = 0{\cdot}35 f_b + 0{\cdot}65 \sqrt{f_b^2 + 4 f_t^2} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (111)$$

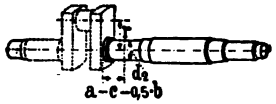
Allowable stress or safe working load—

$$f = 2800 \text{ to } 4200 \text{ lbs. per sq. inch} \quad . \quad . \quad . \quad . \quad (112)$$

6. *At the point where the main bearing journal joins the crank web on the right flywheel side.*

Tangential component t . . . . . equation 18
Resultant load on bearing $\mathfrak{R}_1$ . . . . „ 36

Twisting moment $M_t = tr$ inch-lbs. . . . . (113)
Bending „ $M_b \mathfrak{R}_2 \times (a - c - 0{\cdot}5 \times b)$ inch-lbs. . . (114)



Modulus of resistance to bending—

$Z_b = 0{\cdot}1 d_2^3$ cubic inches . (115)

Modulus of resistance to twisting—

$Z_t = 0{\cdot}2 d_2^3$ cubic inches . (116)

Stress due to bending $f_b = \frac{M_b}{Z_b}$ lbs. per sq. inch . . . .
„ „ twisting $f_t = \frac{M_t}{Z_t}$ „ „ . . . . (117)

Combined stress—

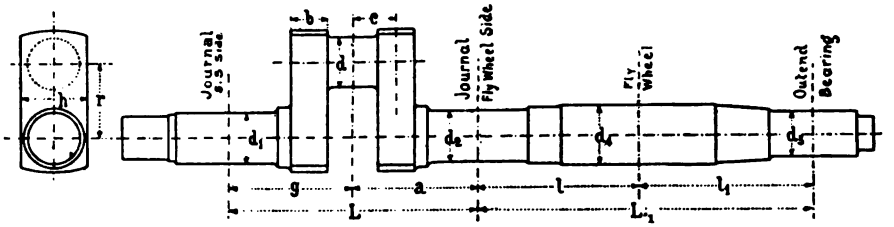
$$f = 0{\cdot}35 f_b + 0{\cdot}65 \sqrt{f_b^2 + 4 f_t^2} \quad . \quad . \quad . \quad . \quad . \quad (118)$$

Allowable stress or safe working load—

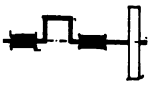
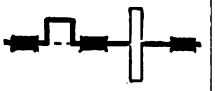
$$f = 11{,}000 \text{ to } 12{,}000 \text{ lbs. per sq. inch} \quad . \quad . \quad . \quad . \quad . \quad (119)$$

To check the above calculations, and verify the dimensions of the crank shaft on which they are based, the results are worked out in Table XXIII., for four different sizes of gas engines, from the equations at pp. 83–89. To calculate a complete example will take a beginner eight hours, a more advanced student five hours. The last column of the table is left blank, for inserting further examples.

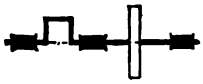
Table VI.—Data for Calculating the Crank Shaft.



FIGS. 29, 30.—Dimensions of the Crank Shaft.

Calculated from equation			Crank shaft with 2 bearings, 1 fly-wheel. 		Crank shaft with 3 bearings, 1 fly-wheel. 		
	Engine	Nominal H.P. =	0·6	17	42	132	H.P.
		Diam. of cylinder D =	$3\frac{1}{4}''$	$10\frac{1}{2}''$	14″	24	inches.
		Stroke S =	5″	$13\frac{3}{8}''$	20″	28″	,,
3, p. 18	Area of piston	A =	8·3	82·5	154	452	sq. ins.
7, ,, 83	Press. on piston, with	p =	3070	30,500	57,000	167,500	lbs.
16, ,, 84	370 lbs. per sq. in.	P =					
	Pressure on piston where max. turning moment occurs	P′ =	2300	23,000	43,000	126,000	,,
	Weight of flywheel from Table I., p. 24	W =	485	2820	10,600	38,400	,,
	The values given are taken from Table II., (p. 73), the dimensions given in this table refer to continental engines built to mm.; for the sake of simplicity the nearest even dimensions in ins. have been taken for these calculations.		$\delta_0 = 1/50$		$\delta_0 = 1/80$		
		r =	$2\frac{1}{2}''$	$6\frac{7}{8}''$	10″	14″	inches
		$a=g$ =	$3\frac{15}{16}''$	11″	$14\frac{3}{4}''$	$23\frac{1}{8}''$	,,
		L =	$7\frac{7}{8}''$	22″	$29\frac{1}{2}''$	$46\frac{1}{4}''$	,,
		l =	$4\frac{1}{2}''$	11	$19\frac{5}{8}''$	$29\frac{1}{2}''$	,,
		l_1 =	—	—	$21\frac{5}{8}''$	$32\frac{5}{8}''$	,,
		L_1 =	—	—	$41\frac{1}{4}''$	$62\frac{1}{8}''$	,,
		d =	$1\frac{5}{8}''$	5	$6\frac{3}{4}''$	$10\frac{1}{4}''$	,,
		$d_1=d_2$ =	,,	,,	$6\frac{1}{2}''$	9″	,,
		d_3 =	—	—	$6\frac{1}{8}''$	$7\frac{7}{8}''$	,,
		d_4 =	$2\frac{3}{16}''$	6″	$7\frac{7}{8}''$	$11\frac{1}{2}''$	,,
		b =	1″	$3\frac{1}{16}''$	$4\frac{3}{16}''$	$7\frac{1}{8}''$	,,
		h =	$2\frac{3}{16}''$	$6\frac{3}{8}''$	$8\frac{1}{2}''$	$12\frac{9}{16}''$	,,
		c =	$1\frac{1}{2}''$	4″	$5\frac{1}{2}''$	$8\frac{3}{4}''$	,,
8, 9, p. 83	Press. on bearings	$A_1=A_2$ =	1535	15,250	28,500	83,750	lbs.
10 ,,	,, ,,	V_1 =	280	1410	Zero *	Zero *	,,

* Compare footnote, p. 94.

From equation		Crank shaft with 2 bearings, 1 flywheel.		Crank shaft with 3 bearings, 1 flywheel.		
						
	Cyl. diam. and stroke of engine.	3¼″ / 5	10¼″ / 13¾	14″ / 20	24″ / 28	
11	Pressure on bearings $V_2=$	760	4,230	5,550	20,000	lbs.
12	,, ,, $V_3=$	—	—	5,020	18,200	,,
13	Resultant pressure on bearings $R_1^*=$	1,560	15,310	28,500	83,750	,,
14	Resultant pressure on bearings $R_2^*=$	1,712	15,800	29,000	86,100	,,
17	Radial component $C=$	1,770	17,700	33,000	97,000	,,
18	Tangential component $t=$	1,470	14,700	27,500	80,600	,,
19	Radial component $C_s=$	270	1,600	5,920	21,500	,,
20	Tangential component $t_s=$	400	2,340	8,800	32,000	,,
21, 22	Pressure on bearings $T_{p1}=T_{p2}=$	885	8,850	16,500	48,500	,,
23	Pressure on bearings $T_{s1}=$	−154	−800	Zero †	Zero †	,,
24	,, ,, $T_{s2}=$	425	2,400	3,100	11,300	,,
25	,, ,, $T_{s3}=$	—	—	2,820	10,200	,,
26	,, ,, $T_1=$	731	8,050	16,500	48,500	,,
27	,, ,, $T_2=$	1,310	11,250	19,600	59,800	,,
28, 29	Pressure on bearings $N_{p1}=N_{p2}=$	735	7,850	13,750	40,300	,,
30	Pressure on bearings $N_{s1}=$	−230	−1,170	Zero †	Zero †	,,
31	,, ,, $N_{s2}=$	625	3,500	4,630	16,800	,,
32	,, ,, $N_{s3}=$	—	—	4,200	15,200	,,
33	,, ,, $N_1=$	505	6,180	13,750	40,300	,,
34	,, ,, $N_2=$	1,360	10,850	18,380	57,100	,,
35	Resultant pressure on bearings $\mathfrak{R}_1^*=$	860	10,150	21,450	63,100	,,
36	Resultant pressure on bearings $\mathfrak{R}_2^*=$	1,892	15,630	26,900	82,700	,,
At the dead point—						
37	1. Crank pin $M_b=$	6,250	168,500	420,000	1,940,000	inch-lbs.
38	$Z_b=$	4·3	12·5	30·7	116 ‡	inches.³
39	$f_b=$	1,450	13,500	13,700	16,700	lbs. per sq. in.
41	2. Crank webs, side-shaft side $M_b=$	3,880	107,000	264,000	1,200,000	inch-lbs.
42	$Z_b=$	366	10	24·8	106	inches.³
43	$f_b=$	10,600	10,700	10,650	11,300	lbs. per sq. in.
44	$f_c=$	690	780	800	930	,, ,,
45	$f=$	11,290	11,480	11,450	12,230	,, ,,

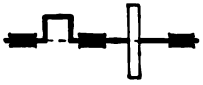
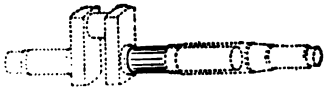
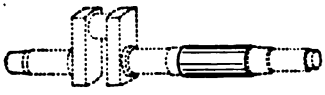
* Compare note, p. 86.

† Compare footnote, p. 94.

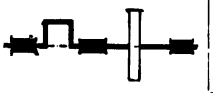
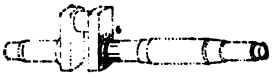
‡ Crank pin, 10¼ inches diameter.

From equation			Crank shaft with 2 bearings, 1 flywheel.		Crank shaft with 3 bearings, 1 flywheel.		
	Cyl. diam. and stroke of engine.		3¼″ / 5	10¼″ / 13¾	14″ / 20	24″ / 28	
47	3. Crank webs, fly-wheel side	M_b =	4,000	108,000	263,000	1,205,000	in.-lbs.
48		Z_b =	0·366	10	24·8	106	ins.³
49		f_b =	10,800	10,800	10,800	11,400	lbs. per sq. in.
50		f_e =	690	780	800	980	,, ,,
51		f =	11,490	11,580	11,600	12,380	,, ,,
	4. Main Bearings.						
55		M_b =	2,220	3,1000	Zero *	Zero *	in.-lbs.
56		Z_b =	4·2	12·3	26·9	87·5	ins.³
57		f_b =	530	2,520	Zero *	Zero *	lbs. per sq. in.
Where greatest turning moment occurs—							
60	1. Crank pin.	M_b =	3,440	112,000	326,000	1,315,000	in.-lbs.
61		M_t =	1,260	42,500	137,500	566,000	,,
62		Z_b =	4·2	12·3	30·2	113†	ins.³
63		Z_t =	8·4	24·6	60·4	226	,,
64		f_b =	820	9,100	10,450	11,600	lbs. per sq. in.
65		f_t =	150	1,730	2,280	2,500	,, ,,
66		f =	852	9,500	11,070	12,260	,, ,,
68	2. Crank webs, side shaft side	M_b =	1,830	56,350	153,000	580,000	in.-lbs.
69		M_t =	1,260	43,200	127,000	580,000	,,
70		Z_b =	0·366	10·0	24·9	106	ins.³
71		Z_t =	0·488	13·3	33·2	141·3	,,
72		f_b =	5,000	5,635	6,150	5,470	lbs. per sq. in.
72		f_t =	2,580	3,260	3,830	4,100	,, ,,
73		f =	6,420	7,570	8,530	8,310	,, ,,
75		M_b =	853	27,000	91,000	357,000	in.-lbs.
76		M_t =	1,260	43,250	127,000	580,000	,,
77		Z_b =	0·8	20·7	50·3	188	ins.³
78		Z_t =	1·07	27·2	67	251	,,
79		f_b =	1,067	1,305	1,800	1,890	lbs. per sq. in.
79		f_t =	1,180	1,590	1,900	2,310	,, ,,
80		f =	2,090	2,690	3,886	3,960	,, ,,

* Compare footnote, p. 94. † At 10½ inches diameter.

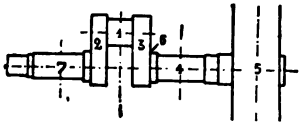
From equation			Crank shaft with 2 bearings, 1 flywheel.		Crank shaft with 3 bearings, 1 flywheel.		
							
	Cyl. diam. and stroke of engine.		$\frac{3\frac{1}{4}''}{5}$	$\frac{10\frac{1}{4}''}{13\frac{3}{4}}$	$\frac{14''}{20}$	$\frac{24''}{28}$	
82	3. Crank webs, fly-wheel side.	$M_b =$	−1360	−51,100	−153,500	−701,000	in.-lbs.
83		$M_t =$	−580	−33,700	−127,000	−582,000	,,
84		$Z =$	0·366	10	24·8	106	ins.³
85		$Z_t =$	0·488	13·3	33·2	141	,,
86		$f_b =$	3720	5,110	6,200	6,570	lbs. per sq. in.
86		$f_t =$	1190	2,530	3,820	4,150	,, ,,
87		$f =$	4160	6,460	8,470	9,180	,, ,,
89		$M_b =$	2889	81,530	230,200	933,800	in.-lbs.
90		$M_t =$	−580	−33,700	−127,000	−582,000	,,
91		$Z_b =$	0·8	20·7	50·3	188	ins.³
92		$Z_t =$	1·07	27·2	67	251	,,
93		$f_b =$	3620	3,940	4,730	4,970	lbs. per sq. in.
93		$f_t =$	−542	−1,240	1,895	2,320	,, ,,
94		$f =$	3710	4,400	5,590	6,150	,, ,,
	4. Main bearings. 						
98–105		$M_b =$	2220	31,000	Zero *	Zero *	in.-lbs.
99		$M_t =$	3670	101,000	275,000	1,128,000	,,
100		$Z_b =$	4·2	12·3	26·9	87·5	ins.³
101		$Z_t =$	8·4	24·6	53·8	175	,,
102		$f_b =$	530	2,520	Zero *	Zero *	lbs. per sq. in.
102		$f_t =$	438	4,100	5,110	6,450	,, ,,
103		$f =$	850	7,900	5,110 *	6,450 *	,, ,,
	5. In the swell for the flywheel. 						
106		$M_b =$	—	—	108,400	595,000	in.-lbs.
107		$M_t =$	1835	50,500	137,500	565,000	,,
108		$Z =$	—	—	48	152	ins.³
109		$Z_t =$	2·1	43·2	96	300	,,
110		$f_b =$	—	—	2,270	3,920	lbs. per sq. in.
110		$f_t =$	875	1,170	1,430	1,860	,, ,,
111		$f =$	875	1,170	3,170	4,570	,, ,,

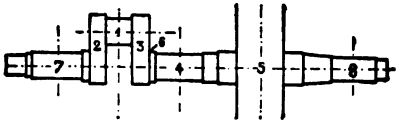
* Compare footnote, p. 94.

From equation		Crank shaft with 2 bearings, 1 flywheel.		Crank shaft with 3 bearings, 1 flywheel.		
						
	Cyl. diam. and stroke of engine.	3¼″ / 5″	10¼″ / 18¾″	14″ / 20″	24″ / 28″	
	6. Part where the main bearing joins the crank webs. Flywheel side.					
114	M_b =	3,420	86,000	145,000	894,000	in.-lbs.
113	M_t =	3,670	101,000	275,000	1,128,000	,,
115	Z_b =	4·2	12·3	26·9	87·5	ins.³
116	Z_t =	8·4	24·6	53·8	175·0	,,
117	f_b =	815	7,000	5,400	8,000	lbs. per sq. in.
117	f_t =	437	4,100	5,100	6,450	,, ,,
118	f =	1,064	9,460	9,500	12,660	,, ,,

To show more clearly the results as worked out for the crank shafts of four different sizes of engine, the stresses only are tabulated in Table VII. The part where greatest danger from stress may be apprehended appears in the larger engines to be the crank pin.

Table VII.—Summary of the Stresses on the Crank Shaft, for Two Positions of the Crank, in Six Different Places.

FIG. 51. 

 FIG. 52.

Size of engine.	Cross-section I.		II.			III.			IV.		V.	VI.	
	Dead point.	34°	Dead point.	34°		Dead point.	34°		Dead point.	34°	34°	34°	
0·6 H.P.	f= 1,450	852	11,290	6420	2090	11,490	4160	3710	530	850	875	1,064	lbs. per sq. in.
17 ,,	f=13,500	9,500	11,880	7570	2690	11,580	6460	4400	2520	7000	1170	9,460	,, ,,
42 ,,	f=13,700	11,070	11,450	8530	3386	11,600	8470	5590	Zero*	5110*	3170	9,500	,, ,,
132 ,,	f=16,700	12,260	12,230	8310	2960	12,330	9180	6150	Zero*	6450*	4700	12,660	,, ,,

* According to Ensslin's method, the stresses due to bending can also be determined in crank shafts with three bearings (Fig. 82, p. 82). These may be stated approximately in the same way as the stresses due to twisting f_t (equation 102), and the combined stress $f = 1·7\ f_t$.

Thus, Column IV. of above table should read—

Dead point.	34°
530	850
2520	7,000
5110	8,700
6450	11,000

In Table VII. the figures printed in *black type* are always the determining factors for the stresses. This summary clearly shows that—

For sections 1, 2, and 3, *i.e.* the crank pin and crank webs (Figs. 51, 52), we must calculate the stresses, in the engines here considered, from the crank at the dead centre, and for sections 4, 5 (*i.e.* the swell for the flywheel) and 6, from the crank when in the position where the greatest turning moment occurs ($a = 34°$, see p. 84). For sections 5 and 6 the dead-centre stresses need not be taken into account at all.

Multiple cranked shafts are only built for special purposes, and to calculate their dimensions and stresses would be outside the scope of this work.

Overheating the Bearings.

The smaller the changes produced by the stresses along the axis of the crank shaft, the less risk will there be of accident or fracture, and the less the tendency of the bearings to become overheated.

Let us first consider a vertical four-cycle engine (Fig. 53), because in this type the load on the piston and weight of the flywheel are exerted in the same direction. The results yielded by calculation of vertical, can also in general be applied to horizontal engines.

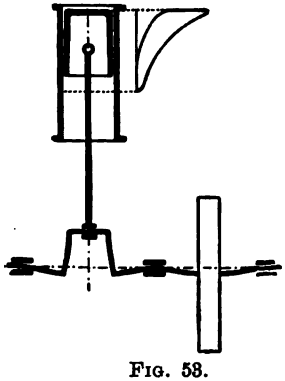


FIG. 53.

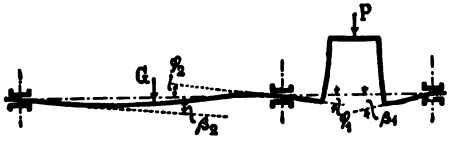


FIG. 54.

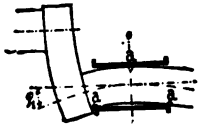


FIG. 55.

The pressure transmitted from the piston to the crank through the connecting rod, which reaches its maximum at the dead centre, seeks to distort the axis of the crank in the direction shown in Figs. 54 and 55.

The weight of the flywheel also produces a deformation of the crank shaft, and the brass only touches the shaft at the points *a a* (see Fig. 55, in which the effect is purposely exaggerated).

The result is local friction, and hence overheating of the bearings.

The so-called "limit of elasticity" is marked by the thick line ——— in Fig. 54. The angles ϕ_1 and ϕ_2 of the main bearing on the flywheel side, and β_1 and β_2 of the bearing on the side shaft side, all of which have to be considered, vary for each position of the piston. These angles can be determined by calculation. (See on this subject Ensslin: "Multiple Cranked Shafts," pp. 84–90.) [Ensslin: "Mehrfach gelagerte Kurbelwellen."]

As ϕ is relatively small, we may write: angle of inclination $\phi = \tan \phi$ in circular measure.

Ensslin has calculated the crank shaft of a 100 H.P. gas engine. From the results he has arrived at the conclusion may be drawn that, for crank shafts of gas engines with three bearings, in which the stress due to bending is 10,000 to 11,500 lbs. per sq. inch, the utmost deflection permissible within the limits of elasticity should be—

$$\phi_1 = \phi_2 = \tfrac{7}{10000}\,;\ \beta_1 = \tfrac{1}{1000}\,;\ \beta_2 = \tfrac{3}{10000}$$

These are the allowable ratios in circular measure, and therefore approximately = tan of the angle.

In Fig. 54 the angle ϕ marks the deviation from the horizontal induced by the bending stress.

A glance at it shows the inaccuracy of the usual method of calculation, according to which the bending moment of the crank shaft at A_2 in the centre of the main bearing (Fig. 56) should be equal to zero.

This method has, however, been retained here, as Ensslin's process of calculation is somewhat complicated and lengthy, but the latter should always be used for large engines.

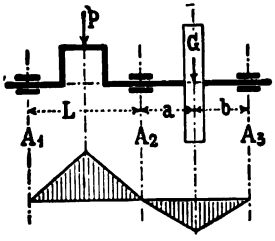


FIG. 56.

FIG. 57.

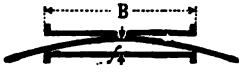


FIG. 58.

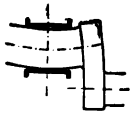


FIG. 59.
Large Diameter,
Short Length.

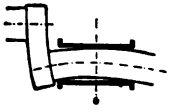


FIG. 60.
Small Diameter,
Long Length.

The longer the bearing, the greater is the deflection of the journal within it (Figs. 57, 58), and the less of its area takes the bearing stress. This is illustrated in Figs. 57–60. The conclusion may be drawn that—

The *crank pin should be as large in diameter as possible*, to withstand the bending stress . . . . . } (120)
The *crank pin should be as short as possible*, to reduce tan ϕ

F = the friction in the bearing as expressed by the product $q \times v^* \times \mu$. As v increases with the diameter of the crank pin, the rule for crank shafts has hitherto been that the—

Diameter of the crank pin should be as small as possible, to reduce the radius of friction
Length of crank pin should be as great as possible, to reduce the pressure per unit of bearing surface . . . . . } (121)

These two rules, 120 and 121, contradict each other.

The length of the bearing should therefore depend, not on its diameter, but upon the pressure on the piston, and therefore on the diameter of the cylinder.

* q = pressure on area of bearing (in lbs. per sq. inch); v = circumferential velocity of the crank pin; μ = coefficient of friction = 0·05 (see p. 98).

Strength and Overheating of the Bearings.

To test the *strength of the journals, etc., does not in itself afford sufficient data* for determining finally the dimensions of the crank shaft. Both the crank pin and the journals should be tested for the pressure on the bearing surface q and the product $q \times v$, for upon this depends the permissible friction in the journals.

The effect of the tension of the belt is very small, and may be neglected. (See the Author's "Steam Engines," 7th Edition, pp. 48 *et seq.*)

(In steam engines it has hitherto been usual to base the calculations on the admission pressure p, although the amount of friction, upon which overheating mainly depends, is chiefly determined by the mean pressure, p_m, above atmosphere.)

In any case the pressure on the bearing surfaces q cannot be determined from the pressure of explosion because, except in the case of double-acting two-cycle gas engines, strokes continually occur during which there is no pressure on the crank pin.

We have therefore to consider—

1. How many strokes intervene between two consecutive working strokes?
2. In what sequence do the working strokes follow each other? (On this point see the developed diagrams, pp. 16 and 17.)
3. In how many cylinders do the pistons work upon the same crank?

In a four-cycle engine we must take count of the inertia curve for the admission and exhaust (Ist and IVth strokes), the different areas must be planimetered, and the arithmetical mean calculated from them. For instance, in a single-acting four-cycle engine with an explosion pressure of 370 lbs. (diagram, Fig. 3, p. 36), we have—

Ist	Stroke	Admission	p_I	= 7 lbs. per sq. inch.	
IInd	,,	Compression	p_{II}	= 24 ,,	,,
IIIrd	,,	Expansion	p_{III}	= 110 ,,	,,
IVth	,,	Exhaust	p_{IV}	= 7 ,,	,,
			Total	= 148 ,,	,,

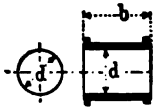
or, pressure per stroke $p_r = \frac{148}{4} = 37$ lbs. per sq. inch.

The figures in Table VIII. have been calculated in this way. Note further that in a double-acting two-cycle gas engine $p_r = p_m$.

Table VIII.—Values of p_r calculated for a Single Crank-pin Bearing.

	For meaning of symbols of gas engines, see p. 3.											Steam engines.				
	Single-acting four-cycle.					Double-acting four-cycle.				Double-acting two-cycle.		Single cylinder.	Tandem.	Compound.	Triple expansion. 3 cranks.	Triple expansion. 2 cranks.
	E 4, 1 c.	G Z 4, 2 c.	Z 4, 2 c.	R 4, 2 c	Z G Z 4, 4 c.	D 4, 1 c.	Z D 4, 2 c.	R D 4, 2 c.	R Z D 4, 4 c.	Koer. D 2, 1 c.	Oech. D 2, 1 c.					
p_r = lbs. per sq. in.	37	57	37	67	57	57	57	85	85	85	55	43	34	17	11	20

In general—



p denotes explosion pressure in lbs. per sq. inch.
p_r „ *mean pressure* for a given stroke in lbs. per sq. inch (see Table VIII.).
A „ sectional area of cylinder in sq. inches (in steam engines area of low-pressure cylinder).
$p \times A$ „ maximum pressure on piston in lbs.
$p_r \times A$ „ mean „ „ „
d denotes diameter, b_l = length of a given bearing in inches.

$v = \dfrac{d \times \pi \times n}{12 \times 60}$ circumferential velocity of the journal or pin in feet per second . . . . . (122)

W denotes weight of the flywheel in lbs.
g, L, and V_0 are determined from p. 83.
$q_{max.}$ denotes greatest pressure on surface of bearing in lbs. per square inch, worked out from $p \times A$.
q „ mean „ „ „ „ „ „ „ $p_r \times A$.
F = friction work = $q \times v \times 0{\cdot}05$ ft.-lbs. per second per sq. inch of bearing (coefficient of friction = 0·05) . . . . . (123)

Table IX.—Values of q and $q \times v$ for Gas Engines.

Bearing pressure per sq. in. = load on bearing / area of bearing =	*Crank Pin.*		*Main Bearings.*			
	$q_{max.} = \dfrac{p \,.\, A}{d \,.\, b}$	$q = \dfrac{p_r \,.\, A}{d \,.\, b}$	$q_{max.} = \dfrac{p \,.\, A \,.\, \frac{g}{L} + V_2}{d \,.\, b}$		$q = \dfrac{p_r \,.\, A \,.\, \frac{g}{L} + V_2}{d \,.\, b}$	
			Crank shaft with—		Crank shaft with—	
Approximately—			2 bearings.	3 bearings.	2 bearings.	3 bearings.
p being = 370 lbs. per sq. in. and $g : L = 0{\cdot}5$			$\dfrac{185A + 1{\cdot}5W}{d \times b}$	$\dfrac{185A + 0{\cdot}6W}{d \times b}$	$\dfrac{0{\cdot}5p_rA + 1{\cdot}5W}{d \times b}$	$\dfrac{0{\cdot}5p_rA + 0{\cdot}6W}{d \times b}$
We ought to have—	$q_{max.} \leqq 1500$	F $\leqq 60{\cdot}6$	$q_{max.} \leqq 570$ lbs. per sq. in.		F = $\leqq 70$	

Example 1.—In a 132 HP engine, E 4, 1 cylinder (with 3 bearings), D = 24″; A = 452 sq. inches; n = 150; ignition pressure p = 370 lbs. per sq. inch (see p. 24). From Table VIII., p_r = 37 lbs. per sq. inch; from p. 24, if $\delta_0 = \frac{1}{80}$, then weight of flywheel W = 38,400 lbs.

Crank Pin.

From Table II., $d = 10\frac{1}{4}''$; $b = 10\frac{1}{4}''$.

$$q_{max.} = \frac{370 \times 452}{10\frac{1}{4}'' \times 10\frac{1}{4}''} = 1600 \text{ lbs. per sq. in.}$$

$$q = \frac{37 \times 452}{10\frac{1}{4}'' \times 10\frac{1}{4}''} = 160 \text{ lbs. per sq. in.}$$

From equation 122—

$$v = \frac{10\frac{1}{4} \times \pi \times 150}{12 \times 60} = 6{\cdot}7 \text{ feet per second}$$

From equation 123—

$$F = 160 \times 6{\cdot}7 \times 0{\cdot}05 = 53{\cdot}6 \text{ ft.-lbs. per second}$$

Main Bearings.

From Table II., $d = 9\frac{5}{8}''$; $b = 18\frac{7}{8}''$.

According to the approximate values given in Table IX.

$$q_{max.} = \frac{185 \times 452 + 0{\cdot}6 \times 38{,}400}{9\frac{5}{8}'' \times 18\frac{7}{8}''} = 590 \text{ lbs. per sq. in.}$$

$$q = \frac{0{\cdot}5 \times 37 \times 452 + 0{\cdot}6 \times 38{,}400}{9\frac{5}{8}'' \times 18\frac{7}{8}''} = \left\{\begin{array}{l}173 \text{ ft.-lbs.}\\ \text{per sq. in.}\end{array}\right.$$

$$v = \frac{9\frac{5}{8}'' \times \pi \times 150}{12 \times 60} = 6{\cdot}3 \text{ feet per second}$$

$$F = 173 \times 6{\cdot}3 \times 0{\cdot}05 = 54{\cdot}4 \text{ ft.-lbs. per second}$$

Thus the values for $q_{max.}$ and F do not exceed the limits laid down in Table IX.

Example 2.—In a 500 HP double acting two-cycle engine, Koert. D 2, 1 cyl., with 3 bearings, let A = 493 sq. inches; n = 100, and W = 26,500 lbs. Table VIII. gives us p_r = 85 lbs. per sq. inch, and by Table IX.—

Crank Pin.

$d = 12\frac{1}{2}''$ dia., $b = 14\frac{3}{8}''$ long

$$q_{max.} = \frac{370 \times 493}{12\frac{1}{2} \times 14\frac{3}{8}} = 1000 \text{ lbs. per sq. in. (approx.)}$$

$$q = \frac{85 \times 493}{12\frac{1}{2} \times 14\frac{3}{8}} = 233 \text{ lbs. per sq. in.}$$

$$v = \frac{12\frac{1}{2} \times \pi \times 100}{12 \times 60} = 5{\cdot}45 \text{ ft. per sec.}$$

$$F = 233 \times 5{\cdot}45 \times 0{\cdot}05 = 63{\cdot}25 \text{ ft.-lbs. per sec.}$$

Main Bearing (flywheel side).

$d = 12\frac{1}{2}$ dia., $b = 20\frac{7}{8}''$ long

$$q_{max} = \frac{185 \times 493 + 0{\cdot}6 \times 26{,}500}{12\frac{1}{2}'' \times 20\frac{7}{8}''} = 410 \text{ lbs. per sq. in.}$$

$$q = \frac{0{\cdot}5 \times 85 \times 493 + 0{\cdot}6 \times 26{,}500}{12\frac{1}{2}'' \times 20\frac{7}{8}''} = 141 \text{ lbs. per sq. in.}$$

$$v = \frac{12\frac{1}{2}'' \times \pi \times 100}{12 \times 60} = 5{\cdot}45 \text{ ft. per sec.}$$

$$F = 141 \times 5{\cdot}45 \times 0{\cdot}05 = 38{\cdot}5 \text{ ft.-lbs. per sec.}$$

Thus the *effect of the weight of the flywheel* on the main bearing in the first example is 0·73, in the second 0·43 of the value of q.

In the bearing on the side-shaft side, which is generally of the same dimensions as the main bearing, q is less, as the effect of the weight of the flywheel is *nil*. For the example given above q = 84 lbs. per sq. inch.

For the out-end bearing only part of the weight of the flywheel need be taken into account, and the pressure on this bearing is approximately = 0·5 × W. For this reason the out-end bearing is generally made somewhat smaller ($\frac{3}{4}$) than the main bearing.

If the same method be used for calculating a steam engine, and the effect of the weight of the flywheel be allowed for, the values given in Table X. are obtained for a 300 HP engine. The figures are taken from Haeder "Steam Engines," 7th Edition.

Table X.—Values of q and F for a 300 HP Steam Engine.

Type.	*Crank Pin.*	*Main Bearing.*
Single cylinder	$q_{max.}$ = 1065; q = 410 lbs. per sq. in., F = 57·5 ft.-lbs./sec.	$q_{max.}$ = 330; q = 114 + 43 = 157 lbs. per sq. in., F = 26·6 ft.-lbs./sec.
Compound . .	,, = 1050; q = 455 ,, ,, F = 55 ft.-lbs./sec.	,, = 330; q = 114 + 57 = 171 ,, ,, F = 25·6 ft.-lbs./sec.
Tandem . . .	,, = 1149; q = 500 ,, ,, F = 60·6 ft.-lbs./sec.	,, = 380; q = 147 + 57 = 204 ,, ,, F = 30·3 ft.-lbs./sec.

Since the differences in working out these calculations for both classes of engines are very slight, it seems desirable to adopt p_r instead of p as a uniform basis of calculation for steam as well as for gas engines.

Section II. The Main Bearings of a Gas Engine.

SMALLER engines are usually built with two bearings only, and as, in order to keep the crank shaft as light as possible, these bearings are placed close against the crank webs, the design of the main bearings should be very carefully considered. The types in most general use are—

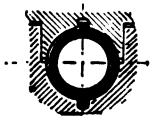


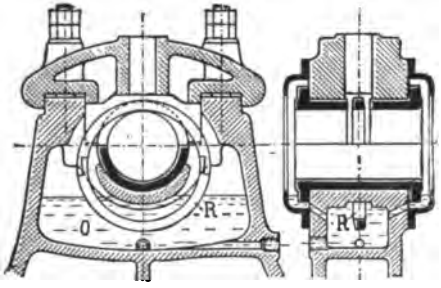
FIG. 62.

FIG. 63.

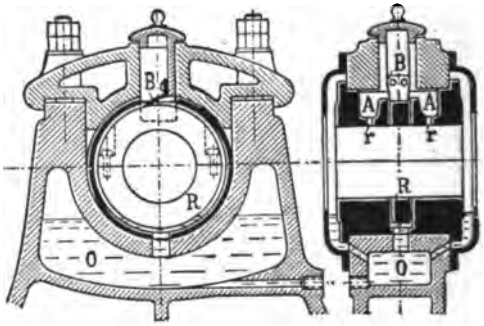
Adjustable bearings with horizontal cap (or keep), or inclined towards the centre (Figs. 62, 63).

Self-lubricating bearings with one or two loose oiling rings (Fig. 64, 65).

Self-lubricating bearings, with fixed oiling ring and wiper (Figs. 66, 67).



FIGS. 64, 65.—Loose Ring.



FIGS. 66, 67.—Fixed Ring (Wülfel).

The oiling ring R (Figs. 64, 65), is rotated with the crank shaft, dips into the reservoir O, and delivers oil to the bearing above.

In Figs. 66, 67, the oil on the surface of the fixed ring R on the shaft is deflected by the wiper plate B into the pockets A (cast in the top of the bearing brass), and flows from thence to the shaft through the holes *r*.

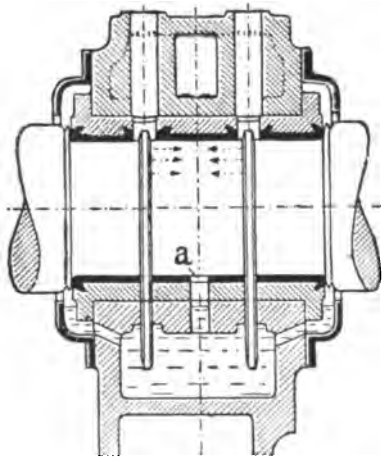


FIG. 68.—Two Loose Rings.

It has sometimes been found that bearings fitted with *two oiling rings* have a tendency, after the engine has been working some time, to run hot in the middle of the journal, *i.e.* in the part between the two rings. Engineers are not yet agreed as to the causes of this phenomenon. Some think that the oil flowing out of the lubricating rings draws out the rest of the oil by suction to the ends of the bearings, thus leaving the part of the journal between the rings dry.

Others are of opinion that, where there are two lubricating rings, the oil between them, as shown at Fig. 68, is driven inwards, and can neither escape nor be replenished.

In either case, to get over the difficulty, a small hole should be drilled in the bottom brass, midway between the rings (*a*, Fig. 68), through which the oil escapes from the middle of the journal.

Upon this subject see a paper in Haeder's "Zeitschrift," 1901, p. 197. Mention is there made of a method of lubricating the crank-shaft bearings of a steam engine by means of two loose oiling rings (Fig. 69), which did not fulfil its object, and the bearings frequently ran hot.

To remedy the evil, two fixed rings were fitted on the shaft, and brasses of the Wülfel type put in (Fig. 70), after which the bearing ran smoothly.

In other cases, however, loose rings have been found quite satisfactory.

The remarks in the author's book "Konstruiren und Rechnen," 3rd Edition, in the chapters on "Main Bearings" and "Transmission" should also be noted.

Four part bearings with wedge (see Fig. 71), or set screw adjustment (see Fig. 72), similar to those customary in steam engine practice, are not used in medium sized and small gas engines.

Where wedges are fitted, they are usually adjusted at the back, and care is taken not to put them in the front, where the pressure due to explosion comes upon the bearing.

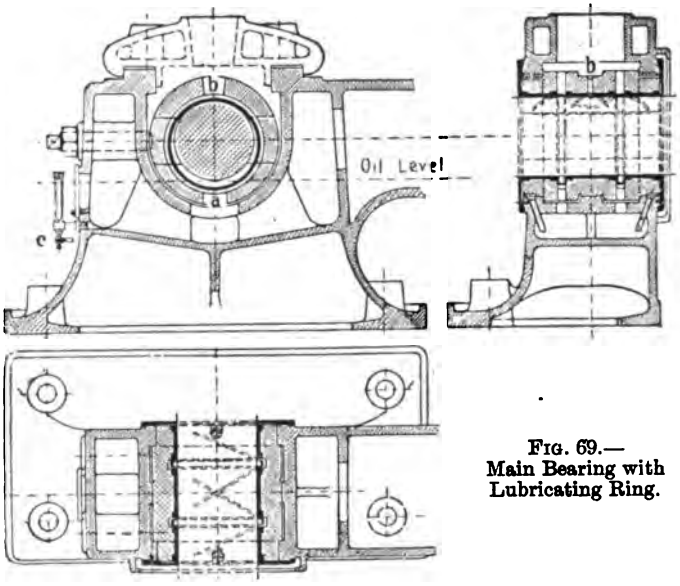


FIG. 69.—Main Bearing with Lubricating Ring.

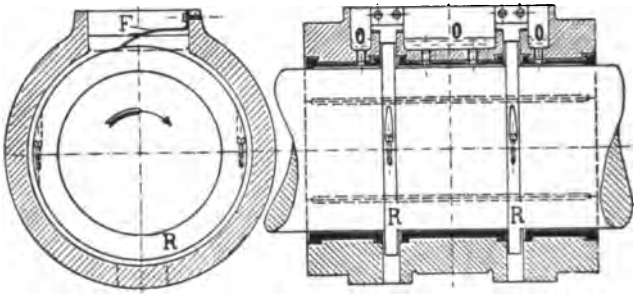
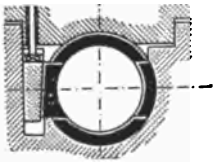


FIG. 70.—Lubricating Ring with Wiper (Wülfel System).

FIG. 71.



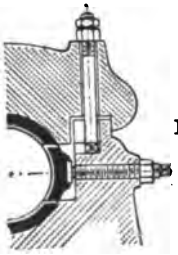


FIG. 72.

Rules for constructing Crank-shaft Bearings.*

1. The *brasses* should have as much *external bearing surface* as possible in the housing (Fig. 73). If they are made with narrow bearing strips, as shown in Fig. 74, the brass

* See also p. 106.

soon works loose in the housing because, owing to the great pressure on the bearing, and especially the shock due to the high explosion pressure, the metal of the brass becomes impressed into that of the housing (see "Die kranke Dampfmaschine," 3 Ed. p. 109).

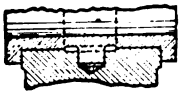


FIG. 73.

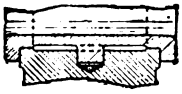


FIG. 74.

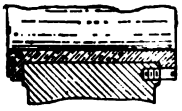


FIG. 75.

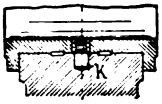


FIG. 76.

FIG. 77.

FIG. 78.

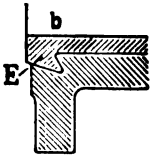


FIG. 79.

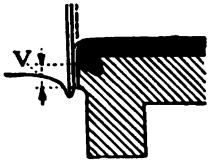


FIG. 80.

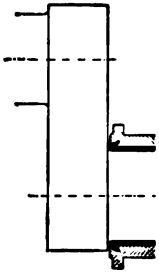
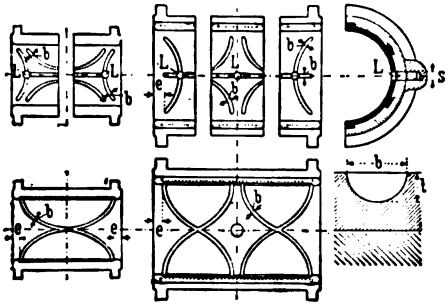


FIG. 81.
Top halves of Brass.



Bottom halves of Brass.

FIGS. 82–88.—Oil Grooves.
L, holes for steady pins to hold the brass in position.

2. It is easy to prevent the *brasses turning round* in the housing, by steady pins in the side flange (Fig. 75), or in the middle of the brass, Fig. 76 (see Table XI., Figs. 83–88).

3. If the brass is of bronze, it should have a *narrow facing* on the flange (Fig. 78), which makes it easier to fit into position than when there is a wide flange, as in Fig. 77.

4. The dove-tailing of the white metal lining of the bearing should be conical (Fig. 79), that it may not be forced out sideways by the pressure on it.

5. If the white metal projects from $\frac{1}{16}$ inch to $\frac{1}{8}$ inch as at E, Fig. 79, and the crank web or oil fender ring presses against it (Fig. 81), the white metal is easily forced out and broken up, and the side play of the shaft rapidly increases, and causes uneasy running of the engine. It seems, therefore, better to design the bearing as at Fig. 80, and not to allow the white metal to project; V should be about $\frac{3}{8}$ inch or $\frac{9}{16}$ inch.

6. *The grooves for the oil* should be shown on the drawing and not left to the caprice of the workman.

Design and dimensions are given in Table XI., Figs. 82–88.

Table XI.—Oil Grooves and Check Screws.
Dimensions in inches.

Diameter of bearing =	2 ins. to 3¾ ins.	4 ins. to 5¾ ins.	5¾ ins. to 7¾ ins.	7¾ ins. to 11¾ ins.	11¾ ins. to 15¾ ins.
	ins.	ins.	ins.	ins.	ins.
Width . . b =	$\frac{3}{16}$	$\frac{7}{32}$	$\frac{1}{4}$	$\frac{5}{16}$	$\frac{3}{8}$
Depth . . t =	$\frac{1}{16}$	$\frac{1}{8}$	$\frac{1}{8}$	$\frac{5}{32}$	$\frac{3}{16}$
Distance from end . . e =	$\frac{5}{16}$	$\frac{7}{16}$	$\frac{3}{4}$	$1\frac{3}{16}$	$1\frac{9}{16}$
Diam. of check screw . . s =	$\frac{3}{8}$	$\frac{1}{2}$	$\frac{5}{8}$	$\frac{5}{8}$	$\frac{3}{4}$

Table XIa.—Oil Grooves and Check Screws.
Dimensions in millimetres.

Diameter of bearing.	50 to 95.	100 to 145.	150 to 195.	200 to 295.	300 to 400.
Width . . . . b =	5	6	7	8	10
Depth . . . . t =	2	3	3	4	5
Distance from end of brass . . . e =	8	12	20	30	40
Diam. of check screw . . . . s =	10	13	16	16	20

7. To prevent waste of the oil running out on either side of the journal, and thrown off from the oil fender ring, it is caught in small *troughs*, F, and returned through the openings *r* into the oil reservoir below.

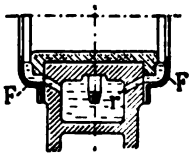
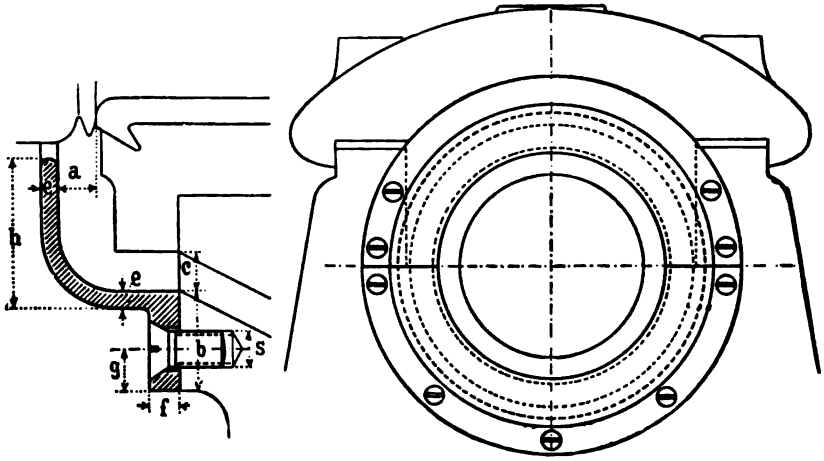


FIG. 89.

Table XII.—Oil Guard for Crank-shaft Bearing (Figs. 90, 91).



Dimensions in millimetres.

Diameter of bearing.	*a*	*b*	*c*	*e*	*f*	*g*	*h*	*s*
100 to 145	10	27	10	5	8	11	30	10
150 „ 195	12	33	12	6	10	14	40	13
200 „ 295	15	42	15	8	13	17	60	16
300 „ 400	20	50	20	10	16	20	80	20

Dimensions in inches.

Diameter of bearing.	*a*	*b*	*c*	*e*	*f*	*g*	*h*	*s*
ins.	ins.	ins.	ins.	ins.	ins.	ins.	ins.	ins.
4–5¾	$\frac{3}{8}$	$1\frac{1}{16}$	$\frac{3}{8}$	$\frac{3}{16}$	$\frac{5}{16}$	$\frac{7}{16}$	$1\frac{3}{16}$	$\frac{3}{8}$
5¾–7¾	$\frac{7}{16}$	$1\frac{1}{4}$	$\frac{7}{16}$	$\frac{7}{32}$	$\frac{3}{8}$	$\frac{9}{16}$	$1\frac{9}{16}$	$\frac{1}{2}$
7¾–11⅝	$\frac{9}{16}$	$1\frac{3}{8}$	$\frac{9}{16}$	$\frac{5}{16}$	$\frac{1}{2}$	$\frac{11}{16}$	$2\frac{3}{8}$	$\frac{5}{8}$
11¾–15¾	$\frac{3}{4}$	2	$\frac{3}{4}$	$\frac{3}{8}$	$\frac{5}{8}$	$\frac{13}{16}$	$3\frac{1}{8}$	$\frac{3}{4}$

The countersunk-headed screws *s* for securing the oil guards to the bearing must be so arranged that they do not penetrate the cap (see Figs. 90, 91), otherwise it would be impossible to remove the bearing cap without first taking out these screws. A sound joint should be made between the lower faces of the guard, as oil readily leaks through a bad joint.

8. The *cap* should only fit the bearing closely at two places, say at *b*, *b* (Fig. 92), and be clear of it at *a*, *a*, otherwise it is difficult to loosen and remove it. If the cap is large, it is easier to lift if two eyebolts, *o*, *o* (Fig. 92), are fitted in it.

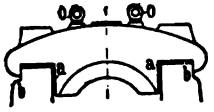


FIG. 92.

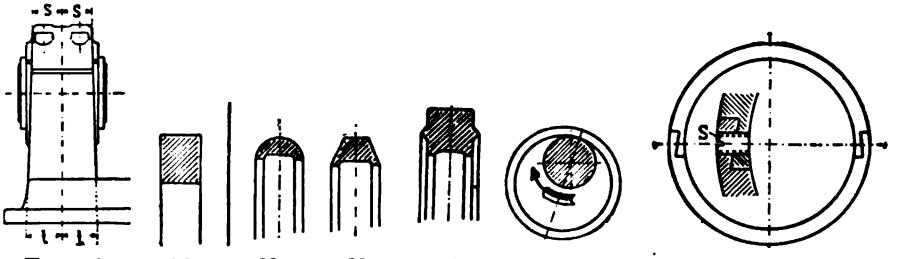
9. The cap is not often shaped as at Fig. 93, but usually as at Figs. 94, 95. The sharper curve in Fig. 95 looks better and stronger than the simple rounded outline at Fig. 94.

In many cases the whole top of the cap is shaped like a parabola. To arch the middle of the cap as much as possible, as in Figs. 105–112, p. 107, gives a pleasing effect.

FIGS. 93, 94, 95.

10. In the same way, to make the bearing and side frame *taper* from the plinth to the top, as in Fig. 96, has a neat and good appearance.

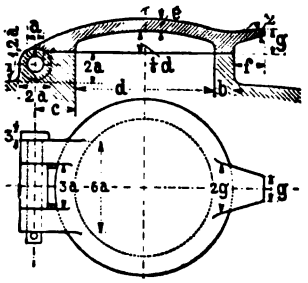
11. The cross section of the *oiling ring* of the crank-shaft bearing should not be rectangular (Fig. 97), otherwise it may stick at the sides, but either semi-circular (Fig. 98), or preferably trapezium-shaped (Fig. 99). When the shaft is rotating rapidly, the ring swings out of the perpendicular (Fig. 101). When in this position, it should be well clear of the housing and bearing; if it touched anywhere, the ring would not rotate with the shaft.



FIGS. 96, 97, 98, 99, 100, 101, 102.

It is advisable to make the oiling ring *in two parts*, as at Fig. 102, otherwise it cannot be easily removed when the engine is erected or dismantled. The two parts are dove-tailed one into the other at the joint, and held together by a small grub screw *s* (Fig. 102).

12. *The lids over the oil boxes* in the main bearing cap should be hinged on to the cap and of a neat pattern, and should have a projecting lip N (Figs. 103, 104) by which to lift them.



FIGS. 103, 104.—Oil-box Lids.

Table XIII.—Oil-box Lids (Figs. 103, 104).
Dimensions in millimetres.

d	*a*	*b*	*c*	*e*	*f*	*g*
20–40	4	6	12	4	10	8
41–70	6	8	16	5	12	10
70–100	8	10	20	6	15	15

Table XIIIa.—Oil-box Lids (Figs. 103, 104).
Dimensions in inches.

d	*a*	*b*	*c*	*e*	*f*	*g*
ins.	ins.	ins.	ins.	ins.	ins.	ins.
$\frac{3}{4}$–$1\frac{1}{2}$	$\frac{5}{32}$	$\frac{1}{4}$	$\frac{7}{16}$	$\frac{1}{8}$	$\frac{3}{8}$	$\frac{3}{16}$
$1\frac{9}{16}$–$2\frac{3}{4}$	$\frac{1}{4}$	$\frac{5}{16}$	$\frac{5}{8}$	$\frac{3}{16}$	$\frac{7}{16}$	$\frac{3}{8}$
$2\frac{3}{4}$–4	$\frac{5}{16}$	$\frac{3}{8}$	$\frac{3}{4}$	$\frac{7}{32}$	$\frac{9}{16}$	$\frac{9}{16}$

Table XIV.—Main Bearing with Oiling Ring. (Ring in two parts.) (Figs. 105–114.)

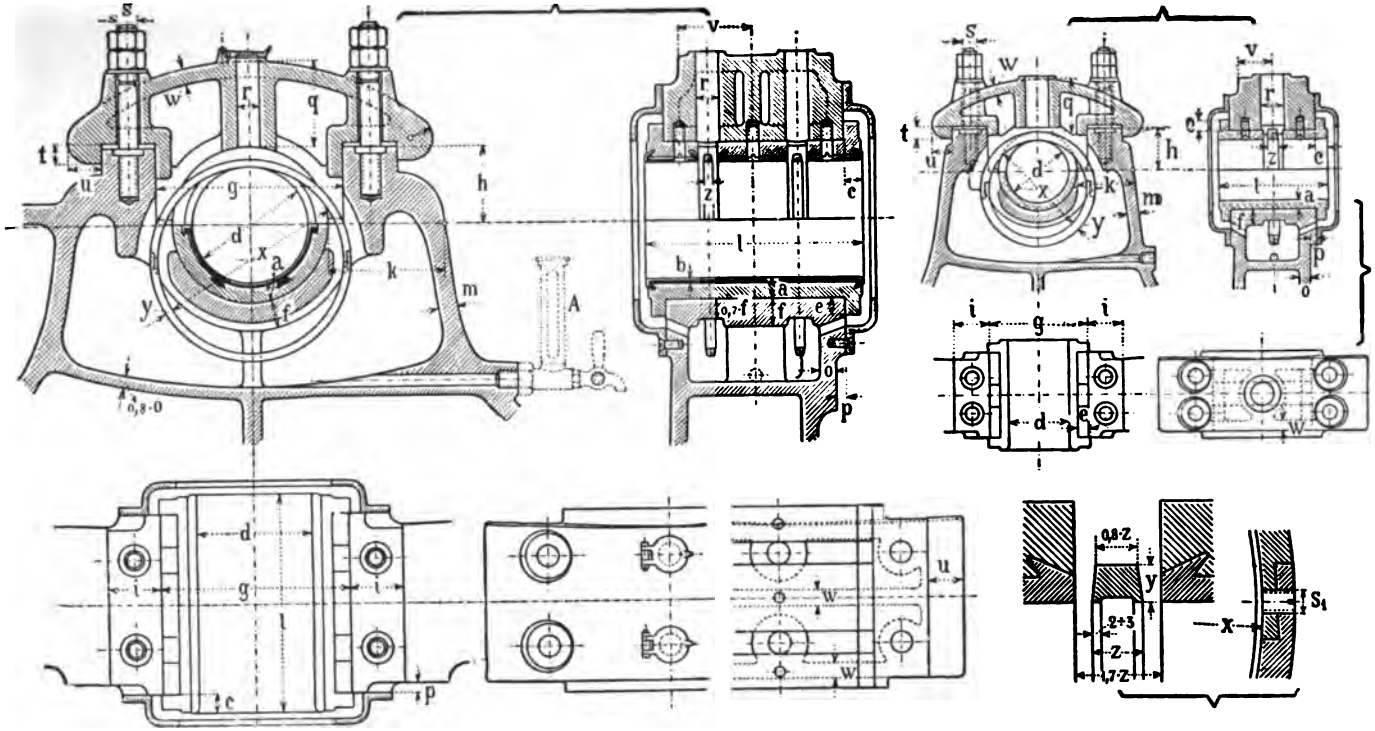
Dimensions in millimetres.

Engine.			Main bearing brass.						Housing of bearing.								Cap of main bearing.								Oiling ring.			
HP	S	D	*d*	*l*	*a*	*b*	*c*	*e*	*f*	*g*	*h*	*i*	*k*	*m*	*o*	*p*	*q*	*r*	*s*	No.	*t*	*u*	*v*	*w*	*x*	*y*	*z*	No.
0·6	125	80	**42**	72	7	—	11	9	14	72	30	40	60	10	10	7	45	22	16	2	7	15	17	—	70	10	14	1
1·2	160	100	**50**	90	8	—	12	10	15	84	35	40	70	11	10	8	52	22	16	2	8	17	21	—	85	10	14	1
2	190	125	**60**	108	9	—	13	10	17	100	42	55	75	12	10	9	58	25	20	2	9	20	27	—	95	12	16	1
3·5	220	145	**70**	125	10	—	14	12	19	114	48	55	80	14	11	9	65	25	20	2	10	23	35	—	110	12	16	1
7	270	185	**95**	170	13	—	16	12	24	146	65	55	85	16	12	10	80	30	20	4	14	30	54	10	150	14	18	1
11	310	220	**110**	195	15	—	18	14	28	170	75	55	90	18	14	11	90	35	20	4	16	35	62	11	175	14	18	1
17	350	260	**125**	230	23	6	20	15	31	204	85	70	100	20	16	12	95	40	26	4	18	40	80	12	200	16	20	1
25	400	300	**145**	265	26	7	22	16	35	234	95	70	120	22	18	13	110	45	26	4	22	45	90	13	230	16	20	1
33	450	330	**155**	290	28	7	24	17	38	250	100	80	135	24	20	14	115	50	29	4	23	47	100	14	250	18	22	1
42	500	360	**165**	305	30	8	26	18	40	270	110	80	160	26	22	15	125	55	29	4	24	50	105	16	265	18	22	1
55	550	400	**180**	340	32	8	28	20	45	290	120	85	180	30	26	16	135	35	32	4	26	55	114	18	290	12	16	2
80	600	480	**200**	395	35	9	32	22	51	324	130	85	210	32	28	18	150	40	32	4	28	60	145	20	320	14	18	2
132	700	600	**245**	480	42	10	35	25	65	390	160	95	230	34	30	20	180	50	35	4	32	70	170	24	390	16	20	2

For bearings up to about 4 inches diameter, the brasses or steps are of gun metal; for diameters of the bearings of 5 inches and upwards, cast iron or cast steel is used for the steps, with white metal linings.

* The bearings illustrated by Figs. 105–114 belong to German engines, and are dimensioned in millimetres in Table XIV. For the convenience of readers not accustomed to working in millimetres Table XIVa. gives the nearest equivalent in inches.

FIGS. 105-108. FIGS. 109-112.



FIGS. 113, 114.—Oiling Ring.

Figs. 105–108.—Main bearing for an engine of 400 mm. or 15¾″ cylinder diameter (D); and 500 mm. or 19¾″. Stroke S. Scale 1 to 12.

Figs. 109-112. „ „ „ 185 „ 7¼″ „ „ „ 270 „ 10⅝″ „ „ „

For a view in perspective of the parts of a self-lubricating bearing with two rings, see p. 108.

For standard measurements of oil guard, see p. 103; of oil-box lids, p. 104; of oil grooves, p. 102.

Parts of a Crank-shaft Bearing with Two Oiling Rings.

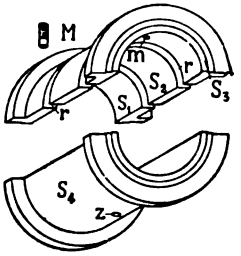


FIG. 115.

Main Bearing Brasses (see p. 107).

S_1 S_2 S_3 Top brasses } see bottom of last page.
S_4 Bottom brass }
r Slots for oiling rings.
M Steady pins for holes *m*.
z Opening in bottom brass, to allow the oil to circulate.

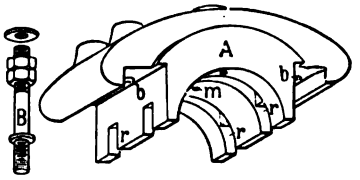


FIG. 116.

Main Bearing Cap or Keep (p. 107).

A Cap or keep.
rr Slots for oiling rings.
B Cap studs for holes *b*.

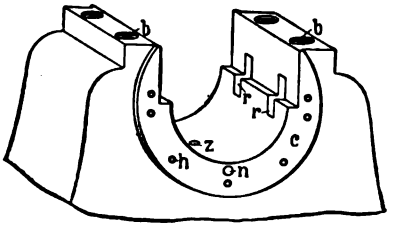


FIG. 117.

Housing of Bearing (p. 107).

b Holes for cap studs B.
rr Slots for oiling rings.
n Opening to allow the oil to pass from the oil guard to the reservoir (see Fig. 89, p. 103).
c Facing for oil guard.
h Tapped holes for studs fastening the oil guard to the bearing.
z Opening to allow the oil to circulate.

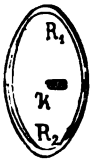


FIG. 118.

Oiling Rings (p. 107).

R_1 R_2 Halves of oiling rings.
K Grub screw, to hold them together.

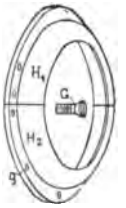
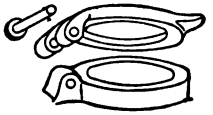


FIG. 119.

Oil Guard (Table XII., p. 103).

H_1 Upper, H_2 lower half of ring.
G Small screws fitting holes *g*.



FIGS. 120, 121, 122.

Hinged Oil-box Lids (Table XIII., p. 104).

To cover the oil boxes on the cap.

Section III. Frame and Cylinder.

Frame.

Gas engines, as already mentioned, are almost exclusively made with cranked shafts (*i.e.* not overhung crank pins). Figs. 123–125 show a usual type for engines up to about 18 inches stroke.

For frames with overhung cylinders (Fig. 123) the same rule holds good as for steam engines, and they are only used for small gas engines, because of the vibration and bending strain upon the cylinder. In larger engines they should be avoided, and the frame should support the cylinder throughout its length (Fig. 126).

R Frame.

E Liner.

L Cap for bearing.

M Water jacket cast with frame.

N Boss for oil-splasher guard (Fig. 141).

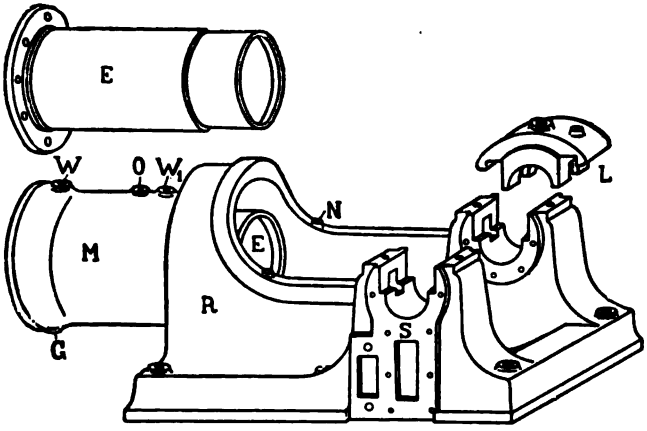
S Facing for side-shaft bracket, and oil-splasher guard protecting the side-shaft worm gear.

G Boss for cock for cooling water to jacket.

W Boss for cock for cooling water passing from breech end to water jacket.

W_1 Cooling water outlet.

O Boss for piston lubricator.



FIGS. 123–125.—Frame and Cylinder for Engines up to 18-inch Stroke.

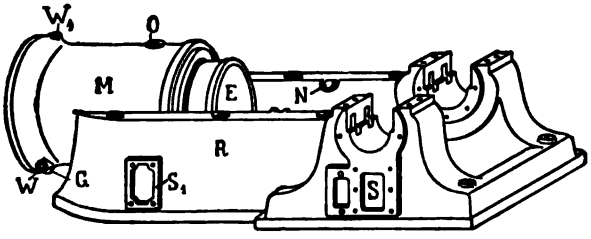


FIG. 126.—Frame and Cylinder for Engines above 20-inch Stroke.

Many makers build the frame as shown at Fig. 127, but the object of enclosing the crank in the casing W is not very apparent. This type of construction, originally adopted only for small power engines, has now been applied to larger sizes.

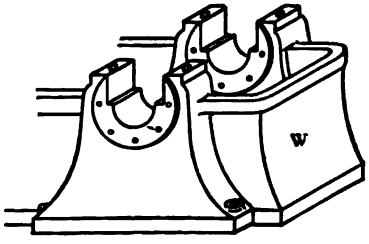


FIG. 127.

Rules for Constructing the Frame.*

1. The side of a frame, *i.e. the part between* the body of the main bearing and the hood of the frame, should be carried up as high as possible. For large engines the arrangement shown at Fig. 128 is not desirable; the heavy bending stresses coming on the side of the frame are more easily taken if the engine is built as at Fig. 129.

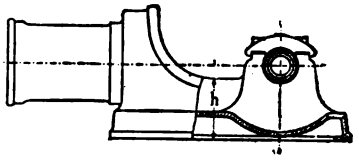


FIG. 128.

The part marked *a* should be as high, and the part marked *g* as shallow as possible.

If this rule be followed, the crank pit in engines of 30 HP and upwards will extend to the depth, *c*, below the bed plate.

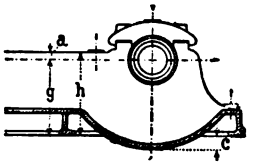


FIG. 129.

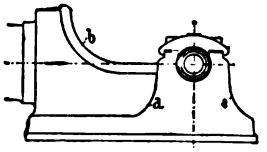


FIG. 130.

2. Attention must also be paid to the lines of the frame, and to make them *pleasing to the eye* they should, as far as possible, be parabolic, as for instance the lower edges *a a*, and the curve at the side, *b*, of the frame.

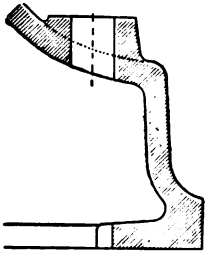


FIG. 131 (wrong).

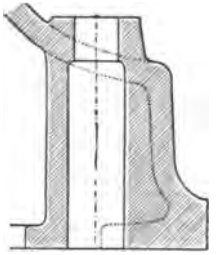


FIG. 132 (right).

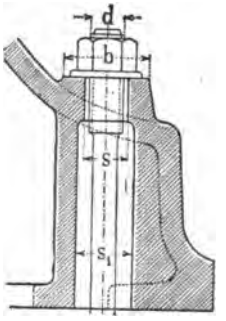


FIG. 133.

Table XV.—Holding down Bolts.

Dimensions in millimetres.

Stroke.	400	500	600	700	800	1000	1250
d =	$1\frac{1}{2}''$	$1\frac{5}{8}''$	$1\frac{3}{4}''$	$1\frac{7}{8}''$	2″	$2\frac{1}{4}''$	$2\frac{3}{4}''$ engl.
s =	48	52	56	60	65	75	87 mm.
s_0 =	60	66	70	76	85	95	115 ,,
s =	100	105	115	125	135	150	170 ,,

Table XVa.—Bolts to hold down the Frame, and Corresponding Holes (Fig. 133).

Dimensions in inches.

Stroke of engine.	16 ins.	20 ins.	24 ins.	28 ins.	32 ins.	40 ins.	48 ins.
	ins.	ins.	ins.	ins.	ins.	ins.	ins.
d =	$1\frac{1}{2}$	$1\frac{5}{8}$	$1\frac{3}{4}$	$1\frac{7}{8}$	2	$2\frac{1}{4}$	$2\frac{3}{4}$
s =	$1\frac{7}{8}$	2	$2\frac{3}{16}$	$2\frac{5}{16}$	$2\frac{9}{16}$	3	$3\frac{7}{16}$
s_1 =	$2\frac{5}{16}$	$2\frac{9}{16}$	$2\frac{3}{4}$	$2\frac{15}{16}$	$3\frac{5}{16}$	$3\frac{3}{4}$	$4\frac{1}{2}$
b =	$3\frac{13}{16}$	$4\frac{1}{8}$	$4\frac{1}{2}$	$4\frac{7}{8}$	$5\frac{5}{16}$	6	$6\frac{3}{4}$

3. *Good material* should be used for the frame, and care taken that the diminution in the thickness of the walls is made gradually, not suddenly. All sharp corners must be avoided. These rules apply specially to engines of *more than* 20″ *stroke.*

4. It is advisable to carry the *metal surrounding the lugs* for the bolts of the frame down to the bed plate, as shown at Fig. 132, to avoid the stresses due to bending which may otherwise be set up in the frame

* The rule given at p. 23 is here of special application.

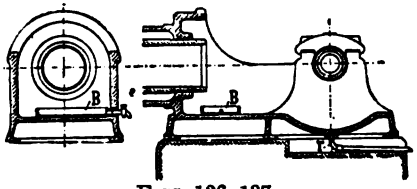
when these bolts are suddenly tightened up. To tighten them two or three men are often required, who use a piece of gas pipe, from 4 to 6 feet long, on the spanner.

In order not to expose the washers, the diameter of the holes may be rather smaller at the top than at the bottom.

5. The frame should have a *fairly large surface* in contact with the *foundation*, and be made as at Fig. 135, and not as at Fig. 134, although the part at the foot of the frame does not add much to its strength. For this reason the metal at *a*, Fig. 135, should not be made too thick.

FIGS. 134, 135.

6. An arrangement should be made to drain off the *lubricating oil* collecting in the crank pit through a small pipe (Fig. 137). The oil from the piston and piston rod should be caught in a separate drip tray B, and discharged, preferably through a cock at the side.



FIGS. 136, 137.

7. To prevent the *lubricating oil* from *leaking* through *to the foundation*, gutters (Fig. 138) should be provided round the bed plate.

8. A facing should be cast on the side of the frame, on to which the side-shaft brackets and worm-gear guard can be bolted (see Fig. 139). For medium sized and larger engines a second facing, S, Fig. 140, is needed to take a second bracket for the side shaft, if one is required, and also, if necessary, for the oil pump and governor.

FIG. 138.

9. An *oil-splasher guard* is fitted over the crank, to prevent the lubricating oil being

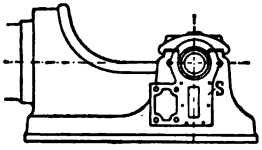


FIG. 139.

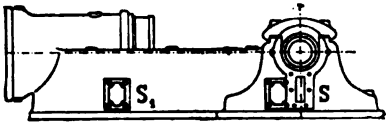


FIG. 140.—Extended Frame.

flung out from the crank pin. It should be of sheet iron, as light as possible, and easily removed. To afford convenient access to the crank shaft it is advisable to have the guard hinged, as at *s*, Fig. 141, that it may be raised into the position shown by the dotted lines.

It has stiffeners at the sides, either riveted (Figs. 142–144), or rolled into it (Fig. 145). Since, as we have said, it should be extremely light, it is best to strengthen it with a beading rolled in it, as at Fig. 145, but this is only applicable to engines up to 20 inches stroke; above that size separate stiffeners are necessary.

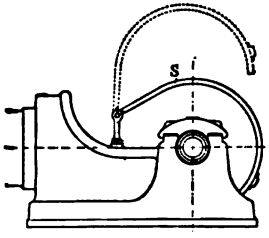


FIG. 141.

FIG. 142.

FIG. 143.

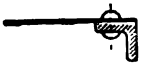
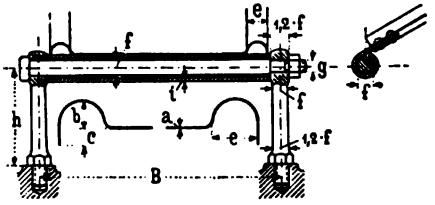


FIG. 144.

FIG. 145.

Table XVI.—Crank Splasher Guard (Figs. 146–148).



FIGS. 146–148.—Splasher Guard.

Dimensions in millimetres.

Stroke.	Splasher guard.				Bolts.		
	a	*b*	*c*	*e*	*f*	*g*	*i*
100–290	0·5	15	10	25	20	16	4
300–440	0·75	20	12	35	24	20	5
450–600	1	25	15	45	28	23	6

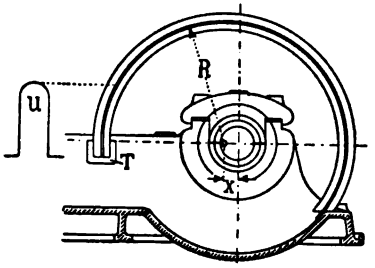


FIG. 149.—$x = 0{\cdot}13 \times$ Stroke. R to be determined later.

Dimensions in inches.

Stroke.	Splasher guard.				Bolts.		
	a	*b*	*c*	*e*	*f*	*g*	*i*
ins.	ins.	ins.	ins.	ins.	ins	ins.	ins.
4–11½	$\frac{1}{64}$	$\frac{9}{16}$	$\frac{3}{8}$	1	$\frac{3}{4}$	$\frac{5}{8}$	$\frac{3}{16}$
12–17½	$\frac{1}{32}$	$\frac{3}{4}$	$\frac{7}{16}$	$1\frac{3}{8}$	1	$\frac{3}{8}$	$\frac{3}{16}$
17¾–24	$\frac{1}{32}$	1	$\frac{9}{16}$	$1\frac{3}{4}$	$1\frac{1}{8}$	$\frac{7}{8}$	$\frac{1}{4}$

The dimensions of B and *h*, Figs. 146–148, must be determined when designing the oil splasher guard in conjunction with the frame.

In *large gas engines* the oil splasher guard cannot be made with a hinge, and to remove it easily it is fitted into two pockets, T, Fig. 149, cast in the sides of the frame.

A slot U is left in it, to clear the connecting rod.

The Cylinder.

The liner and water jacket are either *cast in one* (Fig. 150), or made in *two separate* parts, at at Fig. 151.

In small engines, in which there is no great pressure of water, the jacket is sometimes replaced by a *sheet iron casing* (Fig. 152), but in large engines the pressure of water in the jacket is as much as 14 to 20 lbs. per sq. inch (see chapter on "Water cooling").

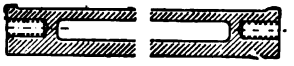


FIG. 150.—Jacket and Cylinder cast in one.

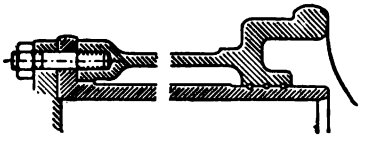


FIG. 151.—Water Jacket and Separate Cylinder Liner.

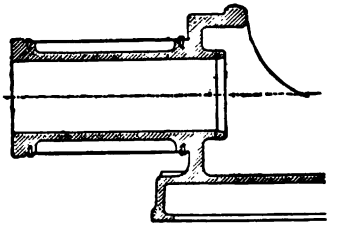


FIG. 152.—Sheet-iron Casing in place of Water Jacket.

The advantage of a *separate liner* is that specially good tough material, preferably close grained cast iron, can be used, and if the liner becomes worn, it can quickly be renewed or re-bored (see Fig. 151).

The cylinder is either bolted to the frame, Figs. 153, 154, or cast in one with it, Fig. 155. For securing the cylinder to the frame the bolts should be few in number, but as strong as possible. The bolts and studs in Figs. 153, 154 are calculated for a pressure on the piston of $D^2\frac{\pi}{4} \times 380$ lbs. per sq. inch, and an allowable stress per section area at bottom of thread of 4200 to 5600 lbs. per sq. inch.*

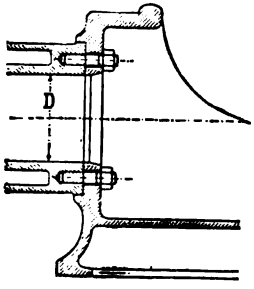


FIG. 153.
Cylinder and Jacket in one, bolted to Frame. Usual type for small engines.

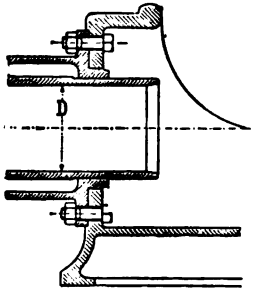


FIG. 154.
Jacket bolted to Frame. Separate Cylinder Liner. Seldom made.

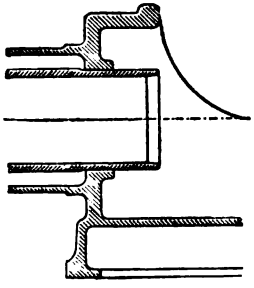


FIG. 155.
Jacket and Frame cast in one. Most usual and popular type.

If there is a separate liner the jacket is generally cast in one with the frame (Fig. 155).
To allow the liner to expand in conformity with the high temperatures developed, it should only be bolted at one end (usually the back), and permitted to expand freely at the other.

In many engines there is no special packing at *a*, Fig. 156, to prevent leakage from the water jacket. The jacket is heated up to 390° Fahr., then shrunk over the liner, which is turned about $\frac{1}{4}$ per cent. larger than the bore of the water jacket. Grooves

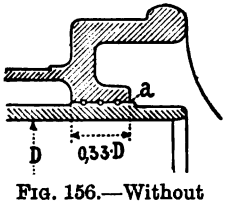


FIG. 156.—Without Packing.

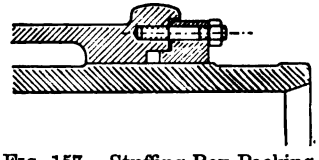


FIG. 157.—Stuffing Box Packing.

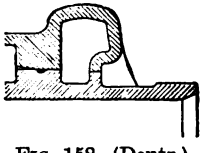


FIG. 158. (Deutz.)

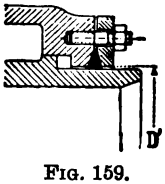


FIG. 159.
Packing of Rubber Rings.

are often turned in the jacket and liner at *a*, which are supposed to check any leakage. In the packing shown at Figs. 157–159, allowance is made for the expansion of the liner.

If the engine has a long stroke, it is not advisable to cast the cylinder and jacket in one, as at Fig. 150, because of the unequal expansion of the walls. Taking 20° C. = 68° F. as the temperature of the jacket, and 100° C. = 212° F. as that of the cylinder liner, these figures represent a difference in temperature of 212° − 68° = 144° F. Thus—

Length of cylinder	20″	30″	40″	60″	80″
Corresponding expansion	0·017″	0·026″	0·035″	0·051″	00·69″

* TRANSLATOR'S NOTE.—5000 lbs. per sq. inch should not be exceeded, if possible. See Table of Bolts at end of book.

Table XVII.—Liner Joint Ring and Stuffing Box, for packing the Joint between the Liner and Jacket, in Larger Single-acting Four-cycle Engines.

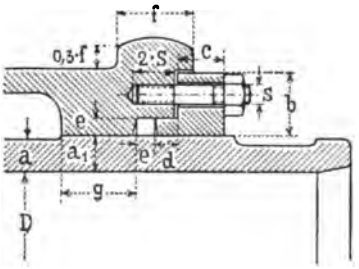


FIG. 160.

Dimensions in millimetres.

D	a	a1	b	c	d	e	f	g	Studs. s	Studs. No.
400	32	36	46	48	14	14	70	75	16	8
450	35	40	48	50	15	15	80	80	16	8
500	38	45	56	58	18	16	90	85	20	9
550	41	50	58	56	20	18	95	90	20	10
600	44	54	60	60	23	20	105	95	20	12
700	50	60	64	66	28	24	120	105	20	14
800	56	70	72	70	30	25	140	110	23	16
1000	68	82	74	78	35	28	160	120	23	20
1200	80	95	82	85	40	30	180	130	26	24

Dimensions in inches.

D	a	a1	b	c	d	e	f	g	Studs. s	Studs. No.
ins.	ins.	ins.	ins.	ins.	ins.	ins.	ins.	ins.	ins.	
$15\frac{3}{4}$	$1\frac{1}{4}$	$1\frac{3}{8}$	$1\frac{13}{16}$	$1\frac{7}{8}$	$\frac{9}{16}$	$\frac{9}{16}$	$2\frac{3}{4}$	3	$\frac{5}{8}$	8
$17\frac{3}{4}$	$1\frac{5}{8}$	$1\frac{9}{16}$	$1\frac{7}{8}$	$1\frac{15}{16}$	$\frac{9}{16}$	$\frac{9}{16}$	$3\frac{1}{8}$	$3\frac{1}{8}$	$\frac{5}{8}$	8
$19\frac{3}{4}$	$1\frac{1}{2}$	$1\frac{3}{4}$	$2\frac{3}{16}$	$2\frac{1}{16}$	$\frac{11}{16}$	$\frac{5}{8}$	$3\frac{1}{2}$	$3\frac{5}{8}$	$\frac{3}{4}$	9
$21\frac{7}{8}$	$1\frac{5}{8}$	2	$2\frac{1}{4}$	$2\frac{1}{16}$	$\frac{3}{4}$	$\frac{11}{16}$	$3\frac{3}{4}$	$3\frac{1}{2}$	$\frac{3}{4}$	10
$23\frac{5}{8}$	$1\frac{3}{4}$	$2\frac{1}{8}$	$2\frac{3}{8}$	$2\frac{5}{8}$	$\frac{15}{16}$	$\frac{3}{4}$	$4\frac{1}{8}$	$3\frac{7}{8}$	$\frac{3}{4}$	12
$27\frac{1}{2}$	2	$2\frac{3}{8}$	$2\frac{1}{2}$	$2\frac{9}{16}$	1	1	$4\frac{3}{4}$	$4\frac{1}{8}$	$\frac{3}{4}$	14
$31\frac{1}{2}$	$2\frac{3}{16}$	$2\frac{3}{4}$	$2\frac{7}{8}$	$2\frac{3}{4}$	$1\frac{3}{16}$	1	$5\frac{1}{2}$	$4\frac{3}{8}$	$\frac{7}{8}$	16
$39\frac{3}{8}$	$2\frac{5}{8}$	$3\frac{1}{4}$	$2\frac{15}{16}$	$3\frac{1}{16}$	$1\frac{3}{8}$	$1\frac{1}{16}$	$6\frac{1}{4}$	$4\frac{3}{4}$	$\frac{7}{8}$	20
$47\frac{1}{4}$	$3\frac{1}{8}$	$3\frac{3}{4}$	$3\frac{1}{4}$	$3\frac{5}{8}$	$1\frac{9}{16}$	$1\frac{3}{16}$	$7\frac{1}{16}$	$5\frac{1}{8}$	1	24

To pack (or stanch) the

Joint between the Breech End and the Liner

requires special care, as it is here that the highest pressures and temperatures are developed.

The joint between the two can be made in various ways.

A few methods are shown below. Scale 1 to 10.

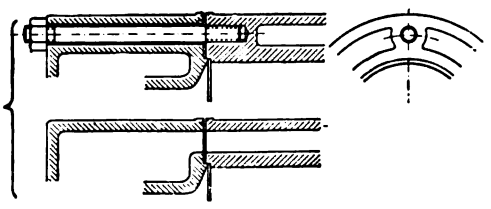


FIG. 161.—(Deutz, 6 HP). Diam. of cyl. (170 mm.) $6\frac{3}{4}''$; 4 studs $\frac{7}{8}''$ diam.

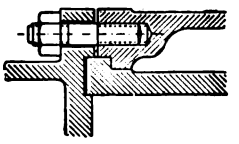


FIG. 162.—(Niel Engine). Diam. of cyl. (350 mm.) $13\frac{3}{4}''$; 8 $1\frac{1}{8}''$ studs.

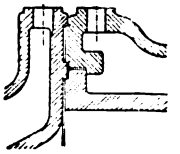


FIG. 163.—(Deutz, 60 HP). Cyl. diam. 390 mm. = $15\frac{5}{16}''$; 10 $1\frac{1}{4}''$ studs.

The design of breech end shown at Fig. 166 has many good points; the walls W of the combustion chamber are quite free of the flange V on the outer shell, so that all strains set up in casting are avoided; the disadvantage is that the liner

joint cannot be tightened after the engine has warmed up. For further particulars see Part B.

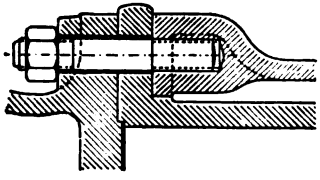


FIG. 164.—(Deutz, 60 HP). Diam. of cyl. (440 mm.) 17⅜"; 12 1⅝" studs.

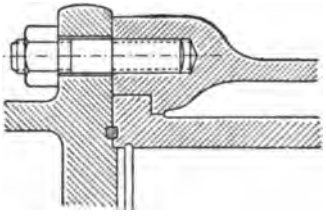


FIG. 165.—Diam. of cyl. (650 mm.) 25½"; 18 1¾" studs.

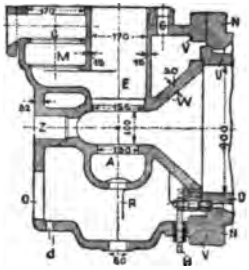


FIG. 166.—(Bollenckx). Cyl. diam. 400 mm. = 15¾".

For this packing only heat-resisting substances, such as asbestos, vulcanised fibre, etc., can, of course, be used.

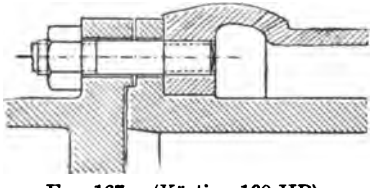


FIG. 167.—(Körting 160 HP). Diam. of cyl. (665 mm.) 26⅛"; 20 1¾" studs.

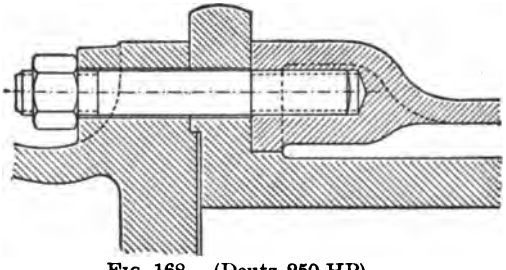


FIG. 168.—(Deutz, 250 HP). Diam. of cyl. (840 mm.) 33"; 24 2¼" studs.

In Figs. 169–172, the packing consists of an intermediate layer of thin *asbestos paper*, about $\frac{1}{25}$ inch thick. A stouter layer should be avoided, because it may easily be blown out

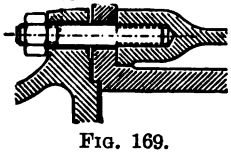


FIG. 169.

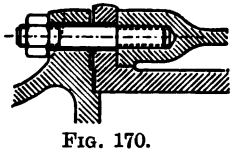


FIG. 170.

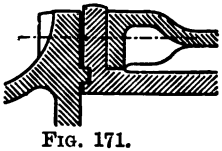


FIG. 171.

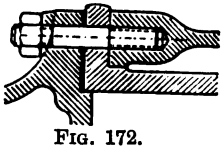


FIG. 172.

by the pressure of the gas, unless the bolts are unnecessarily tightened, to hold it in position.

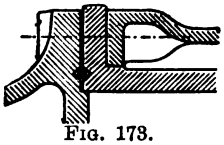


FIG. 173.

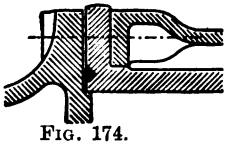


FIG. 174.

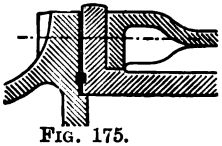


FIG. 175.

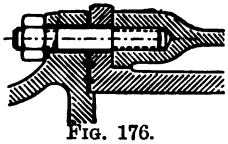


FIG. 176.

Asbestos string is often used for packing (Figs. 173–176).

If a joint is blown out when the engine is running, and a temporary packing has to be put in, *cardboard soaked in linseed oil* has sometimes been used for the purpose, but it does not always answer.

As the *arrangement of the bolts* must depend on the shape of the breech end, the calculation and standard sizes of these bolts will be found under "*Breech ends.*" Whenever possible, the designer should arrange that the pitch circle of the bolts, securing the breech end to the water jacket, should be the same as the mean diameter of the water jacket shell, as shown in Figs. 169–176, to avoid bending stresses on the flange.

For *standard dimensions* of the frame and cylinder, see pp. 116–120.

FIG. 178.

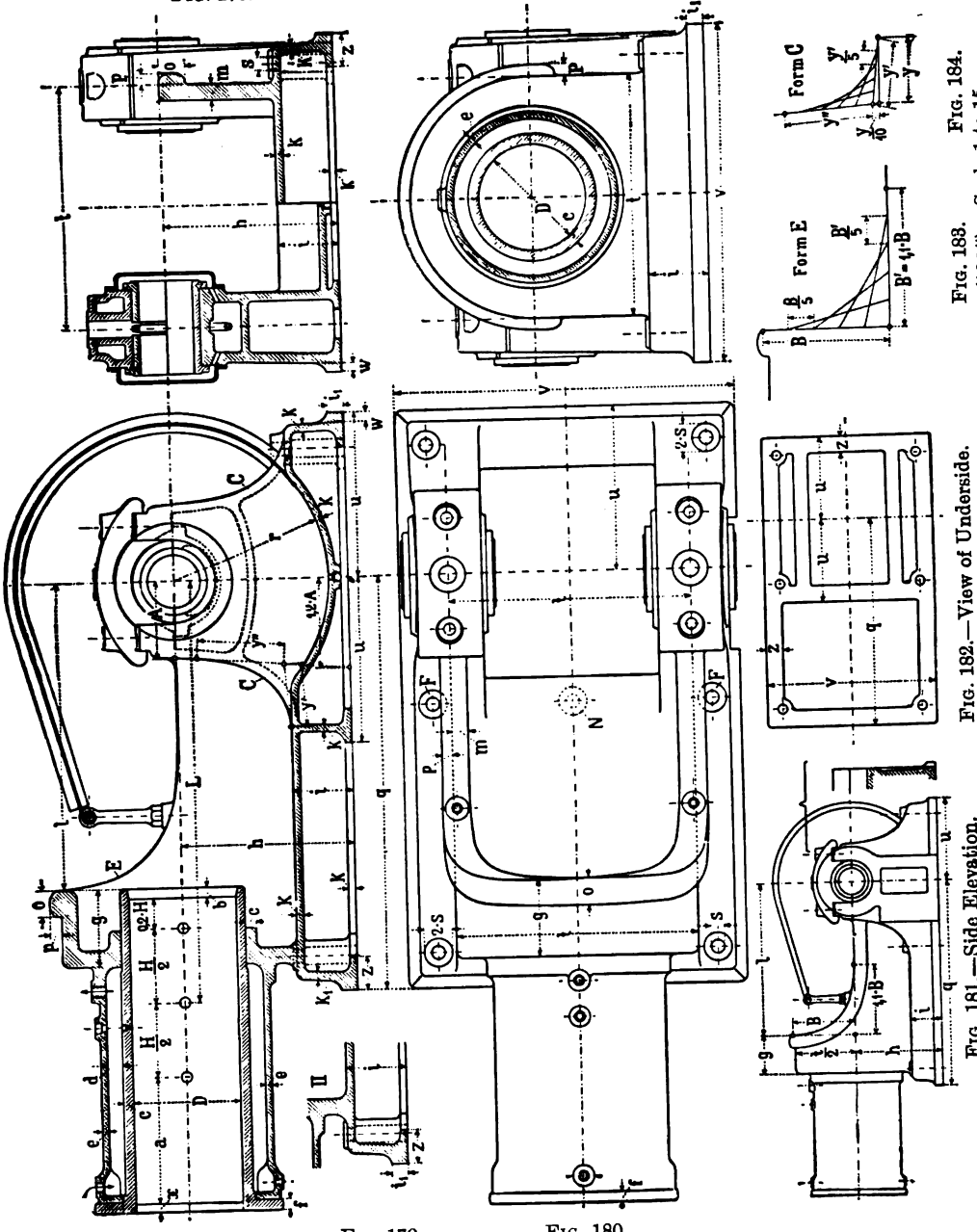


FIG. 177. FIG. 179. FIG. 180.

FIG. 181.—Side Elevation.

FIG. 182.—View of Underside.

FIG. 183.

FIG. 184.

FIGS. 177–184.—Frame and Cylinder of an Engine with D = 220 mm. ($8\frac{5}{8}''$); S = 310 mm. ($12\frac{3}{16}''$). Scale 1 to 15.

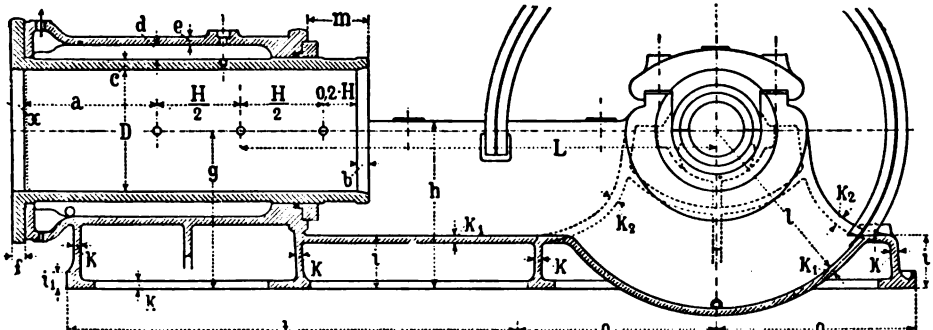
HP	S	D	ing rod. L	a	b	c	d	e	f	g	h	i	i_1	k	k_1	l	m	o	p	q	r*	s	t	u	v	w	z
	ins	ins.	ins.	ins.	ins.	ins	ins.	ins.	ins.	ins.	ins.	ins.	ins	ins.	ins.	ins.	ins.	ins.	ins.	ins.	ins.	ins.	ins.	ins.	ins.	ins	ins.
0·6	5	$3\frac{1}{4}$	$14\frac{3}{4}$	$3\frac{11}{16}$	$\frac{5}{16}$	$\frac{15}{32}$	$\frac{9}{16}$	$\frac{15}{32}$	$\frac{15}{32}$	$2\frac{1}{8}$	$6\frac{11}{16}$	$1\frac{15}{16}$	—	$\frac{3}{8}$	$\frac{15}{32}$	$10\frac{7}{8}$	$\frac{1}{8}$	$\frac{7}{8}$	$\frac{5}{16}$	$15\frac{1}{2}$	$5\frac{7}{8}$	$\frac{15}{16}$	$7\frac{7}{8}$	$5\frac{7}{8}$	10	—	$1\frac{3}{16}$
1·2	$6\frac{1}{4}$	4	$18\frac{1}{16}$	$4\frac{11}{16}$	$\frac{3}{8}$	$\frac{1}{2}$	$\frac{5}{8}$	$\frac{15}{32}$	$\frac{1}{2}$	$2\frac{3}{4}$	$7\frac{7}{16}$	$2\frac{5}{16}$	—	$\frac{3}{8}$	$\frac{15}{32}$	$13\frac{1}{4}$	$\frac{3}{8}$	$\frac{15}{16}$	$\frac{3}{8}$	$18\frac{1}{2}$	$6\frac{7}{8}$	$\frac{15}{16}$	$9\frac{7}{16}$	$6\frac{7}{8}$	$11\frac{3}{4}$	—	$1\frac{3}{16}$
2	$7\frac{1}{2}$	5	$21\frac{3}{8}$	$5\frac{7}{8}$	$\frac{15}{32}$	$\frac{1}{2}$	$\frac{11}{16}$	$\frac{15}{32}$	$\frac{5}{8}$	$3\frac{1}{2}$	$8\frac{13}{16}$	$2\frac{3}{4}$	—	$\frac{7}{16}$	$\frac{1}{2}$	16	$\frac{11}{16}$	$1\frac{3}{16}$	$\frac{7}{16}$	22	$7\frac{7}{8}$	$\frac{15}{16}$	$10\frac{3}{4}$	$7\frac{7}{8}$	$13\frac{3}{4}$	—	$1\frac{3}{8}$
3·5	$8\frac{3}{4}$	$5\frac{3}{4}$	$25\frac{3}{16}$	$6\frac{7}{8}$	$\frac{1}{2}$	$\frac{5}{8}$	$\frac{3}{4}$	$\frac{15}{16}$	$\frac{11}{16}$	$3\frac{15}{16}$	$10\frac{3}{16}$	$3\frac{1}{8}$	—	$\frac{15}{32}$	$\frac{9}{16}$	$18\frac{5}{8}$	$\frac{3}{4}$	$1\frac{3}{8}$	$\frac{15}{32}$	25	$9\frac{1}{4}$	$\frac{15}{16}$	$12\frac{5}{8}$	$9\frac{1}{4}$	$16\frac{3}{8}$	—	$1\frac{9}{16}$
5 } 7 }	$10\frac{5}{8}$	$6\frac{1}{8}$ $7\frac{1}{2}$	$31\frac{1}{2}$	$8\frac{1}{4}$	$\frac{11}{16}$	$\frac{11}{16}$	$1\frac{3}{8}$ $\frac{3}{4}$	$\frac{1}{2}$	$\frac{3}{4}$	$5\frac{1}{8}$	$12\frac{3}{8}$	$3\frac{15}{16}$	—	$\frac{1}{2}$	$\frac{11}{16}$	$23\frac{3}{8}$	$\frac{15}{16}$	$1\frac{9}{16}$	$\frac{5}{8}$	$30\frac{1}{2}$	$11\frac{1}{4}$	$1\frac{1}{16}$	$16\frac{7}{8}$	$11\frac{3}{8}$	$22\frac{3}{8}$	—	$1\frac{15}{16}$
9 } 11 }	$12\frac{1}{4}$	$7\frac{7}{8}$ $8\frac{3}{4}$	$34\frac{7}{16}$	$9\frac{13}{16}$	$\frac{13}{16}$	$\frac{3}{4}$	$1\frac{3}{8}$ $\frac{15}{16}$	$\frac{9}{16}$	$\frac{15}{16}$	$6\frac{1}{4}$	$13\frac{3}{4}$	$4\frac{7}{8}$	$\frac{15}{16}$	$\frac{5}{8}$	$\frac{13}{16}$	25	$1\frac{3}{16}$	$1\frac{15}{16}$	$\frac{3}{4}$	34	$12\frac{5}{8}$	$1\frac{1}{16}$	$19\frac{5}{8}$	$13\frac{3}{8}$	$27\frac{1}{2}$	$\frac{3}{4}$	$2\frac{3}{4}$
13 } 17 }	$13\frac{3}{4}$	9 $10\frac{1}{4}$	40	$11\frac{13}{16}$	1	$\frac{7}{8}$	$1\frac{11}{16}$ $1\frac{1}{16}$	$\frac{5}{8}$	$1\frac{1}{16}$	$7\frac{7}{16}$	$16\frac{1}{8}$	$6\frac{1}{16}$	$1\frac{3}{16}$	$\frac{21}{32}$	$\frac{15}{16}$	$29\frac{5}{8}$	$1\frac{3}{8}$	$2\frac{5}{16}$	$\frac{15}{16}$	$39\frac{3}{4}$	$14\frac{9}{16}$	$1\frac{1}{4}$	22	$15\frac{3}{8}$	$31\frac{7}{8}$	$\frac{15}{16}$	$3\frac{1}{2}$
20 } 25 }	$15\frac{3}{4}$	$10\frac{5}{8}$ $11\frac{7}{8}$	46	$13\frac{3}{16}$	$1\frac{3}{16}$	1	$1\frac{3}{4}$ $1\frac{3}{16}$	$\frac{5}{8}$	$1\frac{3}{16}$	$8\frac{5}{8}$	$18\frac{1}{2}$	$7\frac{1}{4}$	$1\frac{3}{8}$	$\frac{3}{4}$	$1\frac{1}{16}$	34	$1\frac{9}{16}$	$2\frac{3}{4}$	$1\frac{1}{16}$	$45\frac{1}{4}$	$16\frac{7}{8}$	$1\frac{1}{2}$	$25\frac{5}{8}$	$17\frac{3}{4}$	$36\frac{5}{8}$	$1\frac{1}{16}$	$4\frac{5}{16}$

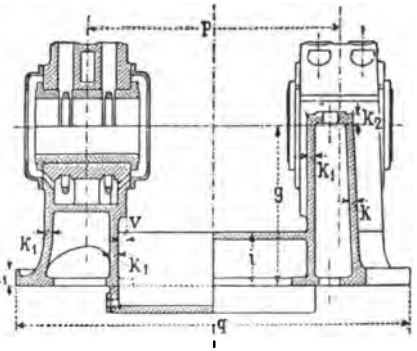
The beading O at the top may be omitted, and the beading at the side thinned off into the upper wall.

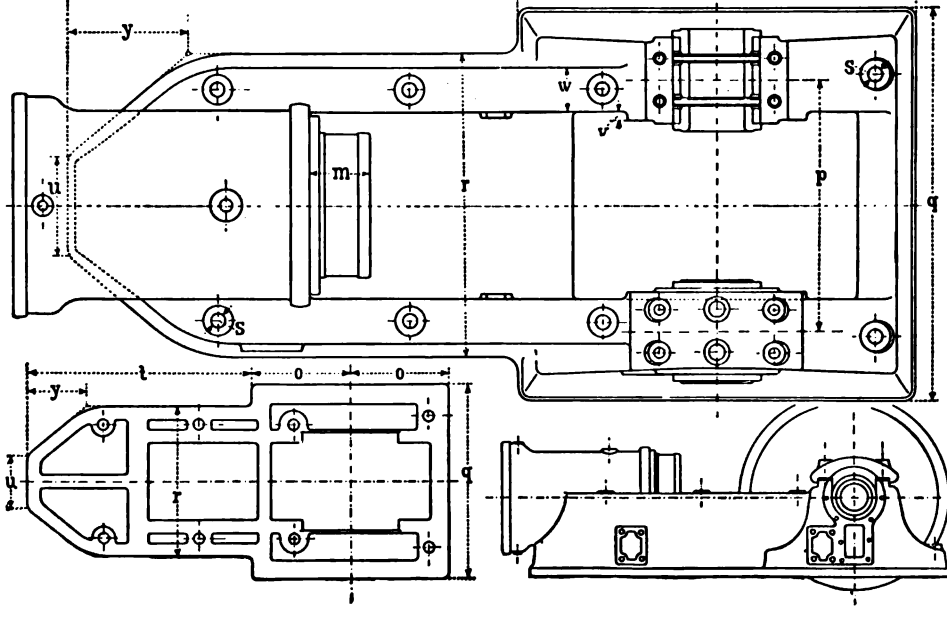
Length x (from back end of piston, at the end of the stroke, to liner joint) will be calculated later, with the dimensions of the breech end.

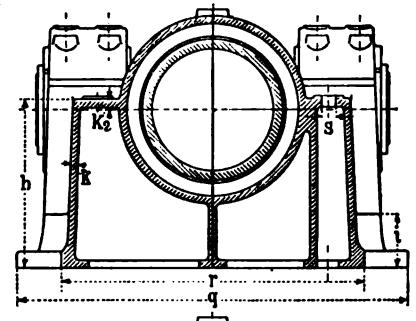
For engines of less than $10\frac{1}{2}$ inches stroke five bolts for holding down the frame are sufficient. As shown at Fig. 179, to the left in p. 116, the two bolts nearest the cylinder are in that case placed behind the frame, the bosses marked F are omitted, and replaced by the boss N, indicated by dotted lines in the centre of the frame.

* The frame can be made much lighter by reducing the dimension h, and allowing the crank pit to project below the base line of the frame; the dimension r can be settled after drawing in the connecting rod in several positions, and allowing good clearance (see next page).









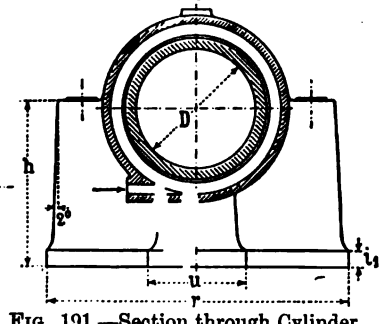


Fig. 189.—View of Under Side. Fig. 190.—Side Elevation. Fig. 191.—Section through Cylinder.

Figs. 185–191.—Frame and Cylinder for an Engine having D = 400 mm. ($15\frac{3}{4}''$); S = 550 mm. ($21\frac{5}{8}''$).

Table XIX.—Frame and Cylinder for Engines E 4 up to 700 mm. ($27\frac{1}{2}''$) Stroke (Figs. 185–191).

Engine.			Conng. rod.	Cylinder.								Frame.																
HP	S	D	L	a*	b	c	d	e	f	g	m	h	i	i_1	k	k_1	k_2	l†	o	p	q	r	s	t	u	v	w	y
29 33	450	310 330	1300	370	32	27	45 35	20	34	450	150	480	135	50	18	20	24	480	550	705	1100	865	42	1220	270	22	120	350
36 42	500	335 360	1450	390	34	29	50½ 38	22	38	480	160	515	150	55	20	22	26	525	600	750	1180	915	46	1380	300	24	130	370
45 55	550	370 400	1575	440	38	32	57 42	24	42	525	180	565	175	60	23	25	28	590	660	830	1300	1010	50	1490	330	25	150	400
65 80	600	430 480	1700	520	42	37	70 45	26	46	600	220	645	200	65	26	28	30	710	760	970	1520	1200	50	1520	380	26	175	500
110 132	700	540 600	1950	630	46	44	78 48	28	48	700	280	750	230	75	28	30	32	890	920	1180	1850	1500	53	1840	460	30	220	620

NOTE.—The frames and cylinders illustrated by Figs. 185–191 are for German engines, and are dimensioned in millimetres; in Table XIXA. the nearest equivalent in inches is given.

Table XIXa.—Frame and Cylinder for E 4 Engines up to $27\frac{1}{2}''$ Stroke (Figs. 185–191).

Engine.			Length of connecting-rod.	Cylinder.								Frame.																
HP	S	D	L	a*	b	c	d	e	f	g	m	h	i	i_1	k	k_1	k_2	l†	o	p	q	r	s	t	u	v	w	y
	ins.	ins.	ins.	ins.	ins.	ins.	ins.	ins.	ins.	ins.	ins.	ins.	ins.	ins.	ins.	ins.	ins.	ins.	ins.	ins.	ins.	ins.	ins.	ins	ins.	ins.	ins.	ins.
29 33	$17\frac{3}{4}$	$12\frac{1}{4}$ 13	$51\frac{1}{4}$	$14\frac{9}{16}$	$1\frac{1}{4}$	$1\frac{1}{16}$	$1\frac{3}{4}$ $1\frac{3}{8}$	$\frac{3}{4}$	$1\frac{5}{16}$	$17\frac{11}{16}$	$5\frac{7}{8}$	$18\frac{7}{8}$	$5\frac{5}{16}$	$1\frac{5}{16}$	$\frac{11}{16}$	$\frac{3}{4}$	$\frac{15}{16}$	$18\frac{7}{8}$	$21\frac{5}{8}$	$27\frac{3}{4}$	$43\frac{1}{2}$	34	$1\frac{5}{8}$	48	$10\frac{5}{8}$	$\frac{13}{16}$	$41\frac{11}{16}$	$13\frac{3}{4}$
36 42	$19\frac{3}{4}$	$13\frac{3}{16}$ $14\frac{1}{4}$	57	$15\frac{5}{16}$	$1\frac{5}{16}$	$1\frac{1}{8}$	2 $1\frac{1}{2}$	$\frac{13}{16}$	$1\frac{1}{2}$	$18\frac{7}{8}$	$6\frac{1}{4}$	$20\frac{1}{4}$	$5\frac{7}{8}$	$2\frac{1}{8}$	$\frac{3}{4}$	$\frac{13}{16}$	1	$20\frac{5}{8}$	$23\frac{5}{8}$	$29\frac{1}{2}$	$46\frac{1}{2}$	36	$1\frac{13}{16}$	$54\frac{1}{2}$	$11\frac{13}{16}$	$\frac{15}{16}$	$5\frac{1}{8}$	$14\frac{9}{16}$
45 55	$21\frac{5}{8}$	$14\frac{9}{16}$ $15\frac{3}{4}$	62	$17\frac{5}{16}$	$1\frac{1}{2}$	$1\frac{1}{4}$	$2\frac{1}{4}$ $1\frac{3}{8}$	$\frac{15}{16}$	$1\frac{5}{8}$	$20\frac{5}{8}$	$7\frac{1}{16}$	$22\frac{1}{4}$	$6\frac{7}{8}$	$2\frac{5}{16}$	$\frac{7}{8}$	$\frac{15}{16}$	$1\frac{1}{16}$	$23\frac{3}{16}$	$25\frac{15}{16}$	$32\frac{5}{8}$	$51\frac{1}{4}$	$39\frac{3}{4}$	$1\frac{15}{16}$	$58\frac{1}{2}$	13	$\frac{15}{16}$	$5\frac{7}{8}$	$15\frac{3}{4}$
65 80	$23\frac{5}{8}$	17 $18\frac{7}{8}$	67	$20\frac{7}{16}$	$1\frac{5}{8}$	$1\frac{7}{16}$	$2\frac{3}{4}$ $1\frac{3}{4}$	1	$1\frac{13}{16}$	$23\frac{5}{8}$	$8\frac{5}{8}$	$25\frac{3}{8}$	$7\frac{7}{8}$	$2\frac{9}{16}$	1	$1\frac{1}{16}$	$1\frac{3}{16}$	$27\frac{15}{16}$	$29\frac{7}{8}$	$38\frac{3}{16}$	60	$47\frac{1}{4}$	$1\frac{15}{16}$	60	$14\frac{15}{16}$	1	$6\frac{7}{8}$	$19\frac{11}{16}$
110 132	$27\frac{1}{2}$	$21\frac{1}{4}$ $23\frac{5}{8}$	$76\frac{3}{4}$	$24\frac{3}{4}$	$1\frac{13}{16}$	$1\frac{11}{16}$	$3\frac{1}{16}$ $1\frac{7}{8}$	$1\frac{1}{16}$	$1\frac{7}{8}$	$27\frac{1}{2}$	11	$29\frac{1}{2}$	$9\frac{1}{16}$	$2\frac{15}{16}$	$1\frac{1}{16}$	$1\frac{3}{16}$	$1\frac{1}{4}$	35	$36\frac{3}{16}$	$46\frac{1}{2}$	73	59	$2\frac{1}{16}$	$72\frac{1}{2}$	$18\frac{1}{16}$	$1\frac{3}{16}$	$8\frac{5}{8}$	$24\frac{3}{8}$

The *bearing surfaces* between the frame and the foundation, especially the part below the main bearing, should be as ample as possible; in other words, the base O must be well splayed out. The outline of the *side of the foot* of the main bearing must be parabolic, as in Fig. 184, p. 116.

For a view in *perspective* of this frame, see Fig. 126, p. 109. For *standard types* and *dimensions* of bearings, see p. 107; of crank shafts, p. 73. The dimensions of x will be given later, with those of the breech end.

In large engines hand holes should be arranged in the water jacket, for removing mud and scale; these holes must be compensated by casting a deep facing around them (equivalent to the metal removed). Cleaning doors should be fitted over these holes, secured by studs; screwed plugs should be avoided.

* The dimensions of a correspond to those in column 3, Table I., p. 140.

† When calculating the length of l, room must be allowed for the connecting rod, that it may not foul the splasher guard anywhere.

Calculation of the Strength of a Frame.

To explain the method adopted in calculating the strength of the frame, we will consider a few very simple types to begin with. The cross section where there is *greatest danger* of fracture is at A (Fig. 192). *All dimensions to be given in inches.*

1. Simplest Type of Frame,

In which the neutral axis of the two beams connecting the sides of the frame to the bearing and cylinder is in the same plane as the middle of the shaft. Thus at cross-section A, Fig. 192 (the metal of the beams being in pure tension)—

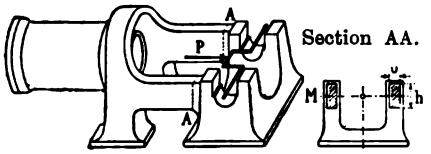


FIG. 192. FIG. 192A. M centre line of crank shaft.

$$\text{Tension } k_t = \frac{P}{2f} \text{ in lbs. per sq. in.} \quad . \quad (124)$$

where $P = \frac{\pi}{4} D^2 p$, the load on the piston in lbs. per sq. inch and f = sectional area of one side frame in sq. inches; if the whole area be taken $f = b \times h$.

The thickness of the shell of the water jacket should also be calculated for the tensile stress, P, without taking into account the foot of the frame, this stress should never exceed 1400 lbs. per sq. inch (the allowable stress is low in order to allow for a slight shifting of the core).

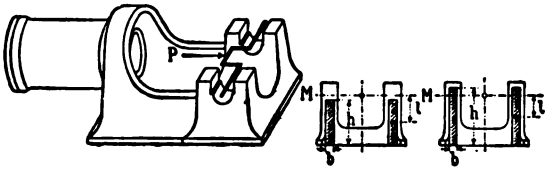


FIG. 193. FIG. 193A. FIG. 193B. MM centre line of crank shaft.

2. Where the neutral axis of the side beams lies below the **centre** or axial line of the **crank shaft.**

Here the dangerous cross section is close to the bearing (Figs. 193–193B). In this case the metal of the frame is exposed to a combined stress due to tension and bending. Let l = *Distance of centre of gravity* of the cross section of the frame from the axis of the crank shaft MM, Z_b = *Modulus of resistance* of the cross section of the frame, in cubic inches, reduced to the horizontal axis of its centre of gravity, f = area of the *cross section* in sq. inches (i.e. $b \times h$).

From Figs. 193–193B. Bending moment $M_b = P \times l$ . . . . . (125)

$$\text{Stress due to bending } k_b = \frac{M_b}{2Z_b} \text{ in lbs. per sq. inch} \quad . \quad . \quad . \quad . \quad . \quad . \quad (126)$$

$$\text{,, ,, tension } k_t = \frac{P}{2f} \text{ ,, ,,} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (127)$$

$$\text{Combined stress } k = k_b + k_t \text{ ,, ,,} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (128)$$

If the cross section were rectangular—

$$k = k_b + k_t = \frac{P \times l}{\frac{2bh^2}{b}} + \frac{P}{2bh} \text{ in lbs. per sq. inch} \quad . \quad . \quad . \quad (129)$$

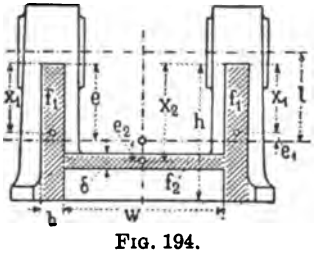


FIG. 194.

3. Where the side frames are connected by a bottom plate, as shown at p. 116.

The stresses are calculated in the same way as before, but to determine the modulus of resistance of the section is more complicated. For instance, the cross section of the frame (Fig. 194) must be divided up into three sections, f_1, f_2, f_1, the area of each of these sections calculated, and the distance of their centres of gravity from the top of the section, i.e. x_1 for f_1 x_2, for f_2, as shown in Fig. 194, must be found. The centre of gravity of the whole cross section will be—

$$e = \frac{2(f_1 x_1) + f_2 x_2}{2f_1 + f_2} \text{ in inches} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (130)$$

The moment of inertia is determined by the equation—

$$I = \frac{2bh^3}{12} + 2bhe_1^2 + \frac{w\delta^3}{12} + w\delta e_2^2 \text{ in inches}^4 \quad . \quad . \quad . \quad . \quad (131)$$

$$\text{Modulus of resistance } Z_b = \frac{I}{e} \text{ in inches}^2 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (132)$$

$$\text{Bending moment} = M_b = Pl \text{ inch-lbs.} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (133)$$

$$\text{Stress due to bending } k_b = \frac{M_b}{Z_b} \text{ lbs. per sq. inch} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (134)$$

$$\text{,, tension } k_t = \frac{P}{\Sigma . f} \text{ lbs. per sq. inch} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (135)$$

$$\text{Combined stress } k = k_b + k_t \text{ lbs. per sq. inch} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (136)$$

4. Hollow cast-iron box frame. The design is taken from Fig. 186, p. 119.

Distance of the centre of gravity (see Fig. 195).

$$e = \frac{f_1x_1 + f_2x_2 + f_3x_3 + f_4x_4}{f_1 + f_2 + f_3 + f_4} \text{ in inches} \quad . \quad . \quad (137)$$

The moments of inertia of the separate areas are determined as follows:—

$$I_1 = \frac{b_1h^3}{12} + b_1he_1^2 \text{ inches}^4$$

$$I_2 = \frac{w_1b_2^3}{12} + wb_2e_2^2 \text{ inches, and so on.}$$

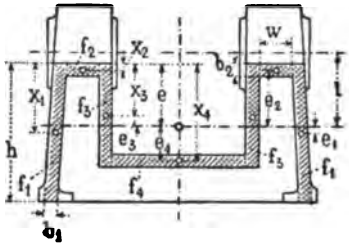


FIG. 195.

The moment of inertia of the whole section is the sum of the separate moments of inertia—

$$I = I_1 + I_2, \text{ etc.} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (138)$$

$$\text{Modulus of resistance } Z_b = \frac{I}{e} \text{ inches}^2 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (139)$$

$$\text{Again, the stress due to tension } k_t = \frac{P}{f_1 + f_2 + \ldots} \text{ in lbs. per sq. inch} \quad . \quad . \quad (140)$$

$$\text{Stress due to bending } k_b = \frac{M_b}{Z_b} = \frac{Pl}{Z_b} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (141)$$

$$\text{Combined stress } k = k_b + k_t \text{ in lbs. per sq. inch} \quad . \quad . \quad . \quad . \quad (142)$$

Allowable stress, or safe working load

$$k = 1400 \text{ to } 2000 \text{ lbs. per sq. inch.*} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (143)$$

Calculation of the Front Part of the Frame (Fig. 196).

In this case—

$$M_b = 0{\cdot}5\ Pl \text{ inch-lbs.} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (144)$$

$$k_b = \frac{M_b}{Z_b} \text{ lbs. per sq. inch} \quad . \quad . \quad . \quad . \quad . \quad . \quad (145)$$

Z_b can either be more accurately calculated from the moment of inertia of this box section, or approximately determined by the formula—

$$Z_b = 0{\cdot}9\ \frac{bh^2}{6} \text{ inches}^2 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (146)$$

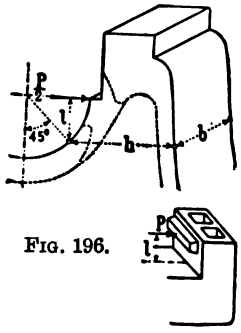


FIG. 196.

Allowable stress or safe working load—

$$k = 1400 \text{ to } 2000 \text{ lbs. per sq. inch} \quad . \quad . \quad . \quad (147)$$

* As explained in more detail in Haeder's "Konstruiren und Rechnen," 2nd Edition, p. 288, a cast-iron frame of the above type gives way when loaded up to 9800 lbs. per sq. inch, whereas a straight bar frame will bear a load up to 34,000 lbs. per sq. inch without breaking.

Calculation of the thickness of the Cylinder Wall.

Let D = diameter of cylinder in inches. p = Explosion pressure in lbs. per sq. inch. k_t = Stress due to tension in lbs. per sq. inch. δ = Thickness of cylinder wall in inches, then according to Bach, "Maschinenelemente"—

$$\delta = \frac{D}{2}\left(\sqrt{\frac{k_t + 0{\cdot}4p}{k_t - 1{\cdot}3p}} - 1\right) \text{ inches} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (148)$$

As the full pressure of explosion only occurs at the back end of the cylinder, where it is stiffened with a deep flange, and as, to increase the cooling action of the jacket, the metal of the cylinder should be as thin as possible, considerable allowance may be made for the stress due to tension, namely—

Allowable stress, k_t or safe working load about = 4200 lbs. per sq. inch . (149)

With a maximum pressure p = 440 lbs. per sq. inch and k_t = 4200 lbs. we get from equation 148—

$$\delta = \frac{D}{2}\left(\sqrt{\frac{4200 + 0{\cdot}4 \times 440}{4200 - 1{\cdot}3 \times 440}} - 1\right) = \text{about } 0{\cdot}5D \text{ inches} \quad . \quad . \quad . \quad . \quad (150)$$

Having regard to a possible reboring of the cylinder, after a few years' wear, then—

$$\delta = \mathbf{0{\cdot}05D} + x \textbf{ inches} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (151)$$

Table XX.—Values for x and δ in Centimetres.

D =	10	15	20	25	30	35	40	45	50	60	70	80	100	120
0·05D =	0·5	0·75	1·0	1·25	1·5	1·75	2·0	2·25	2·5	3·0	3·5	4·0	5·0	6·0
x =	0·9	0·95	1·0	1·05	1·1	1·15	1·2	1·25	1·3	1·4	1·5	1·6	1·8	2·0
δ =	1·4	1·7	2·0	2·3	2·6	2·9	3·2	3·5	3·8	4·4	5·0	5·6	6·8	8·0

Table XXa.—Values of x and δ in Inches.

Diam. of cyl. D =	4″	6″	$7\frac{7}{8}$″	$9\frac{7}{8}$″	$11\frac{7}{8}$″	$13\frac{3}{4}$″	$15\frac{3}{4}$″	$17\frac{3}{4}$″	$19\frac{3}{4}$″	$23\frac{5}{8}$″	$27\frac{9}{16}$″	$31\frac{1}{2}$″	$39\frac{3}{8}$″	$47\frac{1}{4}$″
0·05D =	$\frac{3}{16}$	$\frac{5}{16}$	$\frac{3}{8}$	$\frac{1}{2}$	$\frac{9}{16}$	$\frac{11}{16}$	$\frac{13}{16}$	$\frac{7}{8}$	1	$1\frac{3}{16}$	$1\frac{3}{8}$	$1\frac{9}{16}$	2	$2\frac{3}{8}$
x =	$\frac{3}{8}$	$\frac{3}{8}$	$\frac{3}{8}$	$\frac{7}{16}$	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{9}{16}$	$\frac{9}{16}$	$\frac{5}{8}$	$\frac{11}{16}$	$\frac{13}{16}$
δ =	$\frac{9}{16}$	$\frac{11}{16}$	$\frac{3}{4}$	$\frac{15}{16}$	$1\frac{1}{16}$	$1\frac{3}{16}$	$1\frac{5}{16}$	$1\frac{3}{8}$	$1\frac{1}{2}$	$1\frac{3}{4}$	$1\frac{15}{16}$	$2\frac{3}{16}$	$2\frac{11}{16}$	$3\frac{3}{16}$

In larger cylinders, with diameters of 24 inches and upwards, the thickness of the cylinder may often be reduced gradually, or step by step towards the crank end, to economise weight and quantity of metal. Large cylinders are also often stiffened with circumferential ribs, in which case the thickness of the walls δ is somewhat less than the values given in Table XX.

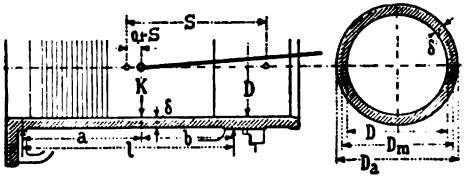


FIG. 198.

As an *open-ended or trunk piston also serves as a cross head*, a further stress due to bending comes upon the cylinder, for which allowance must be made in large gas engines, and especially in engines with a long stroke.

As shown at p. 126, the maximum turning moment under ordinary working conditions occurs when the piston has travelled through 10 per cent. of its stroke from the dead point. In this position the pressure on the piston is about 0·8p., with a maximum explosion pressure p = 440 lbs. per sq. inch.

And from equation 2, p. 126, vertical thrust $k = \frac{\pi}{4}D^2 \times 0{\cdot}8 \times 440 \times 0{\cdot}11 = 38 \times D^2$ in lbs. (152)

Again, $M_b = k\frac{ab}{l}$ in lbs. per sq. inch; $Z_b = 0{\cdot}8D_m{}^2\delta$ in inches^3; $k_b = \frac{M_b}{Z_b}$ in lbs. per sq. inch (153)

Neglecting the weight of the piston itself, deflection $= k \times \frac{a^2b^2}{EI_3l}$ in inches*. . . . (154)

E = 12,000,000 modulus of elasticity for cast iron.
I = 0·05 ($D_a{}^4 - D^4$) moment of inertia in inches.

In ordinary four-cycle engines, with the thicknesses of cylinder given in Table XX., the deflection does not exceed $\frac{1}{5000}$ inch or $\frac{1}{200}$ mm. for small, and $\frac{1}{1250}$ inch or $\frac{1}{50}$ mm. for large engines, and is thus very slight.

* See Haeder's "Konstruiren und Rechnen," 2nd edit., p. 292.

Section IV. The Pistons of the Engine.

The style of piston varies with the type of engine, whether *single* or *double acting*, and the kind of crosshead guide adopted. The most usual types are given below, Figs. 6 to 15, and are shown in outline in Figs. 1 to 5.

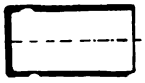


FIG. 1. Open-ended Piston without water cooling.

FIG. 2. Open-ended Piston, water cooled.

FIG. 3. Box Piston, generally water cooled.

FIG. 4. Piston Rod without water cooling.

FIG. 5. Piston Rod, water cooled.

1. Open-ended or Trunk Pistons (see also p. 3).

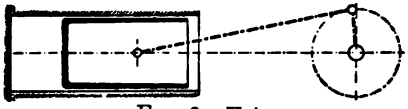


FIG. 6. E 4. Guide inside Cylinder. Most usual type for engines up to 28″ stroke.

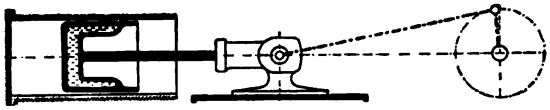


FIG. 7. E 4. Outside Guide (seldom used, and only for large engines).

FIG. 8. E 4.—Outside Crosshead Slipper and Guide, and Guide Piston T inside (large Nuremberg engine).

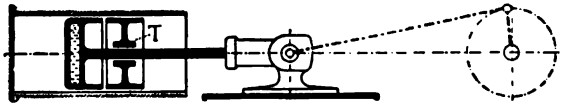


FIG. 9. R 4.—Piston of first cylinder forms guide.

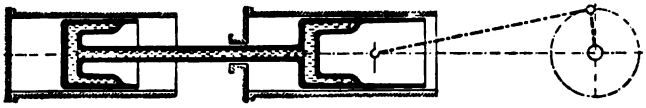
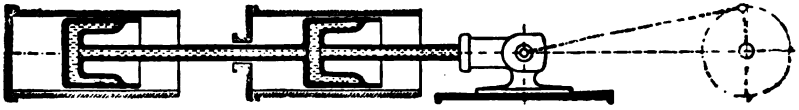


FIG. 10. R 4. Outside Guide.



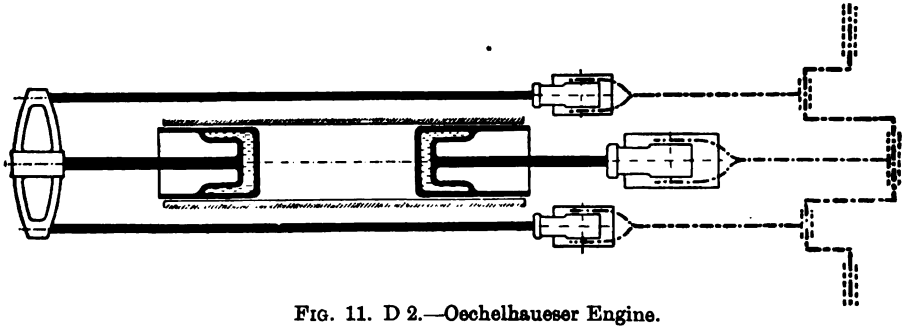


FIG. 11. D 2.—Oechelhaueser Engine.

2. Box Pistons (see p. 3).

Open-ended or trunk pistons are almost universally used for small, and box pistons for larger engines. In double-acting engines the latter are, of course, a necessity.

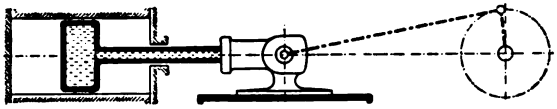


FIG. 12. D 4.—Without Tail Rod and Guide (not often made).

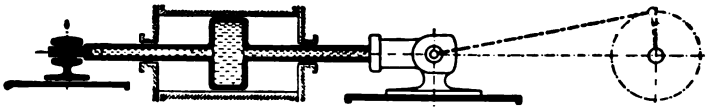


FIG. 13. D 4.—Tail Rod and Guide (usual type).

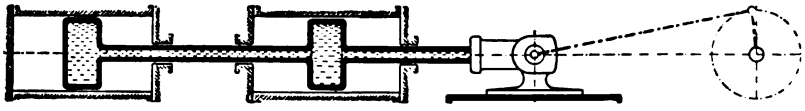


FIG. 14. R D 4.

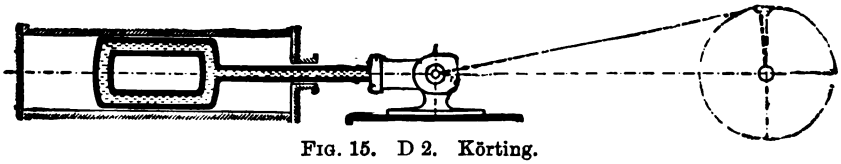
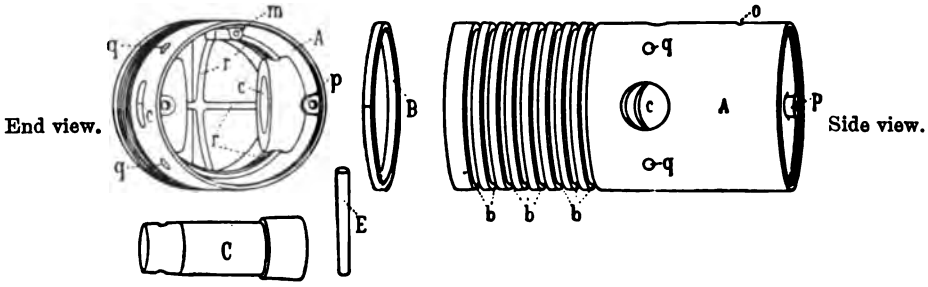


FIG. 15. D 2. Körting.

Open-ended or Trunk Piston for Single-acting Four-cycle Engines.

For engines up to 120 HP an *open-ended trunk piston without water cooling* is almost invariably adopted.

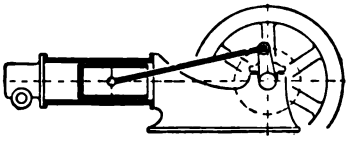


FIGS. 16, 17.—Open-ended Piston.

A Trunk of piston, *B* piston ring, C piston pin or crosshead, *b* grooves for piston rings B, *c* bored hole to receive piston pin C, E taper pin for piston pin C, *m* boss for oil hole (p. 130), *o* oil hole (p. 130), *p* lugs for turning the piston in the lathe, and to take the eye bolts when fitting it into the cylinder, *q* hole for taper pin E, *rr* stiffening ribs.

For standard measurements, see pp. 139, 140.

The piston also acts as a slide block, and to take the vertical thrust due to the obliquity of the connecting rod.



Pressure on the Guide.

The pressure with which the body or trunk of the piston is forced downwards against the liner (Fig. 18) is given by the equation—

$$K = P \tan \beta \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

in which P = load on the piston at a given point in the stroke, in lbs., and
β = angle formed by the connecting rod and the axial line of the cylinder.

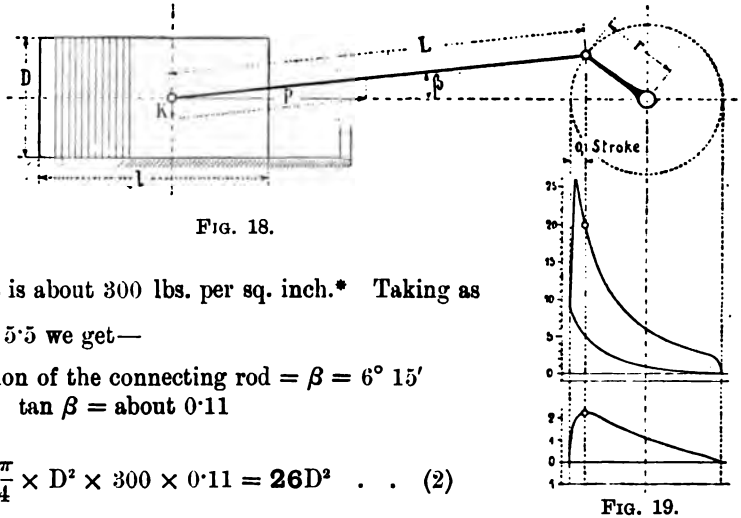


FIG. 18.

FIG. 19.

This pressure K, equation 1, will reach its maximum when the piston has travelled through 10 per cent. of its stroke. According to the diagram, Fig. 19, the pressure in the cylinder at this point is about 300 lbs. per sq. inch.* Taking as the mean value $\frac{L}{r} = 5{\cdot}5$ we get—

Angle of inclination of the connecting rod = $\beta = 6^\circ\ 15'$
$\tan \beta$ = about 0·11

and further—

$$\mathbf{K} = \frac{\pi}{4} \times D^2 \times 300 \times 0{\cdot}11 = \mathbf{26}D^2 \quad . \quad . \quad (2)$$

Assuming that the piston pin is in the centre of the bearing surface of the piston, and taking as the width of the bearing surface—

Width of bearing surface = 0·85D (as the pressure falls only on one-third of the circumference of the piston) (see Fig. 20).

Length of bearing surface = 0·8l (deducting the piston rings) (Fig. 18).

The pressure on the bearing surface will thus be—

$$q = \frac{26D^2}{0{\cdot}85D \times 0{\cdot}8l} = \frac{38{\cdot}2D}{l} \text{ lbs. per sq. inch} \quad . \quad (3)$$

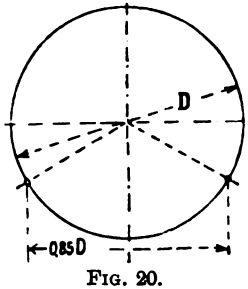


FIG. 20.

* With a pressure of explosion of 380 lbs. per sq. inch.

If a mean ratio be taken for $\frac{D}{l}$ of 0·5 to 0·65, equation 3 gives—

$$q = 19 \text{ to } 25 \text{ lbs. per sq. inch of area of guide} \quad . \quad . \quad . \quad . \quad (4)$$

(In steam engines the pressure on the cross head is usually $q = 30$ to 40 lbs. per sq. inch, if calculated from the pressure over the whole area of the piston, and when the crank is in its highest position.)

Position of the Piston Pin in the Piston.

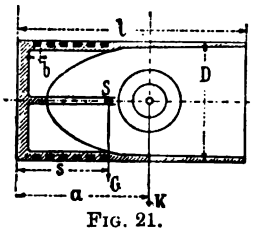


FIG. 21.

In practice the distance from the centre of the piston pin to the back of the piston is—

$$a = 0{\cdot}4l \text{ to } 0{\cdot}7l$$

To maintain a uniform pressure on the surface of the guide the moment to the right must equal the moment to the left (see Figs. 21–23). Therefore—

$$W \times s + K \times a = (K + W) \times 0{\cdot}5\,(l + zb) \quad . \quad . \quad . \quad . \quad (5)$$

where W = weight of the piston in lbs.

s = distance of the centre of gravity of the piston from the back end in inches.

a = distance from the centre of the piston pin to the back end in inches.

K = vertical pressure on the piston pin in lbs.

z = number of piston rings.

b = width of piston rings in inches.

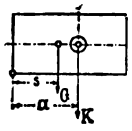


FIG. 22.

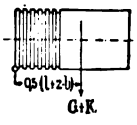


FIG. 23

$0{\cdot}5(l + zb)$ = distance from the back end of the piston to the middle of the bearing surface ($z \times b$ is not reckoned as forming part of the bearing surface, as the piston rings must be free to move in the grooves).

Taking as the approximate mean value—

$$s = 0{\cdot}4l\,; \quad z \times b = 0{\cdot}2l$$

from equation 5 we get—

$$a = l\left(0{\cdot}6 + 0{\cdot}2\,\frac{W}{K}\right) \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (6)$$

As the weight W does not much affect the result, the last part of the equation may be written = zero, whence—

$$a = \text{about } 0{\cdot}6l \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (7)$$

In a water-cooled piston the temperature at the back end is practically the same as that of the cylinder liner (Fig. 25), but when the piston is *not cooled* (*i.e.* cooled only by the air) (Fig. 24), the difference in temperature, assuming the back end of the piston to be 250° C. (= 482° F.) and that of the liner 90° C. (= 194° F.), will be 482 − 194 = 288° F. The piston will consequently expand more than the liner, namely—

$$D \times \frac{0{\cdot}00061 \times 288}{100} = \text{abt. } \frac{1}{570}\,D \quad . \quad . \quad . \quad . \quad . \quad . \quad (8)^*$$

* The expansion of cast iron is about 0·00061 inch per inch of length, for every 100° Fahr. difference of temperature.

As the length affected by this unequal expansion is about $\frac{1}{10}$ of the total length of the piston, it will be advisable to reduce its diameter conically for one-tenth of the length, measured from the back end, by—

$$\frac{1}{570}\text{D at back end} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (9)$$

For example, in a cylinder 40 inches diameter, the diameter of the piston for one-tenth of its length at the back end will be $39\frac{15}{16}$ inches, and the clearance $\frac{1}{16}$ inch. Similarly in a liner of 20 inches, the diameter of the piston for one-tenth of its length at the back end will be $19\frac{31}{32}$ inches, and the clearance $\frac{1}{32}$ inch.

As the back end of such an uncooled piston, where the diameter is reduced, does not, strictly speaking, form part of the bearing surfaces, the piston pin may be moved slightly forward, the length of a increased, and therefore in this case—

Distance between centre of piston pin and back end of piston $a = 0{\cdot}63l$ . . . . . (10)

(In practice $a = 0{\cdot}4l$ to $0{\cdot}7l$.)

FIG. 24.
Piston not cooled.

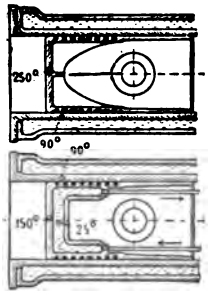


FIG. 25.
Water-cooled Piston.

The nearer the piston pin is to the back end of the piston, the more injurious is the effect upon it of the temperatures in the combustion chamber.

In the accompanying figure (Fig. 26) a better circulation of air to cool the piston is said to be obtained by the fanlike action of the end of the piston rod.

In practice, in engines of large diameter (*i.e.* single-acting four-cycle engines with cylinders above 24 inches diameter), it is always necessary to have a separate water-cooling system for the piston.

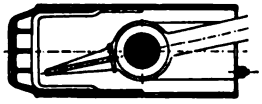


FIG. 26.

Rules for Constructing the Piston.

1. *The material used* is cast iron, or, for pistons of large engines, cast steel lined with white metal.

2. The *weight* of the piston *should be reduced as much as possible*, that is, it should be made no heavier than is usually considered necessary, with ample allowance for stress, and shifting of the core in casting. The weight of the piston affects the balance of the reciprocating parts (see Chapter IV.).

3. Two or more lugs (N N, Fig. 27) should be cast near the front end of the piston, to *hold it when machining it* on the lathe, and to take eye bolts for mounting and withdrawing it from the cylinder.

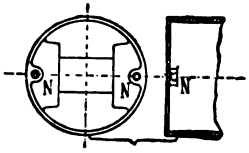


FIG. 27.

4. The piston should be removable at the crank-shaft end of the cylinder (see Fig. 28); this should be borne in mind when designing the frame or bed. It should never be removed at the back end, if it can possibly be avoided, for with pressures of 350 to 450 lbs. per sq. inch *it is difficult*, when replacing it, *to make the joint* between the cylinder and breech end *tight*.

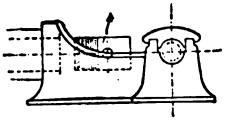


FIG. 28.

5. The piston is *lubricated* by oil delivered under pressure from a pump. The oil pipe should be screwed into the liner but free in the water jacket (to allow for difference in expansion); the oil should be delivered behind the second ring, when the piston is on the outer dead point.

The larger the engine, the more important it is to lubricate the piston efficiently, because the area of the cylinder increases with the diameter × by the stroke, while the proportion

of dust (*i.e.* products of combustion) increases with the charge, or as $D^2 \times S$. Assuming gas of the same composition, and that the exhaust gases in both cases contain the same proportion of dust per unit volume, then the volume of the products of combustion will be *five times greater per square inch of cylinder area* in a cylinder 20 inches diameter than in one 4 inches diameter. It is also more difficult to distribute the oil uniformly over a *large* than over a *small surface.*

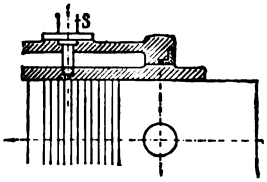
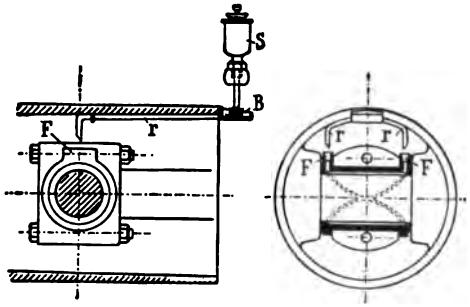
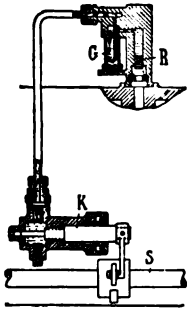


FIG. 29.



FIGS. 30, 31.



Lubricating the Cylinder (Deutz) German Patent. Nr. 97, 144.

This method of lubrication consists of an oil pump with a plunger piston K, shaped like a differential piston, and working to and fro along a screw. The oil in small and variable quantities is forced into the engine cylinder through the glass sight feed G and the non-return valve R behind it. The object of this valve is to check the pressure during the explosion stroke, that the oil drops may be properly formed, and not blown off, and that the sight feed glass, which controls the quantity of oil passing to the cylinder, may not be dimmed.

As the lubricating oil collects in the cylinder and evaporates, it forms a highly inflammable gaseous vapour, which might easily cause pre-ignition. In the lower part of the piston, therefore, five or six grooves 6 inches long are turned, in which the oil is carried off and flung to the bottom of the frame, the piston, of course, being always in a horizontal position.

The Piston and Piston Pin should be separately lubricated.

Fig. 30 shows a trough B at the front end of the piston, with a wiper which wipes off the oil from the wick on the sight feed lubricator S. From the trough the oil passes through small pipes *r r*, on either side of the brasses, into the recesses F F, and is delivered through grooves on to the piston pin.

The long trough (R, Fig. 32) has been found useful for engines with a *short stroke* and high speeds, and the oil does not splash out over the trough, as might have been expected.

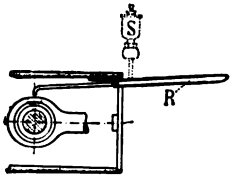


FIG. 32.

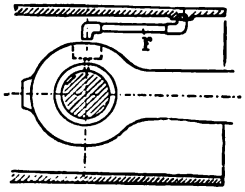


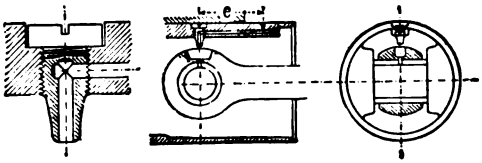
FIG. 33 (Figs. 38, 39 belong).

Lubricating the Piston and Piston Pin from an Oil Pump.

In Fig. 33 the oil from a hole drilled in the upper part of the piston is fed to the piston pin through the pipe *r*.

(To cut a long groove in the top of the piston trunk to take the oil for lubricating the piston pin is said not to work well in practice.)

Instead of the pipe (Fig. 33), *an oil hole* is sometimes drilled in the upper part of the piston trunk, to lubricate the piston pin (Figs. 34–37).



FIGS. 34–36.—Gasmotorenfabrik Deutz 18 HP.

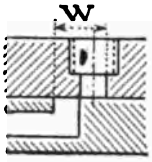
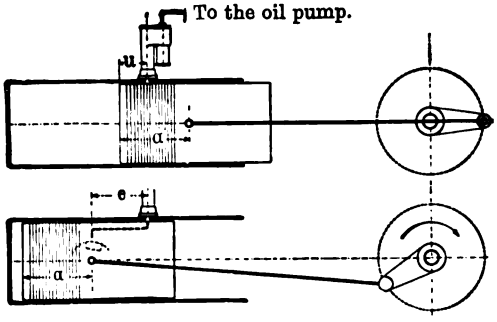


FIG. 37.

Thus the oil passes into the passage at the top of the piston when the speed is at its lowest. This is a very important point, because otherwise with this method of lubrication the pin does not get enough oil.

In order to be quite certain that the oil reaches the pin through this hole, care must be taken to allow the passage between the oil supply and the channel W (Fig. 37) to remain open for an appreciable time. The oil way must be so arranged that the hole in the liner communicates with the one in the piston at about the end of the stroke, when the speed is lowest. This is the object of the oil passage shown at Fig. 35.



FIGS. 38, 39.

Figs. 38, 39, show the position of the lubricator on the cylinder, and the length of the oil pipe or recess, and correspond to Figs. 33–37.

When the crank is on the out stroke dead centre, the oil should be admitted in the space between the *second* and *third* piston rings, and not further back, otherwise it might filter through to the combustion chamber, and be burnt and wasted.

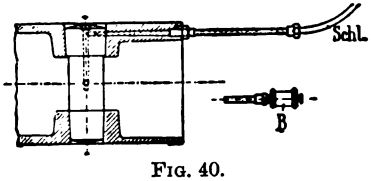


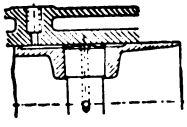
FIG. 40.

Taking approximately $u = 0{\cdot}25a$, and assuming that the oil is admitted to the piston, *i.e.* that the recess overruns the oil hole in the liner when the piston is within 3 per cent. of the outer dead point, then—

$$e = 0{\cdot}97 \times \text{stroke} + u - a \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (11)$$

but it is well to determine these values more accurately from Figs. 38, 39.

For *larger* and vertical engines the oil is delivered through the boss on the piston pin (Fig. 40), either by the leather pipe,* Schl., or through trombone pipes and the spring grease pot B. The liner is in that case separately lubricated.

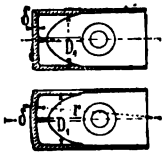


* For various reasons a leather pipe cannot be recommended. It is better to have a groove in the piston and force the oil into it under pressure (see drawing at the side).

6. A hole should be provided, to take *the gear actuating the indicator*, if the lugs mentioned under head 3 do not suffice for this purpose. The indicator is often driven from a pin on the crank shaft.

Calculation of the Piston End.

The end of the piston is usually made flat, but often convex or dished; the curve of the ends, if rounded, is struck from a radius $r = 2D$ or $2{\cdot}5D$.



FIGS. 42, 43.

The stresses coming on the piston are—

(*a*) For *flat-ended* pistons—

$$k_b = 0{\cdot}16D_1^2 \times \frac{p}{\delta^2} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (12)$$

(*b*) For *convex-ended* pistons—

$$k_z = \frac{r}{2} \times \frac{p}{\delta} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (13)$$

Allowable stress, or safe working load, neglecting the stiffening due to the ribs—

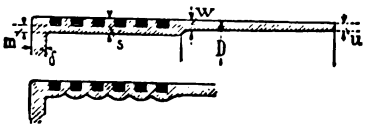
$$k_b = \mathbf{6400} \text{ lbs. per sq. inch}; \quad k_z = \mathbf{4260} \text{ lbs. per sq. inch} \quad . \quad . \quad . \quad (14)$$

If $p = 360$ lbs. per sq. inch, this works out at—

For *flat-ended* pistons $\delta = 0{\cdot}11D_1$ in inches . . . . . (15)

„ *convex-ended* pistons $\delta = 0{\cdot}042r$ „ . . . . . (16)

Number of ribs usually 4; thickness of ribs $0{\cdot}5\delta$.



FIGS. 44, 45.

Body or Trunk of Piston.

As grooves must be recessed to take the piston rings, the thickness of the metal must be more than would otherwise be required. Little is saved by making the piston as at Fig. 45, and it is more difficult to make.

We may take $m = 0{\cdot}04D$; $s = 1{\cdot}6 - 1{\cdot}8 \times$ depth of groove; $w = 0{\cdot}05D$; $u = 0{\cdot}7w$.

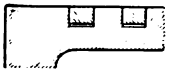


FIG. 46. Sharp edges.

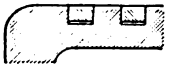


FIG. 47. Rounded corners.

Whether the back end of the piston should be left square (Fig. 46), or the edges rounded off (Fig. 47), is a much debated point. It is generally considered better, however, to round it off, as the burnt products are not so likely to adhere to it as to a piston with sharp corners.

For pistons *not water cooled*, see note on p. 127.

Bore of the Cylinder.

It is not usual in gas engines, as in steam engines, to allow the piston to overrun the working part of the cylinder at the back. The liner is turned of the same diameter up to the breech end (Fig. 48), where it is exposed to the highest temperatures, and therefore to its utmost limit of expansion.

In steam engines the general rule obtains that the cross head is allowed to overrun the guide by $\frac{1}{6}$ the length of the cross-head slipper guide. In gas engines, to avoid rocking, and jamming the piston, care must be taken that the cross-head pin never projects beyond the cylinder. Therefore—

$$x \text{ (Fig. 48) should be} = 0{\cdot}2 + \text{stroke} \quad . \quad . \quad . \quad . \quad . \quad (17)$$

The space between the end of the piston and the breech end of the cylinder, *i.e.* the clearance, depends on various circumstances ; we will for the present represent it by s.

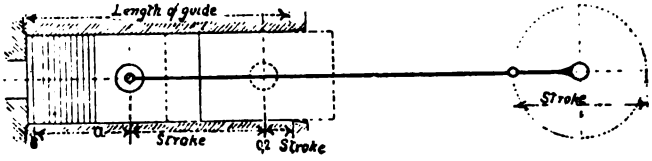


FIG. 48.

Thus—

$$\textit{Working length} \text{ of the cylinder} = s + a + \text{stroke} + x \quad . \quad . \quad . \quad (18)$$

Piston Rings

are usually made of cast iron, occasionally of steel.

The so-called Ramsbottom rings are almost exclusively used. The more rings there are, the less is the pressure they have to exert. The following table gives the tension of these rings.

Table I.—Piston Rings. Empirical values of q. (Pressures on the bearing surface of the rings.)

Number of rings.	Pressure in the cylinder in lbs. per sq. inch.						
z	70	140	210	280	350	420	560 lbs. per sq. inch
2 to 3	$q = 2{\cdot}3$	2·8	3·4	—	—	—	— lbs. per sq. inch
4 „ 5	1·7	2·0	2·4	3·1	3·5	**4·3**	(5·0) „ „
6 „ 8	1·4	1·7	2·1	2·5	2·8	**3·5**	**4·0** „ „

The figures printed in *black type* should be used for gas engines, those, namely, which are applicable if there are six to eight rings, in which case—

$$q = \mathbf{3{\cdot}5} \text{ to } 4{\cdot}3 \text{ lbs. per sq. inch} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

that they may be easily stretched over the body of the piston.

Thickness of Self-adjusting Piston Rings.

These rings are turned and polished before being stretched over the trunk. They should be sufficiently elastic to bend out as much as $D + 2c$ of their diameter, without breaking or losing their spring.

Calculation of the Piston Rings.

D = diameter of the cylinder in inches.

c = depth of the ring in inches.

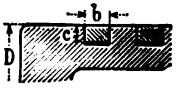


FIG. 1.

q = pressure on the bearing surface of the rings, in lbs. per sq. inch (from Table I. and equation 1).

E = 11,500,000. Modulus of elasticity for cast iron.

e = length of piece cut out of ring in inches.

k_a = *bending stress* on the ring when passing it over the piston body, in lbs. per sq. inch.

k_t = bending stress on the ring under working conditions, in lbs. per sq. inch.

Equations 2 to 4, next page, have been worked out from Reinhardt's calculations * combined with Grashof's theory of bent iron bars.

Table II.—Ramsbottom or Self-adjusting Piston Rings (equations 1 to 4).

	k_t = 8500	11,100	14,000	17,000 lbs. per sq. inch.
$q = 1{\cdot}4$	$\frac{D}{C} = 45$ $k_a = 9000$ $e = 0{\cdot}08D$	52 6800 0·12D	58 5500 0·17D	64 4500 lbs. per sq. in. 0·23 D in ins.
$q = 2{\cdot}1$	$\frac{D}{C} = 36$ $k_a = 13{,}600$ $e = 0{\cdot}07D$	42 10,000 0·1D	47 8300 0·14D	52 6800 lbs. per sq. in. 0·18D in ins.
$q = 2{\cdot}8$	$\frac{D}{C} = 32$ $k_a = 18{,}000$ $e = 0{\cdot}06D$	36 14,000 0·09D	40 11,400 0·12D	45 9000 lbs. per sq. in. 0·16D in ins.
$q = 3{\cdot}5$	$\frac{D}{C} = 28$ $k_a = 23{,}000$ $e = 0{\cdot}05D$	33 16,800 0·08D	37 13,400 0·11D	40 11,400 lbs. per sq. in. 0·14D in ins.
$q = 4{\cdot}2$	$\frac{D}{C} = 26$ $k_a = 27{,}500$ $e = 0{\cdot}05D$	30 20,000 0·07D	33 16,800 0·1D	37 13,500 lbs. per sq. in. 0·13D in ins.
$q = 5{\cdot}7$	$\frac{D}{C} = 22$ $k_a = 36{,}300$ $e = 0{\cdot}04D$	26 25,400 0·06D	29 21,600 0·09D	22 18,000 lbs. per sq. in. 0·11D in ins.

* *Zeitschrift des Vereines deutscher Ingénieure*, 1901, pp. 232 *et seq.*

Bending stress due to stretching the ring over the piston body—

$$k_a = \frac{Ec^2}{2{\cdot}5(0{\cdot}5D)^2} = 18{,}400{,}000\left(\frac{c}{D}\right)^2 \quad . \quad . \quad . \quad . \quad . \quad . \quad (2)$$

Bending stress under working conditions $k_i = \dfrac{12q(0{\cdot}5D)^2}{c^2} = 3q\left(\dfrac{D}{c}\right)^2$. (3)

Length of piece cut out of ring $e = 1''\dfrac{D^2}{c} \times \dfrac{k_i}{E}$ . . . . . (4)

Allowable stress, or safe working load $k_a = 17{,}000$ lbs. per sq. inch
" " " $k_i = 14{,}000$ " " } (5)

Table II. has been calculated from equations 1 to 4.

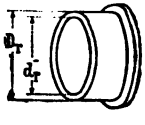
The required thickness of metal c, and the length of the piece to be cut from the ring can be determined from Table II. for a given pressure q on the bearing surface of the ring. For gas engines it is advisable to adopt the figures printed in thick type, namely—

$$c = \frac{1}{33}D\,;\ e = 0{\cdot}08D \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (6)$$

then $q = 3{\cdot}5\,;\ k_i = 11{,}000\,;\ k_a = 16{,}500$ lbs. per sq. inch . . . (7)

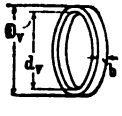
Having calculated the thickness of the ring c, and the length of the piece to be cut out e, the diameter of the cylinder from which the ring is to be cut can next be determined. The various stages in the process of cutting the ring are as follows :—

Let D = *diameter* of the cylindrical piece from which the ring is cut, in inches.
c = *thickness* " " " "
$d = D - 2c$ *inside* diameter in inches of the *finished ring.*



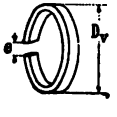
$D_r = D + \frac{e}{\pi} + \frac{3}{8}''$ = outside diameter in inches of the cylindrical piece *before machining* . . . . . (8)

$d_r = d + \frac{e}{\pi} - \frac{3}{8}''$ inside diameter of the same . . . . . (9)



(The flange is for holding the cylindrical piece when turning it.)

$D_v = D + \frac{e}{\pi} + \frac{3}{16}''$ = outside diameter of the *machined ring* . . . . (10)

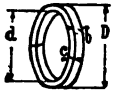


$d_v = d + \frac{e}{\pi} - \frac{3}{16}''$ = inside " " " . . . . (11)

$e = 0{\cdot}08D$ = length of piece cut out.

$D + \frac{1}{8}''$ = outside diameter of the ring when *the two ends are sprung together* . . . . . (12)

$d = D - 2c$ = inside diameter of the finished ring when sprung together (13)



D = outside diameter of the piston ring when sprung together after machining.

$d = D - 2c$ inside diameter of piston ring.

Example.—To calculate the piston rings in a gas engine of $15\frac{3}{4}$ inches diameter. We will take the stress on the ring due to bending, under working conditions $k_1 = 11{,}000$ lbs. per sq. inch ; stress on the ring due to bending, in stretching the ring over the body of the piston $k_a = 17{,}000$ lbs. per sq. inch.

From equation 6 : Thickness of ring $c = \frac{1}{33} \times 15\frac{3}{4}'' = \frac{1}{2}''$ (bare $0{\cdot}475''$).

" " 6 : Length cut out $e = 0{\cdot}08 \times 15\frac{3}{4}'' = 1\frac{1}{4}''$.

(Both these values may be taken from Table II., p. 133.)

Again, from equation 8 : $D_r = 15\frac{3}{4} + \frac{1\frac{1}{4}}{\pi} + \frac{3}{8}'' = 16\frac{1}{2}''$.*

" " 9 : $d_r = (15\frac{3}{4} - 2 \times \frac{1}{2}'') + \frac{1\frac{1}{4}}{\pi} - \frac{3}{8} = 14\frac{3}{4}'' + \frac{1}{16}''$.

" " 10 : $D_e = 15\frac{3}{4} + \frac{1\frac{1}{4}}{\pi} + \frac{3}{16}'' = 16\frac{5}{16}''$ (full).

" " 11 : $d_e = (15\frac{3}{4} - 2 \times \frac{1}{2}'') = \frac{1\frac{1}{4}}{\pi} - \frac{3}{16}'' = 15''$.

Figs. 7 to 10 illustrate the successive operations in machining the ring.

FIG. 7.
Rough casting.

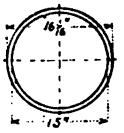


FIG. 8.
Turned and bored.

FIG. 9.
Cut.

FIG. 10.
Sprung together and machined.

Number of Rings.

As the rings must be stretched over the body of the piston, and therefore the pressure on their bearing surfaces cannot exceed $q = 3\frac{1}{2}$ lbs. per sq. inch, several rings must be fitted to the piston. The best and most usual number is—

$$\text{Number of rings} = 7 \text{ or } 8 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (14)$$

To draw the Ring

over the piston body requires great care, as with clumsy handling the limit of elasticity may be exceeded, and the ring break. For this operation Reinhardt † has laid down the following theoretical rule :—

* If the casting is carefully made, it is sufficient to allow $\frac{1}{8}$ inch to $\frac{3}{16}$ inch instead of $\frac{3}{8}$ inch, as given above. The measurements in equations 8 to 11 must be re-calculated, to correspond with this alteration.

† *Zeitschrift des Vereines deutscher Ingénieure*, 1901, p. 376.

If the rings are to be sprung out by hand, and slipped into place, they cannot be properly adjusted unless they are opened to an angle of 30° (Fig. 11). It is better, therefore, especially with rings of large diameter, which it is impossible to stretch by hand, to use Reinhardt's device, as shown in Fig. 11. The angle at which the instrument opens the ring should be set at 30°, as it naturally diminishes when drawing the ring on.

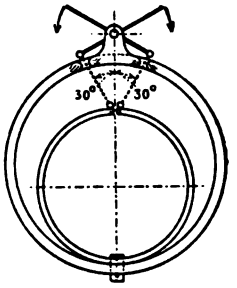
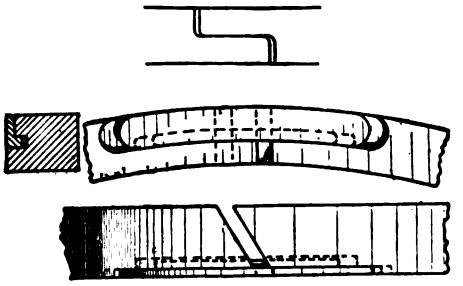


FIG. 11.

FIG. 12.



FIGS. 13–15.—Not often used.

The piece is cut from the ring at right angles to the side edges. To make the ends overlap (Fig. 12), or fit a tongue over the joint, as in Figs. 13–15, does *not give satisfactory results* in gas engine practice.

Care should be taken to prevent the rings turning in their recesses, and the joints should be made at the bottom of the piston (see Fig. 16).

View of underside.

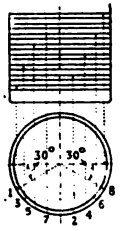


FIG. 16.

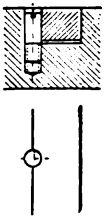


FIG. 17.

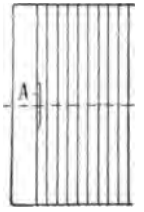
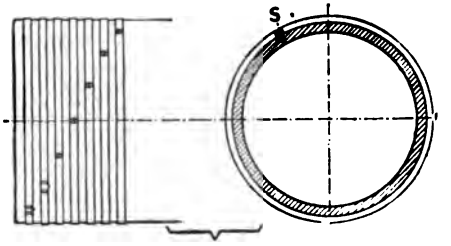
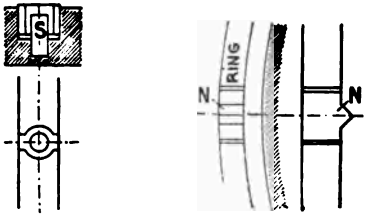


FIG. 18.





FIGS. 19–22.

The simplest way to *secure the rings from shifting* is to fit in a peg (see Fig. 17). Many engine builders cut away a portion of the side of the last piston ring (A, Fig. 18). This allows the pressure to pass behind the ring, and thus the ring is forced outwards against the walls of the cylinder liner.

Another method to secure the piston rings and prevent them from turning is shown in Figs. 19, 20. The rings are held in position by a pin S, screwed into the body of the piston. To increase the tension of the first two rings, they have two steel springs F which cannot twist, but are kept in place by the bosses N riveted to the piston body.

The Piston Pin.

(a) Fitting the Piston Pin in the Boss.

The piston pin must be *very carefully* secured in the bosses cast in the piston to receive it, and also easily taken out, as it has to be removed whenever the connecting rod is disconnected. It can be secured in several ways, as follows :—

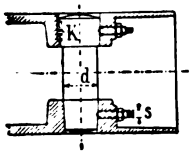


FIG. 1.

1. By *set screws* with pointed ends and square heads locked with check nuts (Fig. 1). Diameter of the set screws $= s = 0{\cdot}3d$. The feather key K prevents the pin from turning round.

2. If the head of the connecting rod is made with a side key, these set screws cause difficulty.

In that case a *taper pin*, *o*, shown in Fig. 2, is used, which prevents the piston pin both from slipping sideways, and from turning.

Diameter of the pin $o = 0{\cdot}2d$.

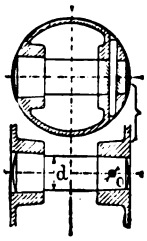


FIG. 2.

Neither of these two methods has proved entirely satisfactory in practice, as it is difficult with either to get the piston pin back into position, if it works slack, or is driven in.

3. In the arrangement shown in Fig. 4 (Deutz), if the cross-head pin is tightened up, it may spring the piston out of shape, if, for instance, there is more wear on the cone of the pin at *a* than at *b*. This is a difficulty which has been experienced with cross heads in steam engines. Fig. 3 shows a locking plate.

4. It is better to make the connection by turning the piston pin conical at each end (Fig. 5). The cross head is secured in the conical end, *c*, by the nut at *d*, and the other end, *e*, is then forced into the boss by the nut at *f*; but on this side allowance must be made for the pin projecting a little way into the boss.

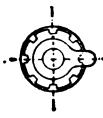


FIG. 3.

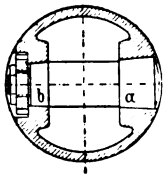


FIG. 4.

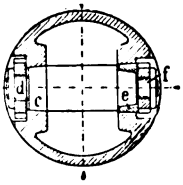


FIG. 5.

If the piston pin is not perfectly tight in the boss, *the shocks on the connecting rod* will be *considerable.*

To put stiffening ribs (*rr*, Fig. 6) between the bosses of the piston pin might counteract the pressure on the piston, but these ribs check the circulation of air, and hinder the cooling of the piston.

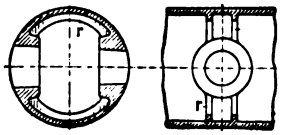


FIG. 6.—Stiffening ribs.

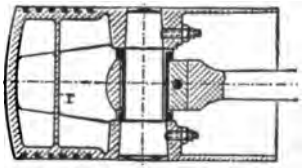


FIG. 7.—Swiderski.

The piston pin being near the working end of the cylinder is exposed to high temperatures, and the difficulty of keeping it in good working order is considerable, especially as it is not easy of access.

In Fig. 7 a partition, *r*, is supposed to prevent the heat reaching the piston pin, and causing the lubricating oil to burn. By piercing a few holes above and below in the wall *r* a circulation of air is set up, which is extremely important for keeping the bottom of the piston cool. These holes are, of course, necessary for removing the cores.

(b) Calculation of the Piston Pin or Cross Head.

The maximum load on the cross head works out at—

$$P = \frac{\pi}{4} D^2 \times 380 = \text{about } 300 \text{ lbs. per sq. inch} \times D^2 \text{ *} \quad . \quad . \quad (1)$$

According to Bach's formula: Bending moment $M_b = \frac{P}{2}\left(\frac{l}{2} - \frac{e}{4}\right)$. . . . (2)

Modulus of resistance to bending for a round cross section—

$$Z_b = 0{\cdot}1d^3 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (3)$$

Stress due to bending (lbs. per sq. inch) $f_b = \frac{M_b}{Z_b}$

Allowable stress or safe working load f_b = **11,000** to **13,000** lbs. per sq. inch (4)

The piston pin must also be tested for pressure on the bearing surface—

$$p = \frac{P}{d \times e} \text{ lbs. per sq. inch} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (5)$$

Safe working load for—

p = **1700** to 2000 lbs. per sq. inch . . . . . (6)

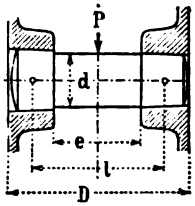


FIG. 8.

(The working load on the cross head pin of a steam engine is usually 1100 to 1400 lbs. per sq. inch.)

In a gas engine a higher load per sq. inch on the piston pin can be allowed for, because the maximum pressure is only developed at every fourth stroke, and for a very short time, whereas in steam engines this pressure is exerted at every stroke. Moreover, it would be difficult to fit a larger pin into the piston. In any case the piston pin is still a weak point in gas engine practice.

Taking the length of the piston pin brass $e = 0{\cdot}5D$, then l = about $0{\cdot}75D$

$$\text{and } M_b = 150\ D^2\left(\frac{0{\cdot}75D}{2} - \frac{0{\cdot}5D}{4}\right) = 37{\cdot}5D^3 \text{ inch-lbs.} \quad . \quad . \quad . \quad (7)$$

Allowing a stress f_b = 12,000 lbs. per sq. inch.

$$d = D\sqrt[3]{\frac{37{\cdot}5}{0{\cdot}1 \times f_b}} = D\sqrt[3]{\frac{37{\cdot}5}{0{\cdot}1 \times 12{,}000}} = \mathbf{0{\cdot}31D} \quad . \quad . \quad . \quad . \quad (8)$$

From these values equation 5 gives—

Pressure on the bearing surface of the piston pin—

$$p = \frac{300{\cdot}D^2}{0{\cdot}5D \times 0{\cdot}31D} = 1940 \text{ lbs. per sq. inch}$$

* The stresses coming on the working parts are generally calculated for an explosion pressure of 380 to 440 lbs. per sq. inch.

Standard Types of Piston and Piston Pin for E 4 Engines.

FIG. 1.

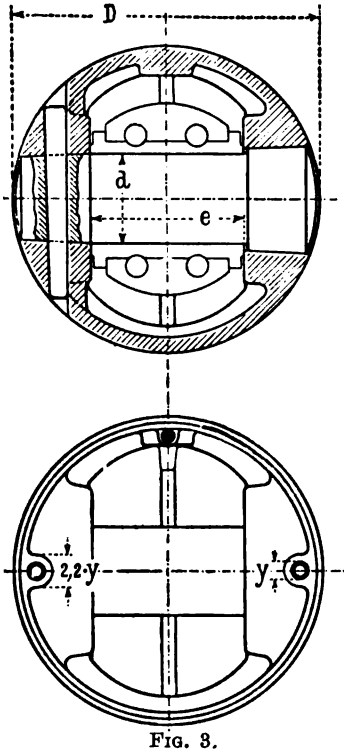


FIG. 2.

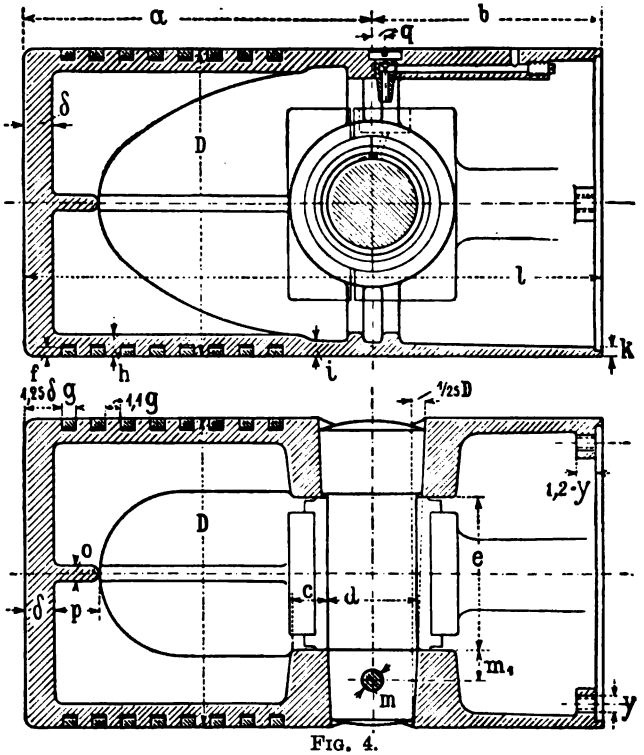


FIG. 3.

FIG. 4.

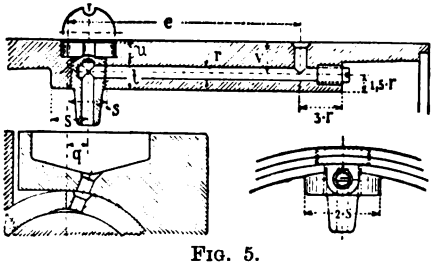


FIG. 5.

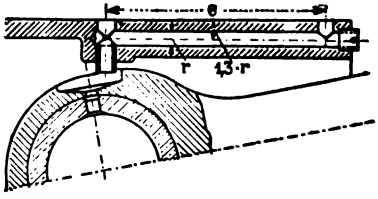
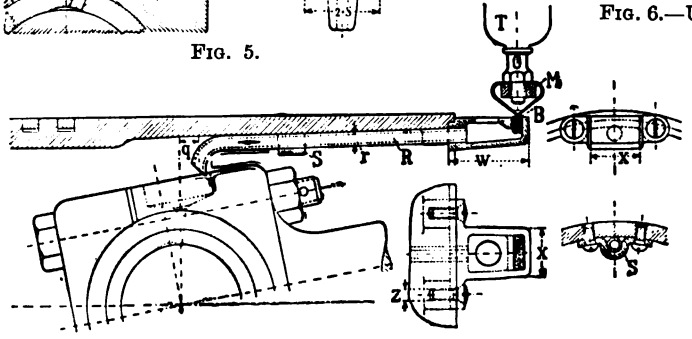


FIG. 6.—Up to 5" Cylinder Diameter.



T, Sight feed lubricator.

M, Bracket fixed on cylinder.

B, Wiper.

S, Bracket to take oil pipe R.

FIG. 7.—With Wiper (Diam. of Cyl. 16". Scale 1 to 8).

Standard.—Lubrication of the Piston Pin (Figs. 5–7, p. 139).

Dimensions in millimetres.

Engine. S	Engine. D	Engine. a	D =	80–125	130–190	200–290	300–390	400–490	500–600
			r =	4	5	6	8	10	12
			s =	—	16	20	26	30	35
			t =	—	14	16	20	24	30
			u =	—	7	8	10	12	16
			v =	—	15	16	20	25	30
130	**80**	95							
160	**100**	120							
190	**125**	150	y =	--	—	80	90	100	110
220	**145**	175	x =	—	—	45	50	55	60
270	**185**	210	z =	—	—	13	13	13	13
310	**220**	250							
350	**260**	300							
400	**300**	335							
450	**330**	370							
500	**360**	390							
550	**400**	440							
600	**480**	520							
700	**600**	630							

IIa.—Lubrication of the Piston Pin (Figs. 5–7).

Dimensions in inches.

Engine. S	Engine. D	Engine. a	4⅞ ins.	5⅛–7 1/16 ins.	7⅞–11⅜ ins.	11 13/16–15 5/16 ins.	15¾–19¼ ins.	19 11/16–23⅝ ins.
ins.	ins.	ins.	ins.	ins.	ins.	ins.	ins.	ins.
5	3¼	3 11/16	3/16	3/16	1/4	5/16	3/8	7/16
6¼	4	4 11/16	—	1/8	3/8	1	1 3/16	1 3/8
7½	5	5⅞	—	9/16	7/8	3/4	1 5/16	1 3/16
8¾	5¾	6⅞	—	1/4	5/8	3/4	7/16	5/8
10⅝	7¼	8¼	—	1/16	1 3/16	3/8	1	1 3/16
12¼	8¾	9 13/16	—	—	3⅛	3½	3 15/16	4 5/16
13¾	10½	11 13/16	—	—	1 3/4	1 15/16	2¼	2⅜
15¾	11⅞	13 7/16	—	—	1/4	1/2	1/4	1/2
17¾	13	14 9/16			1/2		1/2	
19¾	14¼	15 5/16						
21⅝	15¾	17 5/16						
23⅝	19	20 7/16						
27½	23⅝	24¾						

The pistons ng the general methods of lubricating the crank pin or cross head, are enlarged metres in Tabl g (Fig. 35, p. 130). For dimensions of *e*, see Fig. 38 and equation 11.
Table IA. gives of lubrication, and one often used, is shown at Fig. 7. The drops of oil at rt-shaped piece M are wiped off by the wiper B, and pass through trough R

Standard the piston itself is separately lubricated by a mechanical oil pump, the rom the sight feeder to the piston pin can thus be noted.
six rings behind, and two in front of the piston pin, is also often met with, a more equal distribution of the oil. The front ring usually overruns the its width at the end of the stroke.

Cooling the Cylinder.*

(a) Heat Developed in the Cylinder.

It is necessary to bestow special attention on this part of our subject, because an accurate knowledge of the movements of heat is required when designing not only the cylinder and piston, but also the *breech end* and *exhaust valve* of a gas engine (see "Valve gear").

During the working cycle *very high temperatures* are developed in the cylinder of a gas engine, the range of which is more or less in accordance with the following classification.

Table I.—Temperatures in the Working Cylinder.

Period of admission.—The unburnt gases remaining in the cylinder after the exhaust valve has closed are at a temperature of 400° C. The fresh charge is admitted at a temperature of, say, 20° C. Therefore we may assume that during—	Four-cycle engine.	Two-cycle engine.
	Centigrade.	Centigrade.
Admission of the charge of gas and air the temperature falls from	400° to 125°	—
Compression ,, ,, ,, ,, rises ,,	125° ,, 500°	125° to 500°
Ignition (explosion) ,, ,, ,, ,, ,,	500° ,, 1500°	500° ,, 1500°
Expansion ,, ,, ,, ,, falls ,,	1500° ,, 700°	1500° ,, 700°
Exhaust lead (*i.e.* release) ,, ,, ,, ,, ,,	700° ,, 500°	700° ,, 500°
Discharge of exhaust gases ,, ,, ,, ,, ,,	500° ,, 400°	500° ,, 125°

(Compare with the above the following known temperatures of metals: temperature of iron when heated to: dull red, 600° C.; to cherry red, 800° C.; bright red, 1180° C.; white heat, 1500° C.; melting points of zinc, 230° C.; bronze, 900° C.; cast iron, 1150° C.; wrought iron, 1550° C.)

To gain a clear idea of the processes taking place in the cylinder of a gas engine, a knowledge of the *curves* of the *indicator diagram* is as necessary as when studying the working cycle, as explained at pp. 8 *et seq.*

A rough calculation of the temperatures in an E 4 engine (single-acting four-cycle) may be made from Table I. and Fig. 1, as follows :—

Mean temperature during expansion $\frac{1500° + 700°}{2} = 1100°$ C.

,, ,, ,, exhaust $\frac{500° + 400°}{2} = 450°$ C.

,, ,, ,, compression $\frac{500° + 125°}{2} = 310°$ C.

,, ,, ,, admission $\frac{400° + 125°}{2} = 260°$ C.

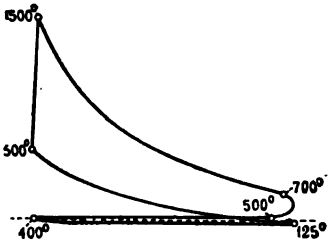


FIG. 1.

Thus the mean temperature of the gases, when in contact with the walls at the explosion end of the cylinder (breech end) would be—

$$T_m = \frac{1100 + 450 + 310 + 260}{4} = \text{about } 530° \text{ C.} \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

* The following pages (142–150) should be taken as the author's contribution towards the solution of this still obscure problem.

(For the front end of the cylinder in a single-acting four-cycle engine (**E 4**) the mean temperature is calculated in a different way, because work is only done in it at certain times, and there is also a different range of temperature in the middle of the cylinder of a double-acting four-cycle engine (**D 4**)).

This rough method of calculation must not, however, be relied on, but a more accurate determination of the temperatures should be made.

For this a clear and comprehensive view may be obtained from the developed diagrams; if the values in Table I. are plotted vertically the temperatures may be read off for each position of the piston, and the mean temperature determined from them by a planimeter.

1. *Temperature Diagrams for the Combustion Chamber and Back End of Piston.*

To illustrate the working cycle better, the diagrams of pressure are shown below the temperature diagrams; *v* denotes the front or crank end, *h* the back or breech end of the cylinder.

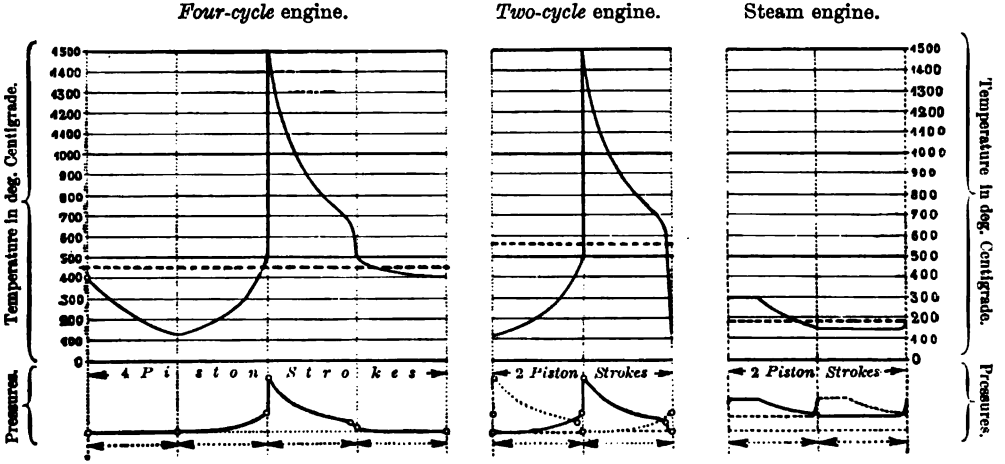


FIG. 2.—E 4 and D 4 Engines. FIG. 3.—D 2 Engine. FIG. 4.—Steam Engine.

Fig. 2 is applicable both to single acting (**E 4**) and double-acting four-cycle engines (D 4); for the latter it shows one side of the piston only.

The *mean temperatures* are indicated in the diagrams by dotted lines, thus .....

2. *Mean Temperatures in the middle of the Cylinder.*

While the back end of the piston is always in communication with the combustion chamber (breech end) of the cylinder, and always exposed to the same high temperatures, the heat is differently communicated to the middle and front end of the cylinder. The temperatures in front of and behind the piston must be considered for every position of the crank, for which the diagrams (Figs. 5 to 8) may be worked out in the same way as before.

Temperature Diagrams for the Middle of the Cylinder.

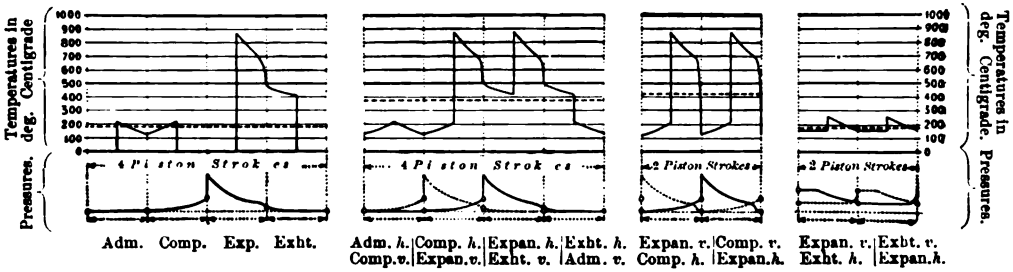


FIG. 5.—E 4 Engine. FIG. 6.—D 4 Engine. FIG. 7.—D 2 Engine. FIG. 8.—Steam Engine.

The mean temperatures are indicated in the diagrams by a dotted line, thus .....

Calculation of the Mean Temperature T_m.

If the temperature diagrams (Figs. 2 to 8) are planimetered, they will give the mean temperatures shown in the following table.

Table II.—Determination of the Mean Temperature T_m.

		E 4 engine.	D 4 engine.	D 2 engine.	Steam engine 570° F. of superheat.
Breech end of cylinder and back end of piston—together . .	Figs. 2 to 4	—	—	—	—
Beginning of stroke	Figs. 5 to 8	460° C.	460° C.	560° C.	190° C.
Middle ,, ...	Figs. 5 to 8	180° C.	360° C.	420° C.	180° C.
End ,, ...	Figs. 5 to 8	Zero	460° C.	400° C.	190° C.
Mean temperature T_m =		320° C.	425° C.	460° C.	185° C.

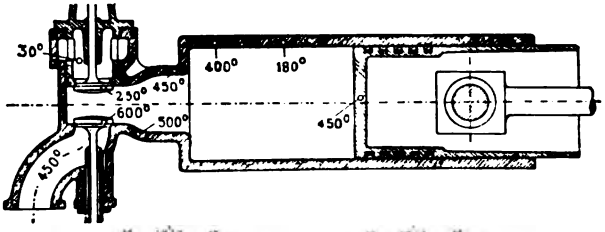


FIG. 9.—Single-acting Engine (E 4), not cooled with water.

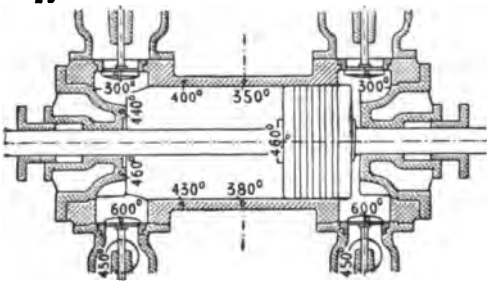


FIG. 10.—Double-acting Engine (D 4), not cooled with water.

In a double-acting four-cycle gas engine the temperature is about twice as high in the middle of the cylinder as in a single-acting engine of the same type, because in the latter

this part is covered by the piston, and thus shielded from the heat of explosion during two out of the four strokes of the cycle.

If during a working cycle heat were not artificially withdrawn from the cylinder of the engine, either by water circulating round it, or by air cooling, the temperatures in it, calculated from Table I., would correspond to those indicated in Figs. 9, 10. In that case only the *admission valve* would be slightly cooled by the incoming charge.

It must be evident that neither cast iron, cast steel, nor other metal could resist *such great heat.* These high temperatures must, as far as possible, be reduced by cooling the cylinder externally, which is done by surrounding it with *water.*

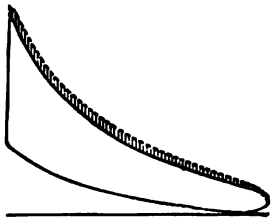


FIG. 11.

A certain *loss of work* is of course entailed by thus cooling it, because the heat carried off in the water cannot do work in the cylinder.

If there were no cooling, the *expansion curve* of the indicator diagram would be higher.

The greater the number of heat units taken up by the cooling water, the larger the shaded area in the diagram (Fig. 11), and therefore the greater is the *loss of energy*, and the less the work done by the engine.

(b) Cooling the Cylinder with Water.

Main Objects aimed at: To reduce the temperature of the cylinder walls within practical working limits; to avoid excessive waste of energy, the quantity of heat withdrawn from the motor cylinder should be as small as possible.

To prevent such a catastrophe as *cracking the breech end*, and to obtain the most uniform distribution of heat throughout the cylinder walls, the water must be admitted in such a way that those parts of the cylinder which, according to Figs. 9, 10, and 15, 16, are exposed to the greatest heat are surrounded by the coldest water.

The following seven parts of the cylinder are cooled in this way:—

Smaller engines E 4 up to 36" stroke, single cylinder, 150 HP.	Besides the parts cooled under heads 1, 2, and 3, the following are also usually cooled in large engines.	In exceptional cases; for example, in the Báuki engine.
1. Water circulation in the jacket 2. Water circulation around the breech end. 3. Water circulation around the exhaust pipe.	4. Piston. 5. Piston rod. 6. Exhaust valve.	7. Cooling the charge by injecting water into the motor cylinder. (Generally done in petrol and naphtha engines of about 8 to 10 HP.)

The Admission Valve.

As the top of the cone of this valve is in contact with the unheated charge of gas and air before explosion, it must be at the same temperature as the breech end of the cylinder (see Temperature Diagrams, Figs. 2 to 4).

Exhaust Valve.

This valve is a source of much trouble and anxiety in gas-engine practice, as the bottom of the cone is always in communication with the exhaust pipe, and therefore exposed to temperatures varying from 400° C. to 500° C.

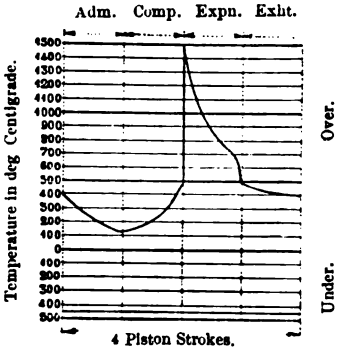


FIG. 12.—Temperature Diagram of the Exhaust Valve.

In Fig. 12 the higher temperatures correspond to those above (cylinder side) and the lower to those below the cone of the valve (exhaust-valve side). The mean temperature of the cone must thus be taken at 600 C. without, and 450° C. with water cooling.

In small valves the valve seat may to a certain extent exert a cooling effect on the cone, but in larger engines

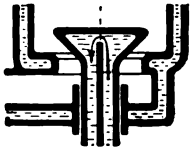


FIG. 13.

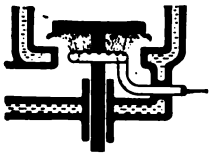


FIG. 14.

the temperature of the cone is always reduced by water cooling (Fig. 13).

Attempts have also been made to cool it by injecting water from below (Fig. 14), but the steam thus generated has given rise to other difficulties.

In E 4 suction gas engines of about 15 HP (stroke = 12″ ; p_c = 140 lbs. per sq. inch ; p = 370 lbs. per sq. inch), it is necessary to cool the cone of the exhaust valve.

(c) The Actual Temperatures.

The temperatures, in actual practice, of the walls of the cylinder and combustion chamber, and of the cone of the exhaust valve, if cooled with water in the usual way, can only be approximately arrived at, as no exact determinations have been made.

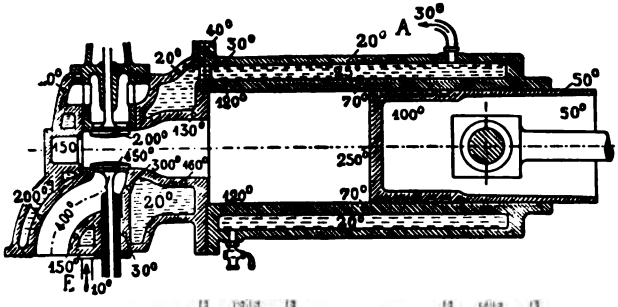


FIG. 15.
Cylinder and Combustion Chamber cooled with Water (Single-acting E 4 Engine).

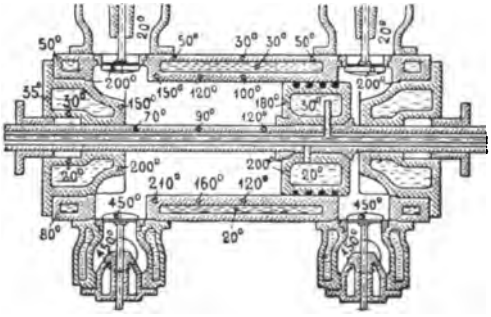


FIG. 16.
Cylinder, Cylinder Cover, Exhaust Valve Box and Pipe, Piston and Piston Rod cooled with water (Double-acting D 4 Engine).

If the cooling system is carefully carried out, the temperatures indicated in Figs. 15, 16, should approximate to those realized in practice.

(d) Calculation of the Quantity of Water required to Cool a Gas Engine.

IHP (Indicated Horse-power). BHP (Brake Horse-power).

M = area in contact with the water, in sq. feet (calculated from the dimensions of the engine). For data for these, see Table III.

Q = quantities of cooling water, in gallons per hour.

t_1 = temperature of cooling water at inlet; t_2 = temperature of cooling water at outlet, in degrees Centigrade.

$t_2 - t_1$ = *difference* in temperature of the water at inlet and outlet.

$t_m = \frac{t_1 + t_2}{2}$ = mean temperature of water varying from 15° C. to 50° C. according to the system of cooling adopted, *i.e.* whether fresh water is run through or water is circulated from cooling tanks.

T_m = mean temperature of the charge, from Table II., p. 144.

$T_m - t_m$ = actual mean difference of temperature between gases on one side and water on the other side of the cylinder (compare equation 2 and Table III.).

k = coefficient of heat transmission, namely, the number of British thermal units transmitted to the water per sq. foot of cylinder area per hour, for each degree Centigrade difference in temperature (k depends upon the kind of metal used, its thickness, and the more or less vigorous circulation of the water).

H = number of British thermal units transmitted from the cylinder to the water per BHP hour.

Therefore—
$$\underbrace{H \times BP}_{\text{Quantities of heat withdrawn from the gases.}} = \underbrace{(T_m - t_m) \times k \times M}_{\text{Quantities of heat transmitted through the cylinder walls.}} = \underbrace{Q \times (t_2 - t_1)}_{\text{Quantities of heat in the cooling water.}} \quad . \quad . \quad . \quad (2)$$

If, as at p. 22, we take the normal, $BHP = C \times D^2 \times S \times n$ . . . . . (3)

then—
$$n \times H = \frac{(T_m - t_m)kM}{C \times D^2 \times S} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (4)$$

Table III.—Number of British Thermal Units per HP hour transmitted to the Water, and Maximum Permissible Number of Revolutions from equation 4. (D and S to be given in feet.)

	E 4 engine.		D 4 engine.	D 2 engine.
	Piston not cooled.	Piston cooled with water.	Piston cooled with water.	Piston cooled with water.
If fresh water be used . . . $T_m - t_m$ =	320° C.	320° C.	425°	460° C.
If circulating water be used . $T_m - t_m$ =	290° C.	290° C.	400°	430° C.
Constant for suction, power, and blast furnace gases . . . . . C =	0·0655	0·0655	0·136	0·285
Approximate value of . . . . . M =	$2{\cdot}2DS + 3D^2$	$2{\cdot}2DS + 3{\cdot}8D^2$	$4{\cdot}8D^2 + 4D^2$	$8{\cdot}65DS + 4D^2$
Usual ratio in large gas engines of S : D =	1·1	1·1	1·2	1·73
British thermal units per HP hour transmitted to the water . . . =	$\frac{23{,}120 \times k}{n \times D}$	$\frac{26{,}670 \times k}{n \times D}$	$\frac{22{,}950 \times k}{n \times k}$	$\frac{11{,}820 \times k}{n \times D}$
If H = 4000, k = 63 . . . . $n_{max.}$ =	$\frac{370}{D}$	$\frac{430}{D}$	$\frac{365}{D}$	$\frac{231}{D}$

If an excessive quantity of water be used (say 22 to 26½ gallons per HP hour), the number of revolutions may under favourable conditions be increased 10 per cent.

In very large blast furnace gas engines the cooling of the engine causes considerable difficulty.

In a Cockerill engine (E 4) D = 51″; S = 55″; n = 90, the consumption of fresh water per HP hour is 24 gallons.

In a Nuremberg engine (E 4) D = 52″; S = 55″; n = 90, the consumption of fresh water per HP hour is 22 gallons.

To run the engine with less water has produced burning of the *piston rings* and *pre-ignition.*

With such a large consumption of water (22 gallons) $t_2 - t_1$ will work out = 12° C., t_m = 15° C.; $T_m - t_m$ will be about 4 per cent., k about 8 per cent., and maximum number of revolutions about 10 per cent. higher.

(e) Consumption of Cooling Water.

Taking from equation 2 B.T.U. × BHP = Q × $(t_2 - t_1)$ and making—

$$\left.\begin{array}{l} H = 4000 \text{ B.T.U. per IHP hour}^{*} \\ H = 4800 \quad \text{,, ,, BHP hour} \end{array}\right\} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (5)$$

then—

$$Q = \frac{222 \times \text{IHP}}{t_2 - t_1} = \text{about } \frac{266 \times \text{BHP}}{t_2 - t_1} \text{ in gallons per hour} \quad . \quad . \quad . \quad . \quad (6)$$

If for the heat transmitted to the water we take figures realized in actual practice they work out as in Table IV.

Table IV.—Consumption of Water per HP per Hour.

$t_2 - t_1$ =	35°	30°	25°	20°	15°	13°	12	11	10 degrees Centigrade
Q : IHP =	6·35	7·4	8·9	11·1	14·8	17·1	18·5	20·2	22·2 gallons per HP hour
Q : BHP =	7·6	8·9	10·6	13·3	17·7	20·4	22·2	24·2	26·6 ,, ,, ,,
With BHP = 10, Q =	12·2	14·3	17·0	21·3	28·4	32·7	35·6	38·7	42·6 cub. ft. per HP hour
,, ,, = 100, Q =	122	143	170	213	284	327	356	387	426 ,, ,, ,,
,, ,, = 500, Q =	610	715	850	1065	1420	1635	1780	1935	2130 ,, ,, ,,

By retaining the number of revolutions per minute given in Table III., we get—

If fresh water be used, $t_2 - t_1 = 30°$ C. therefore consumption of water per BHP hour 8·9 gallons.
If circulating water be used, $t_2 - t_1 = 13°$ C. ,, ,, ,, ,, ,, 20·4 ,,
(If the speed be increased, the consumption of water will also be greater (see bottom of p. 147).

The opening for the inlet and outlet of the water should be calculated for the higher consumption, viz. about 20 gallons; and about ½ gallon to 1 gallon of fresh water per HP hour should be allowed, to replace evaporation, if the water is circulated.

Cooling the Piston.

Of the water consumption given in Table IV., about **70 per cent.** in large gas engines is used for the breech end and the cylinder jacket, and about **30 per cent.** for the piston and exhaust valve. The water in the jacket should be at a pressure of 20 lbs. per sq. inch, that in the piston at 50 to 80 lbs. pressure. If the pressure is less, the necessary quantity of water cannot be forced through the parts which require cooling.

About 120 HP is the limit of size in **E 4** engines, below which it is not necessary to cool the piston.

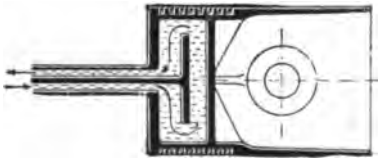


FIG. 1.—Cooling the Piston with Water, *Single-acting* Engine.

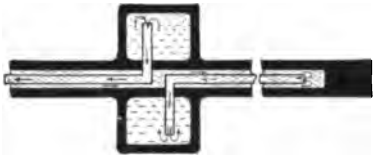


FIG. 2.—Cooling the Piston with Water, *Double-acting* Engine.

Fig. 1 is a diagram of the system of *cooling the piston* in a single-acting engine. To prevent one side of the piston rod getting hot, while the other side is cooled, the water

* H = 4000 B.T.U. is the smallest number of heat units which can be withdrawn. Measurements of the heat under practical working conditions give B.T.U. = 4000 to 6000 for small, and 3600 to 4400 B.T.U. for large engines.

is admitted and discharged through the centre of the rod, as at Fig. 2. It should be admitted at the *bottom* of the piston, and carried off at the *top*, to prevent air collecting in "pockets."

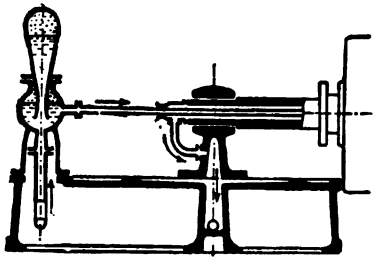


FIG. 3.—Position of the Air Vessel.

As the piston travels to and fro the water in it and in the rod is flung about; an air vessel is therefore placed at the end of the cylinder, to cushion it.

In engines in which the acceleration of the mass of the piston ϕ is great (compare pp. 25 *et seq.*), an air vessel is a necessity, because the water carried to and fro in the piston follows the same laws of acceleration as a solid body.

A usual arrangement is that shown at Fig. 3 in which the water is admitted through "trombone" pipes and a stuffing box. It enters the air vessel in the direction of the arrows, passes thence through the horizontal pipe into the piston, and is finally discharged into the waste pipe through slots at the back of the piston rod guide.

(f) Dimensions of Inlet and Outlet Water Pipes.

If *fresh water* be used (from a spring or from the main), we may take—

Quantity of water required—

$$Q = 11 \text{ gallons per HP hr.}$$

Speed of the water $v = 3{\cdot}3$ ft. per second.

Hence, *Sectional area* of the pipe—

$$\frac{\pi}{4} \times d^2 = \frac{11 \times 1728 \times \text{HP}}{3600 \times 6{\cdot}24 \times 39}$$

$$= \text{about } 0{\cdot}0215 \times \text{HP in sq. inches.}$$

Diameter of the pipe—

$$d = 0{\cdot}165 \times \sqrt{\text{HP}} \text{ in inches.}$$

If the water discharged *be circulated and cooled* and *used over again*, we may take—

$$Q = 22 \text{ gallons per HP hr.} \quad . \quad . \quad (7)$$

$$v = 1{\cdot}5 \text{ ft. per second} \quad . \quad . \quad . \quad . \quad (8)$$

$$\frac{\pi}{4} \times d^2 = \frac{22 \times 1728 \times \text{HP}}{3600 \times 6{\cdot}24 \times 18}$$

$$= \text{about } 0{\cdot}094 \text{ HP in sq. inches.} \quad (9)$$

$$d = 0{\cdot}346 \times \sqrt{\text{HP}} \text{ in inches} \quad . \quad (10)$$

HP here denotes the BHP of the engine for *one* cylinder.

From the above equation (10), the values in Table V. may in round numbers be calculated.

Table V.—Diameter of Cooling Water Pipes for Engines up to 110 HP.

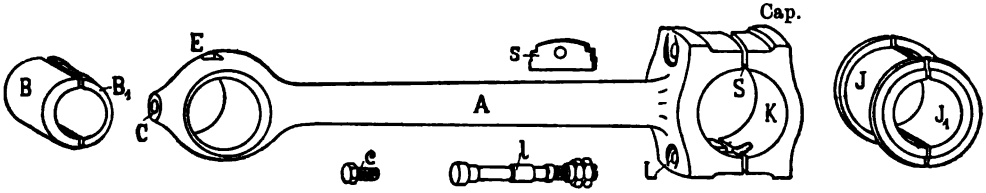
Horse-power of engine,	HP =	1	2	3	6	10	15	22	28	35	45	65	110
Diameter of pipes if fresh water be used,	d =	$\frac{1}{2}''$	$\frac{1}{2}''$	$\frac{1}{2}''$	$\frac{1}{2}''$	$\frac{3}{4}''$	$\frac{3}{4}''$	$1''$	$1''$	$1\frac{1}{4}''$	$1\frac{1}{4}''$	$1\frac{1}{2}''$	$1\frac{3}{4}''$
Diameter of pipes if circulating water be used.	d =	$\frac{1}{2}''$	$\frac{1}{2}''$	$\frac{3}{4}''$	$1''$	$1\frac{1}{4}''$	$1\frac{1}{2}''$	$1\frac{3}{4}''$	$2''$	$2\frac{1}{4}''$	$2\frac{1}{4}''$	$2\frac{3}{4}''$	$3\frac{1}{2}''$
Diameter of pipe connection,	d =	$\frac{1}{2}''$	$\frac{1}{2}''$	$\frac{3}{4}''$	$1''$	$1\frac{1}{4}''$	$1\frac{1}{2}''$	$1\frac{3}{4}''$	$2''$	$2\frac{1}{4}''$	$2\frac{1}{4}''$	$2\frac{3}{4}''$	$3\frac{1}{2}''$

In larger engines, in order to regulate the supply carefully, the water is admitted and discharged through separate pipes to the breech end of the cylinder and to the jacket. The sectional area of each pipe connection is $\frac{3}{4}$ the value found by equation 9.

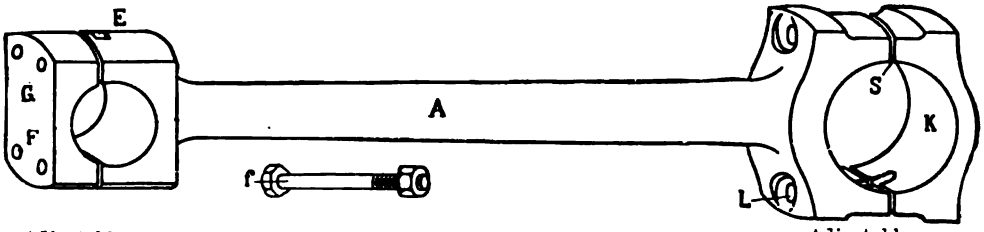
In the smaller engines the water enters at the breech end, and passes out from the jacket, and therefore only one inlet and outlet pipe are necessary.

Section V. The Connecting Rod.

Gas engines are always built with *cranked shafts* (*i.e.* not with overhung crank pins), and therefore the big end or crank-shaft end of the connecting rod is fitted with a *loose* or *adjustable cap or keep*. The *marine type* (Fig. 12) is a favourite design with engine builders for this end of the rod. The small or piston end has sometimes a cap, sometimes a solid forged eye.



FIGS. 1-4.—Design for Smaller E 4 Engines.



FIGS. 5, 6.—Design for Larger E 4 Engines.

A = Shaft or body of connecting rod.
Small end (piston end): BB_1 = brasses; C = tapped hole for adjusting screw *c*; *c* = adjusting screw for brasses; E = oil hole; F = holes for cap bolts *f*; *f* = small end cap bolts; G = cap or keep.
Big end (crank-pin end): JJ_1 = big end brasses; K = cap or keep; L = holes for big end cap bolts *l*; *l* = cap bolts; S = liner (see Fig. 12).

Rules for Constructing the Connecting Rod.

1. The ratio $\frac{\text{length of connecting rod L}}{\text{radius of crank } r}$ varies from 6 in smaller to 5·5 in larger engines. The smaller this ratio L : *r*, the greater is the pressure on the cross-head guide, and therefore on the surface of the trunk piston in four-cycle **E 4** engines (see p. 126).

2. Whether it is better to fit a solid forged eye or a cap or keep at the small end of the piston rod will depend on the available space, which in smaller engines is limited.

The cost of manufacture must also be considered. In general it is advisable to have a forged eye for engines up to about 12 inches cylinder diameter, and a cap (or keep) for engines above that size.

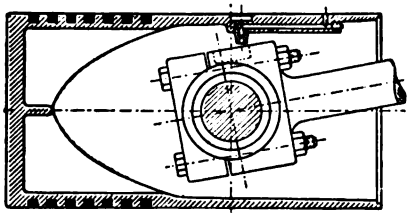


FIG. 7.—Small End of a Connecting Rod for a Trunk Piston.

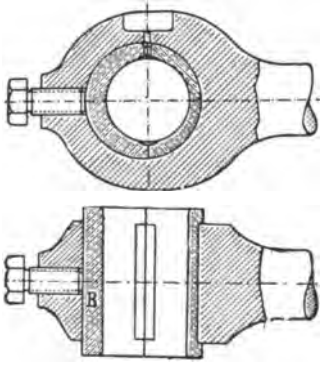
In any case, to make sure that ample room is allowed for the connecting rod in all its various positions, care must always be taken to design the piston *with the connecting rod attached* (Fig. 7).

From lack of space, and to reduce the diameter of the cap, it is often necessary to have four cap bolts (Fig. 10).

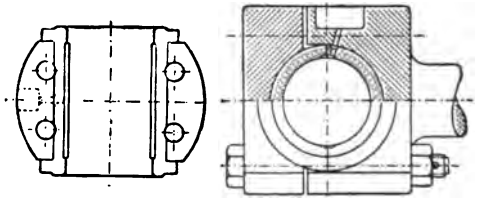
3. *Adjusting the brasses.* To detect and provide for the wear on the brasses of both the piston pin and the crank pin, they should be readjusted from time to time. The usual practice, to take up the slack between the brasses and the pin, is to *file away the surfaces* where the brasses butt.

(In steam engines it was formerly customary to leave a space between the joints of the brasses. This space was filled up with a liner made up of several plates, so that one of these could be removed and the brasses could at any time be tightened up by the cap bolts, without having to be filed. The joints are now always butted, and tightly bolted up.)

A simple method of adjusting the *piston-pin brasses* by means of a *set screw*, where there is a solid forged eye, is shown in Figs. 8, 9. The disadvantage of this type of piston-rod end is that the brasses can only be adjusted when the connecting rod is taken



FIGS. 8, 9.—Adjusting the Bearings for a Connecting Rod End with Solid Eye.



FIGS. 10, 11.—Small End with Cap or Keep.

out and a strong set screw is also necessary. The back end brass B must be *fairly thick* (Fig. 9), as it bears on the flat end of the adjusting screw after the bearings are fitted together; if this brass were thin it might bend or give way when the screw was tightened up.

In the small end of a connecting rod with cap or keep, shown in Figs. 10, 11, the brasses are let together, *i.e.* the ends butted and the cap bolts tightened up.

In the marine type of connecting-rod end there is a *thin liner z* (Fig. 12) which can easily be taken out, filed, and replaced in position.

In most cases these liners are made up of *several thin metal plates*, for instance, say five sheet copper plates $\frac{1}{125}$ inch thick, four of $\frac{1}{64}$ inch thickness, two of $\frac{1}{32}$ inch, and one brass plate $\frac{1}{8}$ inch thick, making a total thickness of $\frac{1}{2}$ inch. As the brasses wear away, one or more of these thin metal plates are removed, and filing is thus avoided.

If the liner is shaped as at Fig. 13, one or more of the thin plates can be slipped out of place without drawing the cap bolts.

4. To prevent the greatest stress coming on the threaded part, both in the marine and the cap type of end, the *body of the cap bolts* is turned down to the same diameter as that at the bottom of the thread (see Fig. 12).

Care must be taken to prevent the bolts *turning;* this is usually done by putting a small snug key under the head, the recess for which should not be inside, as at Fig. 14, but outside the head, as at Fig. 15.

Here, or at the side, the recess is easily made, and the metal cut away in this part of the jaw is not of much value.

5. Nor must the brasses be allowed to turn in the jaw ; this is usually avoided by driving the cap bolt into the brass (see Figs. 16, 17.)

6. In *large* marine type ends four cap bolts, two on either side, are usually fitted. This arrangement gives a neater-looking end than where there is only one cap bolt on each side.

7. For connecting rods with marine ends, the *nuts for the cap bolts* are very deep, and check nuts are often used to prevent the main nuts from slacking back (Fig. 19).

* The end and the shaft are forged in one piece; machined and parted by planing through at Z (Fig. 12).

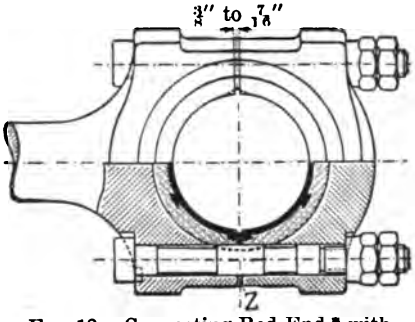


FIG. 12.—Connecting Rod End * with Liner.

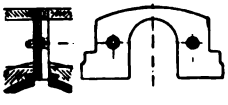


FIG. 13.—Connecting Rod Liner.

FIG. 14.—Wrong.

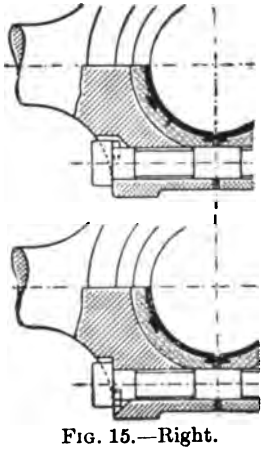


FIG. 15.—Right.

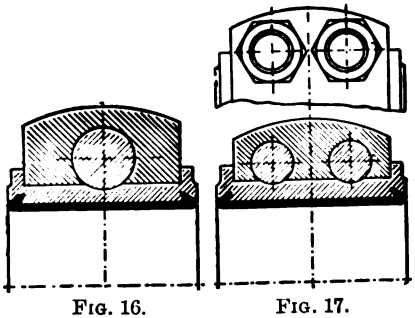


FIG. 16. FIG. 17.

Where the small end is fitted with a cap the same result is obtained if the cap bolts are made with deep nuts, and split pins in the ends of the bolts (Fig. 18).

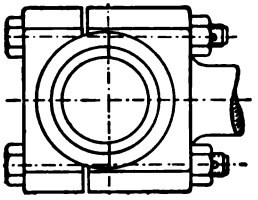


FIG. 18.—Small End with Cap.

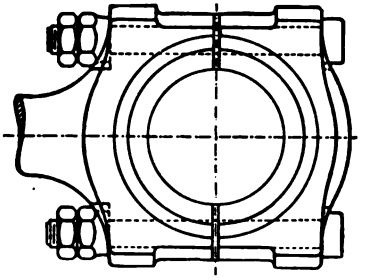


FIG. 19.—Marine Type End.

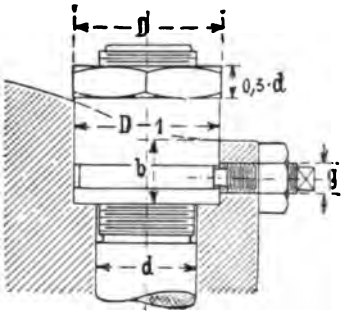
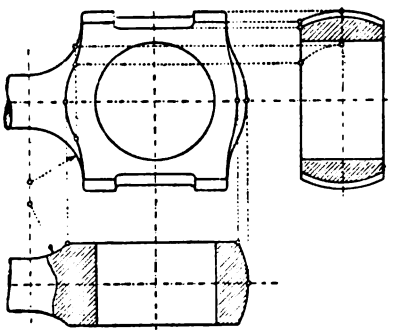


FIG. 20.

8. The same arrangement serves for *locking the nuts on the cap bolts on marine type connecting rods* (p. 152), or special recessed nuts and check screws are fitted, as in Fig. 20. It is a good plan to use a gas thread, as the screws can be more firmly tightened up, and do not easily wear loose.

Table I.—Measurements of Recessed Nuts and Check Screws for Cap Bolts in Marine Type Ends of Connecting Rods.

Diameter of bolt d =	$\frac{1}{2}''$	$\frac{5}{8}''$	$\frac{3}{4}''$	$\frac{7}{8}''$	$1''$	$1\frac{1}{8}''$	$1\frac{1}{4}''$	$1\frac{3}{8}''$	$1\frac{1}{2}''$	$1\frac{5}{8}''$	$1\frac{3}{4}''$	$2''$	$2\frac{1}{4}''$	$2\frac{1}{2}''$	$2\frac{3}{4}''$	$3''$	$3\frac{1}{4}''$
	ins.	ins.	ins.	ins.	ins.	ins.	ins.	ins.	ins.	ins.	ins.	ins.	ins.	ins.	ins.	ins.	ins.
D =	$\frac{29}{32}$	$1\frac{3}{32}$	$1\frac{19}{64}$	$1\frac{31}{64}$	$1\frac{43}{64}$	$1\frac{55}{64}$	$2\frac{3}{64}$	$2\frac{7}{32}$	$2\frac{13}{32}$	$2\frac{37}{64}$	$2\frac{3}{4}$	$3\frac{5}{32}$	$3\frac{35}{64}$	$3\frac{57}{64}$	$4\frac{3}{16}$	$4\frac{17}{32}$	$4\frac{57}{64}$
b =	$\frac{7}{16}$	$\frac{9}{16}$	$\frac{9}{16}$	$\frac{5}{8}$	$\frac{3}{4}$	$\frac{3}{4}$	$\frac{7}{8}$	$\frac{13}{16}$	$\frac{7}{8}$	$\frac{15}{16}$	1	$1\frac{1}{8}$	$1\frac{3}{16}$	$1\frac{1}{4}$	$1\frac{3}{8}$	$1\frac{7}{16}$	$1\frac{1}{2}$
g =	$\frac{1}{4}$	$\frac{1}{4}$	$\frac{5}{16}$	$\frac{16}{16}$	$\frac{16}{16}$	$\frac{3}{8}$	$\frac{3}{8}$	$\frac{3}{8}$	$\frac{3}{8}$	$\frac{3}{8}$	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{5}{8}$	$\frac{5}{8}$	$\frac{5}{8}$



FIGS. 21–23.

9. The method of lubricating the piston pin is explained in the chapter on "Pistons" (pp. 128 *et seq.*).

The crank pin is usually oiled by a centrifugal lubricator (pp. 75 *et seq.*).

10. When the brasses get hot (especially if of bronze), they have a tendency to *close in at the joints* and *grip the pin*. It is usual, therefore, not to allow the brass to *bear right round* the pin, and to bore it out about 4 per cent. larger than the pin.

11. It is essential, when designing the end of the connecting rod, to determine the exact shape it will have when machined. Figs. 21–23 show how this is done, and need no further explanation.

12. Connecting rods are usually *forged* from wrought iron or mild steel.

13. *Calculating the strength of the connecting rod.* To make this calculation it is

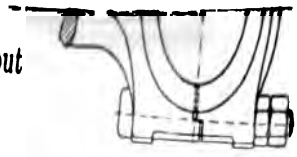
Table II.—Dimensions of Connecting Rod (in millimetres) (Figs. 24–38).

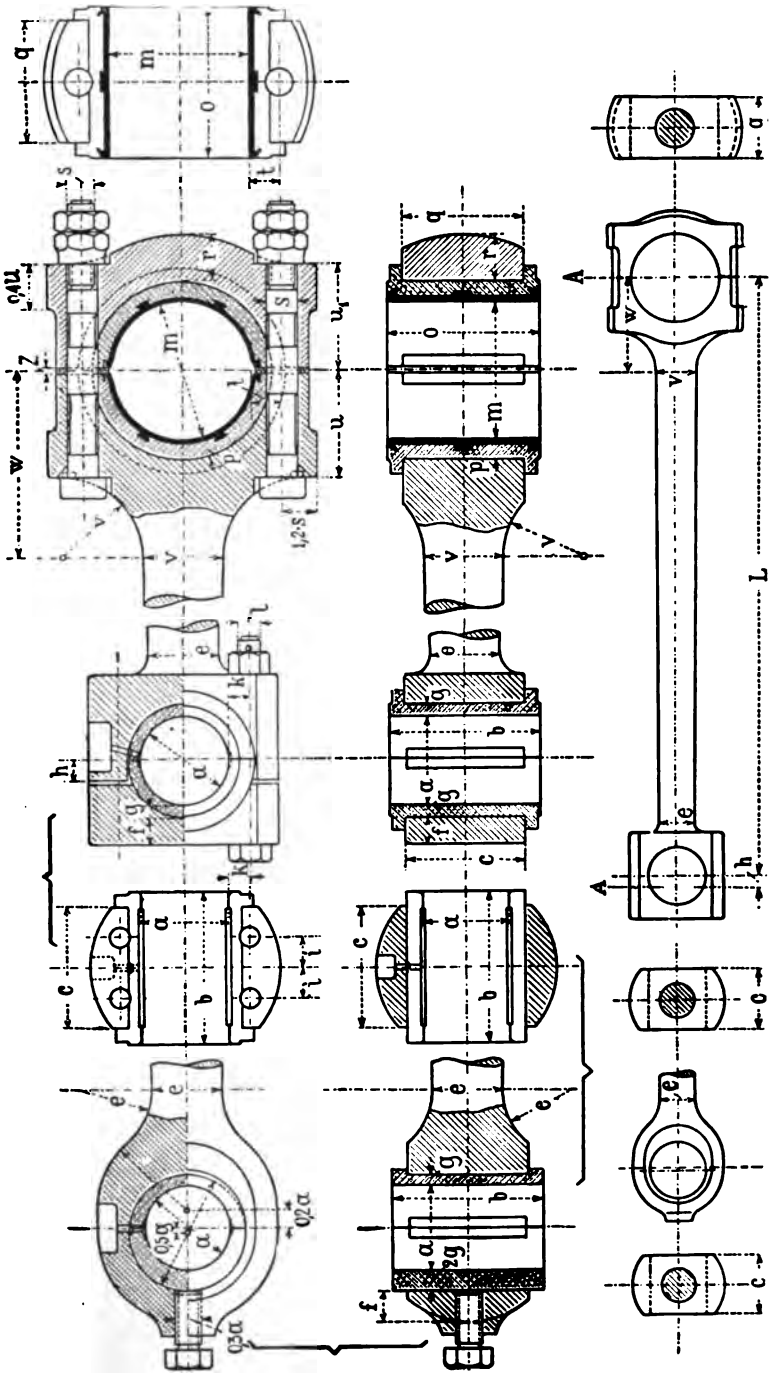
Engine.			Piston end (small end).											Crank end (big end).										
HP	S	D	L	*a*	*b*	*c*	*e*	*f*	*g*	*h*	*i*	*k*	*l*	*m*	*o*	*p*	*q*	*r*	*s*	*t*	*u*	u_1	*v*	*w*
0·6	**130**	80	375	30	40	30	25	6	5	—	—	—	—	42	50	8	40	16	13	12	40	38	30	70
1·2	**160**	100	460	35	50	40	30	8	6	—	—	—	—	50	58	10	45	18	13	13	47	44	35	82
2	**190**	125	550	45	60	45	35	10	6	—	—	—	—	60	65	11	50	20	16	15	54	50	40	94
3·5	**220**	145	640	50	70	55	40	15	7	—	—	—	—	70	80	12	60	22	16	16	60	56	45	105
7	**270**	185	800	60	90	70	45	18	8	—	—	—	—	95	105	15	80	26	20	21	78	72	55	133
11	**310**	220	875	70	110	90	55	20	9	—	—	—	—	110	120	17	95	30	23	24	90	81	65	155
17	**350**	260	1025	85	130	105	65	22	10	—	—	—	—	125	125	19	100	34	26	27	100	90	75	175
25	**400**	300	1175	95	150	125	70	24	12	—	—	—	—	145	145	21	115	38	30	30	115	103	85	200
33	**450**	330	1300	105	165	135	80	26	14	24	34	22	23	155	155	23	120	42	32	33	122	110	95	217
42	**500**	360	1450	115	180	150	85	28	16	26	38	25	26	170	170	26	130	46	35	37	135	122	110	245
55	**550**	400	1575	130	200	160	95	32	18	30	40	28	29	185	185	28	140	52	40	42	150	135	115	265
80	**600**	480	1700	150	240	190	105	38	20	36	48	32	32	215	215	32	165	63	48	46	170	155	125	295
132	**700**	600	1950	190	300	230	140	48	24	45	58	36	35	260	260	38	200	80	58	55	210	190	160	370

The connecting rods illustrated in Figs. 24–38 belong to German engines, and are dimensioned in millimetres

pin is must be fitted to prevent the brasses from turning.

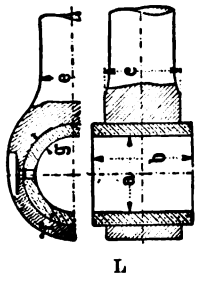
Smaller connecting rods, i.e. for engines of less than 8¾″ stroke, have gun metal bearings without white metal.





Figs. 24-36.—Connecting Rod for Engines with Cyl. Diam. 400 mm. ($15\frac{3}{4}$''), Stroke 550 mm. ($21\frac{5}{8}$'').

Figs. 37, 38. For smaller connecting rods (up to about 2 HP), as the space in the piston is very cramped, the small end is made as at Figs. 37, 38.



necessary to divide the connecting rod into several parts, and consider the strength of each part separately. We will deal with—

1. The shaft or body of the rod (*a*).
2. The big or crank end (*b*).
3. The small or piston-rod end (*c*).

Table III. shows the maximum loads coming on the connecting rod, on which the calculations are based.

Table III.—Maximum Stresses on the Connecting Rod.

	Single-acting engine.	Double-acting engine.
Maximum stress due to *compression*	$P = p \times A =$ about $380 \times A$	$P = p \times A =$ about $380A$
" " " *tension* . .	$P = 0{\cdot}1W\frac{n^2 \times r}{78} + (1 - p_s)A = 64A$	$P = p \times A =$ about $380A$

For meaning of the different symbols used, see p. 59.

It will be seen from this table that the stress due to compression in the rod of a single-acting **E 4** engine is as great as that on the rod of a double-acting **D 4** engine, but there is a great difference in the stress due to tension in the rods of the two engines.

(a) Shaft or Body of the Connecting Rod.

The shaft or body of the rod is usually of round or rectangular cross section (Figs. 42–48). (For very small engines cast steel I-shaped rods are often used.)

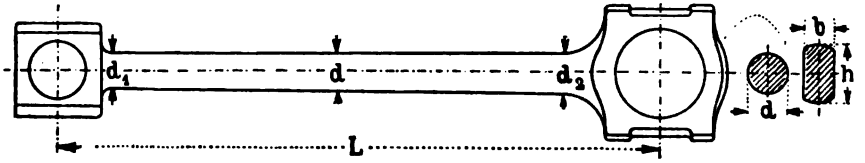


FIG. 39. FIGS. 40, 41.

The *shaft* is first treated as a *strut*, and its strength *calculated* under a breaking load. Referring to Fig. 39—

Let d_1 d d_2 = diameter of a round shaft in inches (Fig. 39).

b = width, h_1 h h_2 = depth of a rectangular shaft in inches (Fig. 46).

L = length of rod in inches (*i.e.* distance between crank pin and piston pin centres).

J = moment of inertia of a round shaft having cross section $\frac{\pi}{4} d^2$; or a rectangular shaft $b \times h$ in inches.

E = modulus of elasticity in lbs. per sq. inch (for wrought iron E = 29,000,000).

m = constant for different piston speeds.

P = maximum load due to compression in lbs. (Table III.).

For a round cross section $J = 0{\cdot}05d^4$ in inches^4 . . . . (1)

" rectangular " $J = \frac{1}{12}b^3h$ " . . . . . (2)

Constant for different piston speeds $m = \frac{10 \times J \times E}{P \times L^2}$ . . . . . (3)

Table for the Conversion of Round into Rectangular Cross Section.

$h : b$	1·5	1·75	2	2·25	2·5
$b : d$	0·79	0·76	0·74	0·72	0·7
$h : d$	1·19	1·33	1·48	1·62	1·75

The constant m depends upon the number of revolutions or piston speed. Table IV. gives its usual values.

Table IV.—Constant m.

Piston speed in ft. per minute c =	200	400	600	800
Constant m =	30	20	15	10

For the smallest cross section of the rod we must also know—

$$\text{Stress due to } \textit{compression } k_c = \frac{\text{compression load by Table III.}}{\text{smallest cross section of rod}} \text{ in lbs. per sq. in.} \quad (4)$$

$$\text{Stress due to } \textit{tension } k_t = \frac{\text{tension by Table III.}}{\text{smallest cross section of rod}} \quad \text{" "} \quad (5)$$

Allowable stress or safe working load $k_t = 7000$ lbs. per sq. in. . . (6)

In *single*-acting engines the stress due to *tension* is generally so small that it may be neglected.

The stress due to the inertia of the rod may be determined from the formula (in Haeder's "Konstruiren und Rechnen," 3rd Ed.)—

$$\text{For circular section } k_b = 0{\cdot}49\left(\frac{n}{100}\right)^2 \times r \times \frac{L^2}{d} \text{ in lbs. per sq. inch} \quad (7)$$

$$\text{For rectangular section } k_b = 0{\cdot}366\left(\frac{n}{100}\right)^2 \times r \times \frac{L^2}{h} \quad \text{" "} \quad (7A)$$

when n = revs. per min.; r = radius of crank in feet; L = length between centres of connecting rod in inches; d = diameter of connecting rod in inches; h = depth of connecting rod; b = breadth of connecting rod in inches.

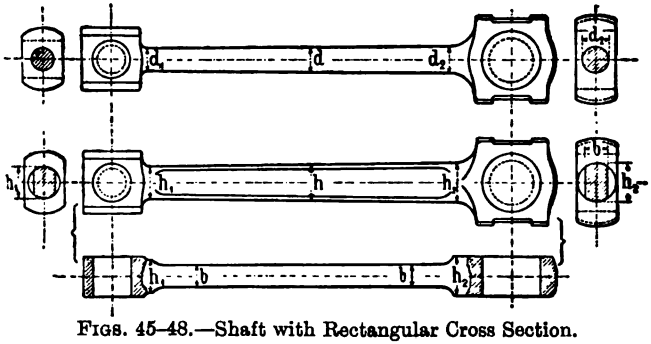
The combined stress works out at $k = k_c + k_b$ . . . . . (8)

Allowable stress or safe working load for wrought iron $k = 8400$ lbs. per sq. inch. (9)

It has a neater appearance if a round rod is made to taper from a larger diameter at the crank end to a smaller one at the piston end (Figs. 42–44), thus—

$$\left.\begin{aligned} d_1 &= 0{\cdot}9d \\ d_2 &= 1{\cdot}1d \end{aligned}\right\} \quad . \quad . \quad . \quad (10)$$

FIGS. 42–44.—Shaft with Round Cross Section.



FIGS. 45–48.—Shaft with Rectangular Cross Section.

For engines above 50 HP the shaft of the rod is often made rectangular in section, as in Figs. 45–48, where—

$$h : b = 1{\cdot}5 \text{ to } 1{\cdot}7 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (11)$$

$$\left.\begin{aligned} h_1 &= 0{\cdot}8h \\ h_2 &= 1{\cdot}2h \end{aligned}\right\} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (12)$$

Example.—Required to find the size of the shaft or body of a connecting rod for a single-acting four-cycle gas engine (**E4 1 cyl.**) having a cylinder of $15\frac{3}{4}''$ (or 400 mm.) diameter; stroke = $21\frac{5}{8}''$ (or 550 mm.); running at n = 175 revolutions per minute.

From Figs. 39, 40, and Table II. and IIA. (pp. 154–157), $d_1 = 3\frac{3}{4}''$ (or 95 mm.); $d_2 = 4\frac{1}{2}''$ (or 115 mm.)

$d = \frac{3\frac{3}{4}'' + 4\frac{1}{2}''}{2} = 4\frac{1}{8}''$; L = 62″ (or 1575 mm.); and from Table II. (p. 28) W = 1165 lbs. (or 530 kilos).

From Table III.: Stress on the rod due to *compression* $= 380 \text{ lbs.} \times \frac{\pi}{4} \times 15\frac{3}{4}^2 = 194{\cdot}8 \times 380 = 74{,}000$ lbs.

" " *tension* $= 64 \text{ lbs.} \times \frac{\pi}{4} \times 15\frac{3}{4}^2 = 194{\cdot}8 \times 64 = 12{,}500$ lbs.

From equation 1: Moment of inertia $J = 0{\cdot}05 \times 4\frac{1}{8}^4 = 0{\cdot}05 \times 290 = 14{\cdot}5$

" 3: Constant $m = \frac{10 \times 14{\cdot}5 \times 29{,}000{,}000}{74{,}000 \times 62^2}$ = about 15

" 4: Stress due to *compression* $k_c = \frac{74{,}000}{\frac{\pi}{4} \times 3\frac{3}{4}''^2} = 6700$ lbs. per sq. inch.

" 5: *Tensile* stress $k_t = \frac{12{,}500}{\frac{\pi}{4} \times 3\frac{3}{4}''^2} = 1130$ lbs. per sq. inch.

" 7: $k_b = 0{\cdot}49 \times \left(\frac{175}{100}\right)^2 \times 0{\cdot}9' \times \frac{62''^2}{4\frac{1}{8}''} = 1260$ lbs. per sq. inch.

" 8: $k = 6700 + 1260 = 7960$ lbs. per sq. inch.

(b) Big End (or Crank-pin End) of the Connecting Rod.

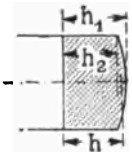


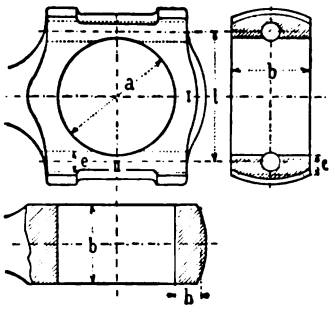
FIG. 49.

As the engine is usually made with a cranked shaft, there is always a cap or keep to the head of the connecting rod.

This is generally turned at the end, but in order to determine the cross section and modulus of resistance quickly and simply, it is usual to consider the actual section as if it were rectangular (Fig. 49) and so take the mean depth at—

$$h = \frac{h_1 + h_2}{2} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (13)$$

1. The Marine Type Big End, with Cap (Figs. 50–52).



FIGS. 50–52.
End with z = 2 Cap Bolts (p. 160).

The stresses coming on the cap due to bending (Section I.) must be determined from the maximum load P, Table III. In a single-acting engine this load is very small, and the cap or keep may be made lighter than that of a steam engine.

With reference to Figs. 50–52, let—

a = diameter of jaw (outside diameter of brasses) in inches.

l = distance between centres of bolts in inches.

b = width of cap in inches.

h and c = thickness of respective cross sections, determined from equation 13 and Fig. 49, in inches.

P = maximum load due to *tension*, from Table III.

For *Cross Section I.*, according to Bach's formula—

$$\text{Bending moment } M_b = 0{\cdot}5\,P\left(\frac{l}{2} - \frac{a}{4}\right) \text{ inch-lbs.} \quad . \quad . \quad . \quad . \quad (14)$$

$$\text{Modulus of resistance } Z_b = \frac{bh^2}{6} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (15)$$

$$\text{Stress due to bending } k_b = \frac{M_b}{z_b} \text{ lbs. per sq. inch} \quad . \quad . \quad . \quad . \quad (16)$$

$$\text{Allowable stress or safe working load } k_b = 8500 \text{ lbs. per sq. inch} \quad . \quad . \quad . \quad . \quad (17)$$

The tensile stress on the *cap bolts* at both ends of the rod must also be calculated, and varies from zero (during the working stroke) to stress P, Table III.

If s = diameter of the bolt at the bottom of the thread in inches, and z = number of bolts e, then—

$$\text{Tensile strength } k_t = \frac{P}{z\frac{\pi}{4}s^2} \text{ in lbs. per sq. in.} \quad . \quad . \quad . \quad . \quad (18)$$

$$\textit{Allowable stress, } k_t = 4200 \text{ to } 5600 \text{ lbs. per square inch} \quad . \quad . \quad . \quad (19)$$

(c) Small End, or Piston Pin End.

Here, as already mentioned, we shall have to deal with both the solid forged eye type, and the type with adjustable cap or keep.

1. Small End of Connecting Rod (Solid Forged-eye Type).

Cross section I. must be calculated for stress due to bending.

Cross section II. must be calculated for stress due to *tension*, from the tensile stress P, Table III.

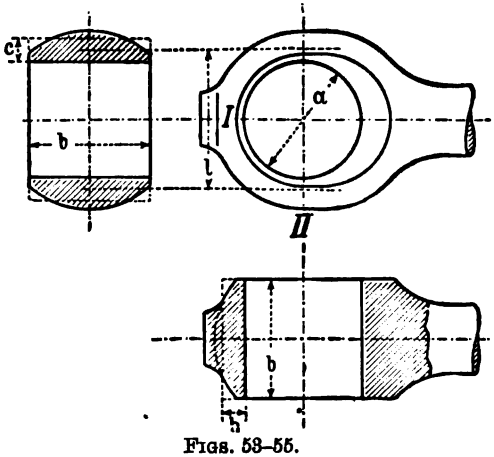
In Figs. 53–55 let—

a = diameter of jaw in inches.

$l = a + c$ = length of corresponding arm of beam in inches.

b = width of connecting rod end, inches.

h and c = thickness of respective sections determined from equation 13 and Fig. 49.



FIGS. 53–55.

Then for *Cross Section I.*—

$$\text{Bending moment } M_b = 0{\cdot}5\,P\left(\frac{l}{2} - \frac{a}{4}\right) \text{ inch-lbs.} \quad . \quad . \quad . \quad . \quad (20)$$

$$\text{Modulus of resistance } Z_b = \frac{bh^2}{6}\,;\; k_b = \frac{M_b}{Z_b} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (21)$$

For *Cross Section II.*—

$$\text{Tensile stress } k_t = \frac{P}{2bc} \text{ in lbs. per sq. inch} \quad . \quad . \quad . \quad . \quad (22)$$

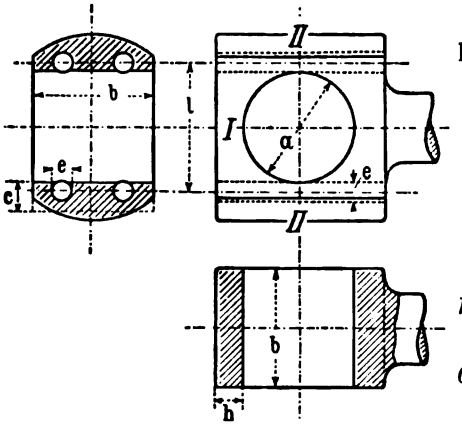
Allowable stress or safe working load—

$$k_b = 8500 \text{ lbs. per square inch} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (23)$$

$$k_t = 4200 \quad \text{"} \quad \text{"} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (24)$$

2. Connecting Rod Small End, with Adjustable Cap.

Figs. 56–58 illustrate a type of end similar in principle to that given in Figs. 50–52. Keeping the same lettering as at page 160, the dimensions can be determined in the same way.



FIGS. 56–58. End with $z = 4$ bolts.

Thus taking the tensile stress P, Table III., p. 157, for *Cross Section I.*, we get—

$$M_b = 0{\cdot}5P\left(\frac{l}{2} - \frac{a}{4}\right) \text{ inch-lbs.} \quad . \quad . \quad (25)$$

$$Z_b = \frac{bh^2}{6} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (26)$$

$$k_b = \frac{M_b}{Z_b} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (27)$$

Allowable stress—

$k_b = 8400$ lbs. per sq. in.; $k_t = 4200$ lbs. per sq. in. . . . . (28)

$$\textit{Cap bolts } k_t = \frac{P}{z\frac{\pi}{4}s^2} \text{ in lbs. per sq. in.} \quad . \quad (29)$$

Allowable stress, 4200 to 5600 lbs. per sq. in. (30)

Example.—Required to design a connecting rod with adjustable cap at each end for a four-cycle gas engine (**E4** 1 cyl.) with cylinder diameter 400 mm. = $15\frac{3}{4}''$, stroke 550 mm. = $21\frac{5}{8}''$.

The *crank pin* is 185 mm. or $7\frac{1}{4}''$ diameter, and 185 mm. or $7\frac{1}{4}''$ long.

The *piston pin* is 125 mm. or $5\frac{1}{8}''$ diameter, and 200 mm. or $7\frac{7}{8}''$ long.

According to the example given on p. 159, the stress due to compression P will be 74,000 lbs., and the stress due to tension 12,500 lbs. per sq. inch.

(a) Marine Type End.

Referring to Figs. 50–52 and pp. 155, 156, we get—

$a = 185 + 2 \times 28 = 241$ mm., or $7\frac{1}{4}'' + 2 \times 1\frac{1}{8} = 9\frac{1}{2}''$.
$b = 140$ mm. or $5\frac{1}{2}''$; $e = 40$ mm., or $1\frac{9}{16}''$.
$l = 185 + 2 \times 42$ mm. $= 269$ mm., or $7\frac{1}{4}'' + 2 \times 1\frac{5}{8}'' = 10\frac{1}{2}''$.
Number of bolts $z = 2$.

From above figures $c = 43$ mm., or $1\frac{5}{8}''$; $h = 45$ mm. or $1\frac{3}{4}''$.

Wherefore by equation 14: $M_b = 0{\cdot}5 \times 12{,}500\ (5\frac{1}{4}'' - 2\frac{3}{8}'') = 18000$ inch-lbs.

" " " 15: $Z_b = \dfrac{5\frac{1}{2} \times (1\frac{3}{4})^2}{6} = 2{\cdot}8$ cubic inches

" " " 16: $k_b = \dfrac{18{,}000}{2{\cdot}8} = \mathbf{6450}$ lbs. per sq. inch.

Allowable stress from equation 17, 8500 lbs. per sq. inch.

From p. 155, area of the bolts at the bottom of the thread $s = 1{\cdot}472$ sq. inches (*i.e.* $1\frac{5}{8}''$ bolt).

Therefore by equation 18—

$$k_t = \frac{12{,}500}{2 \times 1{\cdot}472} = \mathbf{4250} \text{ lbs. per sq. inch.}$$

Allowable stress or safe working load, from equation 19, up to 5600 lbs. per sq. inch.

(b) Small End (Piston End) with Adjustable Cap.

From Table II., p. 155, and Figs. 56–58—

$a = 130 + 2 \times 18 = 166$ mm. or $5\frac{1}{3}'' + 2 \times \frac{11}{16}'' = 6\frac{1}{2}''$ diameter.

$b = 160$ mm. or $6\frac{5}{16}''$; $e = 29$ mm. or $1\frac{1}{8}''$.

$l = 125 + 2 \times 28$ mm. $= 181$ mm. or $5\frac{1}{8}'' + 2 \times 1\frac{1}{3}'' = 7\frac{3}{8}''$.

$h = 32$ mm. or $1\frac{1}{4}''$; c (from the drawing) $= 35$ mm. or $1\frac{3}{8}''$.

Number of cap bolts $z = 4$.

Wherefore by equation 25: $M_b = 6250\,(3\frac{5}{8}'' - 1\frac{5}{8}'') = 12{,}500$ inch-lbs.

" " 26: $Z_b = \frac{6\frac{5}{16} \times (1\frac{1}{4})^2}{6} = 1{\cdot}65$ cubic inches.

" " 26: $k_b = \frac{12{,}500}{1{\cdot}65} = \mathbf{7570}$ lbs. per sq. inch.

Allowable stress 8500 lbs. per sq. inch, from equation 28.

From p. 155: Area of each *cap bolt* at the bottom of the thread $s = 0{\cdot}697''$.

By equation 29: $k_t = \frac{12{,}500}{4 \times 0{\cdot}697} = \mathbf{4500}$ lbs. per sq. inch.

Allowable stress from equation 30, 5600 lbs. per sq. inch.

Explanation of Table IV., p. 163.

To make the subject clearer the dimensions of the *connecting rod*, *crank-shaft bearings*, *crank pin*, *piston* and *piston pin*, or cross head, have been tabulated in Table IV. Besides the *maximum piston loads* (see Table III., p. 157), this table also contains the constant m for the connecting rod, and the values of F, q, and q max., which determine the pressure on the bearing, causing it to wear or to run hot.

With the help of this table, if the maximum load on the piston is known, it is possible to determine the corresponding dimensions of the engine. If the number of revolutions in a given engine differs from the values in column 4, the dimensions of the crank-shaft bearings and the crank pin are determined from the friction work F (*i.e.* load on the bearings per sq. inch × circumferential velocity of the pin in feet per second × by the coefficient of friction = 0·05). Thus, taking n_1 as the speed of the engine in question—

$$\frac{n_1}{n} \times \text{F ft.-lbs. per sec. per sq. in. of bearing} \quad . \quad . \quad . \quad . \quad (31)$$

(n = number of revolutions given in the table, F to be taken from the table for dimensions of crank pin and maximum piston load corresponding to this speed.)

The figures thus obtained when checked by the calculations at p. 97 *et seq.*, show whether the safe working load has been exceeded or not.

As regards the figures printed in italic type, note that they refer only to engines built in special workshops, where engines of this size exclusively are made. In general it is advisable, for engines the dimensions of which are printed thus, to take the dimensions of the engine of the next size larger, having the same stroke.

Table IV.—Connecting Rod, Crank-shaft Bearings, Crank Pin, Piston Pin (Cross Head), and Piston.

For *Single-cylinder, Four-cycle Engines,* with an explosion pressure = 370 lbs. per square inch (see explanation, p. 162).

Engine. Column 1 gives power developed with lighting gas. With suction or power gas, 25 per cent. less.			Revolutions per minute.	Load on piston.	Connecting Rod. (For method of calculation, see p. 157.) (For standard dimensions, see p. 154.)				Main Bearings. (For method of calculation, see p. 97.) (For standard dimensions, see pp. 106, 107.)					Crank Pin. (For method of calculation, see p. 97.) (For standard dimensions, see p. 73.)					Piston. (For method of calculation, see p. 126.) (For standard dimensions, see p. 140.)			Piston Pin or Cross Head. (For method of calculation, see p. 138.) (For standard dimensions, see p. 140.)		
					Diameter.																			
HP	Stroke S ins.	Diam. D ins.	n	P max. lbs.	Piston end ins.	Crank-pin end ins.	L ins.	Constant m.	d ins.	l ins.	q max. lbs. per sq. in.	q lbs. per sq. in.	F.	d ins.	l ins.	q max. lbs. per sq. in	q lbs. per sq. in.	F.	Mean pressure K lbs.	l ins.	q lbs. per sq. in.	d ins.	l ins.	q lbs. per sq. in.
0·6	5	3¼	240	3,060	1	1 3/16	14¾	31	1⅝	2⅞	426	132	11·2	1⅝	2	940	94	·8	275	6¼	20	1 3/16	1 9/16	1650
1·2	6¼	4	240	4,650	1 3/16	1⅜	18⅛	27	1 15/16	3⅛	425	117	11·9	2	2¼	1030	103	10·8	416	7⅞	20	1⅜	2	1700
2	7½	5	235	7,250	1⅜	1 9/16	21⅝	22	2 15/16	4¼	455	122	14·4	2⅜	2 9/16	1200	120	14·6	650	9⅞	20	1¾	2⅜	1750
3·5	8¾	5¾	230	9,600	1 9/16	1⅞	25¼	21	2¾	4⅞	440	118	16·3	2¾	3 3/16	1100	110	15	857	11⅜	20	2	2¾	1750
5 }	10⅝	6⅛ }	230	10,900	1¾	2 3/16	31 1/16	20	3 3/16	5¾	352	84·2	13·5	3⅛	3 9/16	980	98	15·3	980 }	13¾	17	2⅜	3 9/16	1800
7 }		7¼ }		15,300	2	2⅜	31½	20	3 11/16	6⅝	397	98	18·1	3¾	4⅛	990	99	18·6	1,365 }		20			
9 }	12¼	7⅞ }	225	18,000	2¼	2 9/16 }	34½	21	4	7⅛	373	88	17·3	4	4⅜	1030	103	20·2	1,610 }	16½	18	2¾	4 5/16	1880
11 }		8¾ }		22,300	2 5/16	2¾ }		21	4 5/16	7⅞	398	94	19·7	4 5/16	4¾	1090	109	23	1,990 }		20			
13 }	13¾	9 }	220	23,500	2⅜	3 }	40⅜	18	4½	8⅛	378	90	19·6	4½	4¾	1100	110	24	2,110 }	19⅝	18	3⅜	5⅛	1760
17 }		10¼ }		30,500	2 9/16	3 3/16 }		19	4⅞	9 1/16	405	94·5	22·2	5	5	1220	122	29·3	2,730 }		20			
20 }	15¾	10⅝ }	210	32,700	2¾	3⅜ }	46¼	17	5⅛	9½	390	88	20·7	5⅛	5⅛	1240	124	29·2	2,940 }	22⅛	18	3¾	6	1830
25 }		11⅞ }		41,000	2⅞	3½ }		16	5 11/16	10 7/16	408	98	25·5	5¾	5¾	1240	124	32·6	3,660 }		20			
29 }	17¾	12¼ }	195	43,600	3	3¾ }	51¼	16	5⅞	10⅞	410	103	25·8	6	6	1210	121	30·8	3,900 }	24⅜	19	4⅛	6½	1830
33 }		13 }		48,800	3⅛	3⅞ }		16	6 1/16	11⅜	426	108	27·8	6⅛	6⅛	1300	130	34	4,400 }		20			
36 }	19¾	13 3/16 }	185	50,600	3⅜	4 }	57⅛	15	6¼	11⅞	415	108	27·3	6¼	6¼	1300	130	32·7	4,500 }	25⅝	20	4½	7 1/16	1820
42 }		14¼ }		59,000	3⅜	4⅛ }		15	6½	12	460	119	31·2	6¾	6¾	1300	130	35·4	5,300 }		21			
45 }	21⅝	14 9/16 }	175	61,300	3⅝	4⅜ }	62	15	6⅝	12⅝	450	123	31·1	6⅞	6⅞	1300	130	34	5,500 }	28¾	20	5⅛	7⅞	1790
55 }		15¾ }		72,000	3¾	4½ }		15	7 1/16	13⅜	472	128·5	34·7	7¼	7¼	1370	137	38	6,450 }		21			
65 }	23⅝	17 }	165	84,000	4	4¾ }	67	13	7½	14½	487	141	38	7⅝	7⅝	1440	144	39·6	7,500 }	33⅞	19	6	9½	1850
80 }		18⅞ }		103,000	4⅛	5 }		13	7⅞	15½	536	156	44·3	8½	8½	1420	142	43·4	9,250 }		21			
110 }	27½	21¼ }	150	131,000	4½	5½ }	76¾	12	8¾	17	580	183	52·3	9¼	9¼	1530	153	46	11,750 }	41⅜	20	7½	11 13/16	1830
132 }		23⅝ }		162,000	4¾	6 }		12	9⅝	18⅞	575	172	54·2	10¼	10¼	1540	154	51·6	14,600 }		22			

Section VI. The Flywheel.

It is often said that a gas engine requires a very much *heavier flywheel* than a steam engine, and can never be run with the same degree of uniformity. For the same reason the mechanical efficiency of a gas engine is less than that of a steam engine.

The subject cannot be dismissed in such a cursory way, and a distinction must be made between the four following working conditions :—

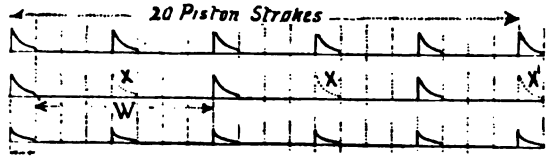
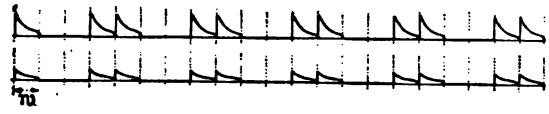
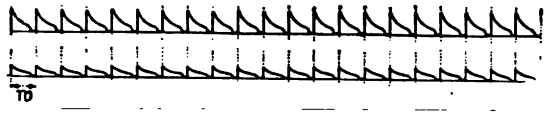
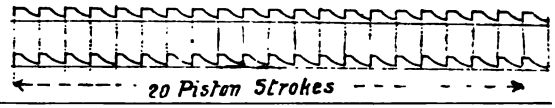
1. Degree of irregularity (or of uniformity) in running at *full load, the load remaining constant.*
2. Degree of uniformity at less than full load, the load remaining constant.
3. Fluctuations in the speed, with *variations in the load.*
4. „ „ „ rapidly repeated *variations in the load.*

1. Degree of Irregularity in running at Full Load, the Load remaining Constant.

The method of governing has a great influence on the degree of irregularity δ_0 (at constant load), and therefore on the weight of flywheel required. The following considerations will make this clearer.

We will suppose that several developed diagrams taken from the same engine are plotted consecutively, as shown in Table I. The more regular the curves (that is, the more closely they approximate to a straight line), the lighter will be the flywheel required.

Table I.—Diagram showing the Fluctuations in the Power developed in a Single-cylinder Gas Engine.

	Type.	*Method of Governing.*	*Load.*	
Gas Engines.	Single-acting *Four*-cycle.	Hit-and-miss, Quantitative or Qualitative	Full.	
			Half.	
			Half.	
	Double-acting *Four*-cycle.	Quantitative or Qualitative	Full.	
			Half.	
	Double-acting *Two*-cycle with separate charging pump.	Quantitative or Qualitative.	Full.	
			Half.	
	Steam Engine.	Expansion Gear.	Full.	
			Half.	

Although hit-and-miss governing gives a more economical consumption of gas, in an engine running at normal or light load, the explosions when produced being always of

the same strength, to govern by varying the quantity or quality of the charge is more usual, because it gives a greater degree of uniformity, as shown in Table I.

Table Ia.—The Degree of Irregularity δ_0 is expressed in—

Decimals, *i.e.* δ_0 =	0·005	0·007	0·01	0·012	0·015	0·02	0·025	0·03	0·035	0·04
Percentages „ =	0·5%	0·7%	1%	1·2%	1·5%	2%	2·5%	3%	3·5%	4%
Fractions „ =	$\frac{1}{200}$	$\frac{1}{143}$	$\frac{1}{100}$	$\frac{1}{83}$	$\frac{1}{66}$	$\frac{1}{50}$	$\frac{1}{40}$	$\frac{1}{33}$	$\frac{1}{29}$	$\frac{1}{25}$
Degree of *Uniformity* 1 : δ_0 =	200	143	100	83	66	50	40	33	29	25

The degree of irregularity in running affords a scale for determining the variations in the speed ; δ_0 shows these variations for a *single* stroke of the piston, and is determined, under given working conditions, by means of the *tangential pressure diagram*, pp. 34 *et seq.* For this calculation the main dimensions of the engine, weight of the flywheel, and an indicator diagram to scale are required.

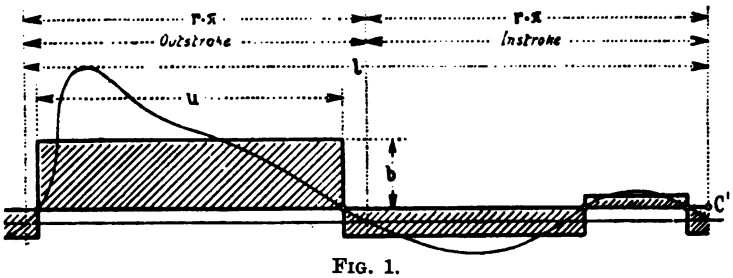


FIG. 1.

According to the data given on pp. 34 *et seq.*—

ub = equivalent rectangle.

$u\frac{S\pi}{l}$ = length of base of the rectangle reduced to the circle described by the crank in feet.

b = height of rectangle in lbs. per square inch.

Let A denote sectional area of the cylinder in square inches.

S „ stroke of the engine in feet.

n „ number of revolutions per minute.

V „ circumferential velocity of centre of gravity of the rim in feet per second by equation 2, p. 39.

IHP „ maximum output of the engine (indicated horse-power).

BHP „ „ „ „ (brake horse-power).

δ_0 „ degree of irregularity in running, to be allowed for, at maximum load; δ_0 only shows the fluctuations in the speed when the *power developed is a constant*, not at varying loads.

Therefore by equation 4, p. 39, weight of flywheel rim $W = 92 \times \frac{u}{l} b \frac{AS}{V^2} \times \frac{1}{\delta_0}$

Inserting in this equation the value $A = \frac{550\,IHP}{p_i \times \frac{2Sn}{60}}$ . . . . . (1)

the weight of the rim works out at $W = 1{,}518{,}000 \frac{u}{l} \times b \times \frac{1}{p_i} \times \frac{IHP}{nV^2} \times \frac{1}{\delta_0}$ . . . (2)

(Here $p_i = \frac{z}{T} \times pm$ is the mean pressure on one working face of the piston throughout the stroke. See Table II., p. 20.)

To avoid very high numbers in Table II., let—

$$100 \times i = 1{,}518{,}000 \times \left(\frac{u}{l} \times b\right) \times \frac{1}{p_i} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (3)$$

and the result is the same equation as in the Author's book on "Steam Engines."

$$\text{Weight of the rim } W = 100 \times i \times \frac{IHP}{n \times V^2} \times \frac{1}{\delta_0} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (4)$$

If, as at p. 173, we use the expression denoted by $\mathfrak{M} \times \mathfrak{D}^2$ and $V = \frac{\mathfrak{D}\pi n}{60}$ be inserted in equation 4, then—

$$\mathfrak{M} \times \mathfrak{D}^2 = \frac{400}{n^3} \times 100 \times i \times \text{IHP} \times \frac{1}{\delta_0} \text{ in lbs.-feet}^2 \quad . \quad . \quad . \quad (5)$$

V = circumferential velocity in circle described by centre of gravity of rim section in feet per second.

IHP = indicated horse-power ; n revolutions per minute ; δ_0 degree of irregularity ; $\frac{1}{\delta_0}$ degree of uniformity ; $\mathfrak{D}$ diameter of circle described by centre of gravity of the rim in feet.

$\mathfrak{M}$ = about 1·1W, weight of the masses in rotation in lbs., from p. 173.

The values calculated with the help of the diagram of tangential pressures at p. 34 *et seq.* work out as follows :—

Table II.—Values of *i* for calculating the Flywheel according to Equations 4 and 5, taking account of the Degree of Irregularity δ_0.

	Four-cycle (gas engine).						Two-cycle (gas engine).			Steam engine.				
	Single-acting.			Double-acting.			Single-acting.	Double-acting.		Single cyl.	Tandem.	Compound.	Triple.	
													2 crks.	3 crks.
Number of cylinders.	1	2*	4	1	2	4	1	1	2	1	2	2	3	3
$\frac{u}{l} \times b =$	28·4	24·5	6·25	38·5	9·5	5·4	2·5	—	—	5·4	2·85	1·85	1·4	0·85
$\frac{1{,}518{,}000}{100} \times \frac{u}{l} \times b =$	432,000	372,000	94,500	584,000	144,000	82,000	380,000	—	—	82,000	43,200	28,000	21,300	12,900
p_i=lbs. per sq. in.	20·6	41·1	82·2	41·1	82·2	164	41·1	82·2	164	40	22·7	22·7	25·5	25·5
i calculated for IHP	21,000	9050	1150	1420	1750	500	9230	360	46	2050	1902	1233	836	506
i calculated for BHP	25,000	10,440	1310	1650	1990	550	—	—	—	2280	2120	1375	930	562

Example from Table II.—To calculate the weight of the flywheel rim, in a double-acting four-cycle two-cylinder gas engine, it would be necessary to make *i* (equation 4) = 1750 if the IHP, and = 1990 if the BHP be taken.

The *difference between steam engines* and *gas engines* in this respect is shown by the values of *i* in Table II., and still more clearly in Table III. (next page).

First Example from Table III.—To make a single-cylinder four-cycle gas engine (type *a*) run with the same degree of uniformity as a *single-cylinder steam engine*, the flywheel would have to be 9·3 times heavier, or have a circumferential velocity $\sqrt{9{\cdot}3}$ = 3·05 times greater than that of the *steam engine.*

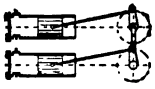
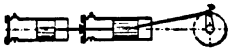
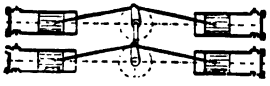
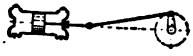
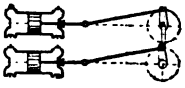
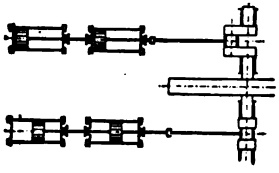
Second Example.—A two-cylinder four-cycle gas engine (type *c*) requires a flywheel seven times heavier than a *compound steam engine.*

Third Example.—A four-cylinder double-acting four-cycle gas engine—
with 90° angle of the crank requires a flywheel 0·9 times the weight } of the flywheel of a
" 180° " " " " 2·9 " " } triple *steam engine.*

* A twin *vis-a-vis* four-cycle gas engine (Type *b*, Table III.) has a flywheel of the same size as a double-acting four-cycle engine (Type *f*, Table III.).

Table III.—Comparative Weights of the Flywheel in Gas and Steam Engines.

(The numbers show how many times greater the weight of the flywheel must be in gas engines than in steam engines, for the same power IHP, circumferential velocity V, and number of revolutions *n*.)

		Type.	Steam Engines. Single cylinder.	Tandem.	Compound.	Triple.
Four-cycle Gas Engine.	Single acting.	(*a*)	9·3	10	16	38
		(*b*)	6·3	6·7	11	26
		(*c*) 	4	4·3	7	16·5
		(*d*) 				
		(*e*) 	0·5	0·55	0·9	2
	Double-acting.	(*f*) 	6·3	6·7	11	26
		(*g*) 	0·86	0·84	1·35	3·2
		(*h*)				
		(*i*) 	0·22 (0·7)	0·24 (0·76)	0·38 (1·2)	0·9 (2·9)

The upper numbers apply to cranks set at an angle of 90°.
The lower " " " 180°.

2. Degree of Irregularity at Light Loads, the Load remaining Constant.

(a) Hit-and-miss Governing.

This (the oldest method of governing, and the most economical as regards the consumption of gas) is only applied to gas engines for industrial purposes; for driving dynamos it would require an impossibly heavy flywheel. With this system, as already explained at p. 19, when the governor is in its highest position the gas valve is held closed from time to time; no gas can enter, and no explosions can occur. The time

elapsing, in other words, the number of piston strokes intervening between two explosions, is thus increased. In an E 4 engine the explosions at full load succeed each other every four strokes, or, what is the same thing, each working cycle comprises two revolutions of the crank shaft.

Supposing, for instance, that such an engine is running at half-load, a certain number of explosions must naturally be missed, and in theory each ignition should be followed by a missfire.

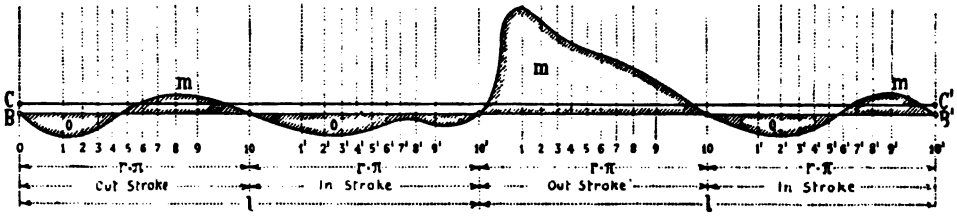


FIG. 6.—Diagram of Tangential Pressures (or Turning-moment Diagram).

Fig. 6 shows the tangential pressure or turning moment diagram of an **E 4** single-cylinder engine, for one working cycle (see Fig. 9, p. 38). At full speed, therefore, this diagram would be consecutively reproduced, and each cycle of work would be spread over four piston strokes. If the engine is run at half load, the cycle of work will occupy twice the time, and be distributed over eight strokes (Fig. 7). To draw correct conclusions from the work of the engine, we must assume that it is governed with absolute regularity, and that missfires alternate invariably with explosions. Deviations from this regular sequence of operations would render a proper study of the conditions impossible.

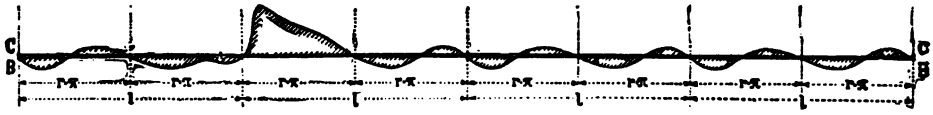


FIG. 7.—Diagram of Tangential Pressures for Eight Piston Strokes (at Half Load).

In this pressure diagram (Fig. 7) the mean resistance BC is naturally less than at full load, that is, the line CC_1 approximates more closely to the line of no pressure BB_1, and thus the equivalent rectangles are larger. According to the method described at pp. 39 *et seq.*, the largest of these rectangles is taken as the basis for calculating the weight of the rim for a given degree of irregularity, δ_o. The larger this rectangle, therefore, for the same δ_o, the heavier will be the flywheel required (see equation 4, p. 39), or, if the weight of the flywheel is not increased, the greater will be the degree of irregularity δ_o, *i.e.* the less satisfactorily will the engine work.

If the diagram of tangential pressures (or turning-moment diagram) be drawn for the engine at full load, the increase in the size of the equivalent rectangles, due to diminutions in the load, and hit-and-miss governing, may be numerically determined as follows :—

The work done in the cylinder during one stroke to and fro is taken at—

$$X_2 = A \times p_m \times 2r \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (6)$$

The work of resistance R, which of course $= X_z$, is in an E4 single cylinder engine spread over a distance of $2 \times l = 4r\pi$ (Fig. 6), and if the length BC = mean resistance per square inch p_w, we get—

$$X_z = R = Ap_w 2l = Ap_w 4r\pi \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (7)$$

$$\text{then } Ap_m 2r = Ap_w 4r\pi \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (8)$$

and at full load—

$$p_w = \frac{p_m}{2\pi} \text{ for E 4 single-cylinder engines} \quad . \quad (9)$$

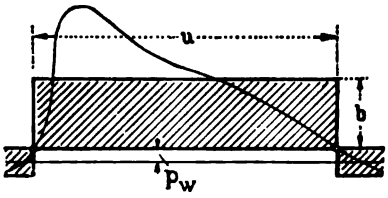


FIG. 8.

For double-acting and multi-cylinder engines equation 9 is written thus—

$$p_w = \frac{z}{T} \times \frac{p_m}{2\pi} \quad . \quad . \quad . \quad . \quad . \quad (10)$$

where z = sum of the cylinder ends in which work is done, and—

T = number of strokes (see page 20).

(The same values are obtained if the curve of tangential pressures is planimetered, as described in Chapter V.)

If the engine is run under light load p_w becomes less, because the distance over which R is distributed is greater (compare Fig. 7). Thus at *half* load, according to Fig. 7, we get—

$$X_z = R = Ap_w 4l = Ap_w 8r\pi \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (11)$$

and hence—

$$p_w = \frac{p_m}{4\pi} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (12)$$

Example.—Let us take the example given at p. 40, and determine the degree of irregularity δ_0, the mean pressure on the piston, with a pressure of explosion of 370 lbs. per sq. inch being $p_m = 90$ lbs. per sq. inch.

$$\text{From equation 9 at full load } p_w = \frac{90}{2\pi} = 14\tfrac{1}{2} \text{ lbs. per sq. inch.}$$

$$\text{,, ,, 12 at half load } p_w = \frac{90}{4\pi} = 7\tfrac{1}{4} \text{ lbs. per sq. inch.}$$

The largest equivalent rectangle (see Fig. 8) will thus be $(14\tfrac{1}{2} - 7\tfrac{1}{4}) \times u$ larger in ft.-lbs.

According to the example given at p. 40, $u = 0{\cdot}131$ ft., $b = 65$ lbs. per sq. inch; therefore the equivalent rectangle at half load will be—

$$u \times b + (14\tfrac{1}{2} - 7\tfrac{1}{4}) \times u = 0{\cdot}131 \times 65 + 7\tfrac{1}{4} \times 0{\cdot}131 = 9{\cdot}45 \text{ lbs.* per ft.}$$

$$\text{As equation 4, p. 39, gives } \delta_0 = 92 \times \frac{ub}{l} \times \frac{AS}{V^2} \times \frac{1}{W} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (13)$$

therefore at half load—

$$\delta_0 = 92 \frac{9{\cdot}45}{0{\cdot}29} \times \frac{113 \times 1{\cdot}33}{72^2} \times \frac{1}{6280} = \frac{1}{72} \text{ as against } \frac{1}{80} \text{ at full load.}$$

(For meaning and value of the different symbols, see pp. 39 and 40.)

Table IV. shows the effect on δ_0 of various loads, the load being assumed to be constant at each of the different powers developed.

Table IV.—Values of δ_0 at less than Full Load, the Load remaining Constant at each Change.

		Full load	0·85	0·75	0·60	0·40	0·20 of full load
Hit-and-miss governing .	Degree of irregularity =	δ_0	$1{\cdot}03\delta_0$	$1{\cdot}06\delta_0$	$1{\cdot}09\delta_0$	$1{\cdot}14\delta_0$	$1{\cdot}18\delta_0$
Quantitative or qualitative governing . . .	Degree of irregularity =	δ_0	$0{\cdot}97\delta_0$	$0{\cdot}94\delta_0$	$0{\cdot}91\delta_0$	$0{\cdot}86\delta_0$	$0{\cdot}82\delta_0$
Under unfavourable conditions it is possible to have		$1{\cdot}8\delta_0$	$1{\cdot}7\delta_0$	$1{\cdot}6\delta_0$	$1{\cdot}5\delta_0$	$1{\cdot}4\delta_0$	$1{\cdot}2\delta_0$

* This value should be used for $u \times b$ at full load.

In actual practice, however, when governing is by hit-and-miss, the ignitions do not follow each other in regular sequence. Not unfrequently also, when the engine is running under light load, ignitions are missed, even when the governor acts by varying the quantity and quality of the charge. It is to these spasmodic ignitions that the degree of irregularity shown in the last line in Table IV. refers.

(b) Quantity or Quality Governing.

With this method of governing the working conditions are different to those described under head *a*. The ignitions always follow each other in the sequence prescribed by the working cycle, but the areas of the diagram are reduced in proportion to the smaller output of the engine. The equivalent rectangles also vary and become smaller, the degree of irregularity is less, and the efficiency is improved (compare bottom line of Table IV.).

If an engine with *hit-and-miss* governing is not to exceed a certain degree of irregularity even when run at *light loads*, and if, as is usual, the weight of the flywheel is determined from the maximum output, a lesser value of δ_0 must be chosen from Table IV. than that corresponding to full load, in order to obtain the requisite steadiness in running at light loads. With quantity or quality governing it is sufficient to take the degree of irregularity at maximum load as a basis for determining the weight of the flywheel, because, as shown in Table IV., δ_0 improves as the load diminishes.

3. Fluctuations in the Speed with Variations of Load.

The degree of irregularity δ_0 affords an index to the fluctuations in the speed when the *power developed* by the engine *remains constant*. In engines in which the output varies more or less rapidly, δ_0 does not form a guide for regulating the speed.

The *kinetic energy* stored up in the flywheel, in other words, the

Energy of the Flywheel

here plays an important part. Let—

V be the circumferential velocity at normal speed, in feet per second.

v „ „ „ at diminished speed, in feet per second.

t „ time in seconds during which the speed diminishes, in other words, the time elapsing while the circumferential speed falls from V to v.

Then $E = \frac{\mathfrak{W}}{g} \times \frac{V^2}{2}$ in ft.-lbs. is the kinetic energy of the rotating masses of the flywheel at normal speed (see p. 173) . . . . . (14)*

$HP_s = \frac{1}{550} \times \frac{\mathfrak{W}}{g} \times \frac{V^2 - v^2}{2} \times \frac{1}{t} = HP$ developed by the flywheel while the speed is diminishing . . . . . (15)

(Thus, to determine HP_s it is necessary to know the time t and the difference between the highest and lowest speeds $V - v$.)

Let us take, for instance, a modern steam *rolling mill*, in which the output varies greatly. Taking the *normal power* developed = 1500 HP, maximum power 1·35 × 1500 = about 2000 HP, $n = 100$, weight of the flywheel, including that of the arms $\mathfrak{W}$ = 88,000 lbs., circumferential speed of the rim V = 130 ft. per second. For the work done by the flywheel as the power required for the rolling mill increases, we get the following :—

* For meaning of $\mathfrak{W}$, see p. 173.

Table V.—Work done by the Flywheel, $\mathfrak{M}$ = 88000 lbs. V = 130 ft. per second. *

(Energy of the Flywheel $\frac{88,000}{32} \times \frac{130^2}{2} = 23,300,000$ ft.-lbs.)

		Speed falling from —				
		100 to 90 revs.	100 to 80 revs.	100 to 50 revs.	100 to 30 revs.	100 to zero revs p. m.
Change of speed =		10 %	20 %	50 %	70 %	100 %
		HP_e =	HP_e =	HP_e =	HP_e =	HP_e =
Time in which the change of speed is effected.	1 second	8050	15,300	31,800	38,600	42,400
	3 seconds	2690	5,100	10,600	12,866	14,130
	5 ,,	**1610**	3,060	6,360	7,720	8,480
	10 ,,	805	**1,530**	3,180	3,860	4,240
	20 ,,	402	765	**1,590**	1,930	2,120
	30 ,,	269	510	1,060	1,286	1,413

If, for instance, the speed of the engine here considered falls in five seconds from 100 to 90 revolutions, the flywheel, according to Table V., will develop 1640 HP.

In practice it sometimes happens that the work done by the flywheel is occasionally much greater † than the normal output of the engine; and for this reason the HP printed in thick type in Table V. have often been obtained under working conditions.

Practically at the same time (after perhaps a second's interval) the governor falls to its lowest position, showing that the maximum load has been thrown upon the engine. Thus with 10 per cent. variation in the speed we get—

$$\text{Mean output} = \frac{1 + 0{\cdot}9}{2} = 0{\cdot}95$$

Work of the engine $0{\cdot}95 \times 1{\cdot}35 \times 1500$ = about 1920 HP
,, ,, flywheel . . . . . 1610 ,,
Total . . . 3530 ,,

If the engine were required to develop this power without a flywheel, the sectional area of the motor cylinder would have to be $\frac{3530}{1920} = 1{\cdot}8$ times larger, whether the engine were driven by steam or gas.

* Translator's Note.—These flywheels are of course built-up steel flywheels; the maximum rim velocity for a built-up cast iron wheel is 90 ft. per second.

† The figures printed in black type in Table V. occur in ordinary practice in every rolling mill. It has been observed, however, that the work of the flywheel may be as much as eight times greater than that in the motor cylinder of a steam engine.

The power developed by the engine and the energy of the flywheel must *supplement each other*, that is, the less the work done by the engine the greater must be the energy of the flywheel, and *vice versâ*. But in any case the amount of work done by the engine, when running light, must be sufficient, when combined with the work of the flywheel, to allow the engine, within the allotted time, to resume its normal speed. To determine this point observations must be made on engines while running.

Degree of Uniformity $\frac{1}{\delta_0}$ and Energy of the Flywheel.

An engine may work very steadily, for instance with a variation, as is often required for driving dynamos, of $\frac{1}{\delta_0} = 250$, and yet be of no use if run with rapidly varying loads, say in a rolling mill.

What is the reason of this?

Because it is not only the degree of uniformity, but as already shown, the kinetic energy stored up in the flywheel, which must be studied to explain the anomaly.

According to equation 14, the kinetic energy in the rim of the flywheel at normal load, and thus the—

$$\text{Energy of the flywheel per HP is } \frac{E}{HP} = \frac{n^2}{23{,}500} \times \frac{\mathfrak{M} \times \mathfrak{D}^2}{IHP} \text{ in ft.-lbs.} \quad (16)$$

$$\text{Degree of uniformity } \frac{1}{\delta_0} = \frac{n^3}{40{,}000} \times \frac{1}{i} \times \frac{\mathfrak{M} \times \mathfrak{D}^3}{IHP} \quad . \quad . \quad (17)$$

Thus the energy of the flywheel increases as the square of n, the degree of uniformity as the cube of n.

What conclusions respecting gas engines may be deduced from our study of flywheels in the preceding pages (164–171)?

1. If a given *degree of uniformity* $\frac{1}{\delta_0}$ is required (as for driving dynamos, spinning mills, etc.), gas engines must have heavier flywheels than steam engines (see Tables II. and III., pp. 166, 167).

2. In engines run *under varying loads*, but in which the degree of uniformity is not a point of great importance, the flywheel need not be heavier than that of a steam engine, as it is the energy stored up in the flywheel which has to be considered. The output of the engine should be rated from 1·3 to 1·4 times higher than the normal output of a *steam engine* of the same dimensions. As far as the action of the flywheel is concerned, there is *nothing to prevent* the use of blast furnace gases for driving rolling mills, if the power developed by the engine is rated from 30 per cent. to 40 per cent. higher than that of a *steam engine* doing the same work, and having a flywheel of the same dimensions.

3. For engines for power purposes, with *varying loads*, and which are used at the *same time* to drive *dynamos* for electric light, the rule given under head 1 applies, since in both cases the degree of uniformity δ_0 is the main point to be considered.

Further, if n and δ_0 be given, the energy of the flywheel can be directly calculated by combining equations 16 and 17, thus—

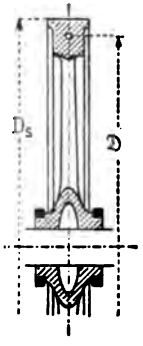
$$\text{Energy per IHP, } E \div IHP = \frac{40 \times i}{n} \times \frac{1}{\delta_0} \text{ in ft.-lbs. per HP} \quad . \quad . \quad . \quad . \quad . \quad (18)$$

Here again it is obvious that the energy of the flywheel $E \div HP$ and degree of uniformity $\frac{1}{\delta_0}$ do not stand in direct ratio to each other, but are affected by the number of revolutions n.

The method of calculation in use hitherto, namely, to take the degree of uniformity $\frac{1}{\delta_0}$ as a basis for determining the weight of the flywheel, even in engines run at varying loads, must yield *erroneous* and *misleading* results.* For this reason the energy of the flywheel necessary to develop the HP required in different industries, and the values of $\frac{1}{\delta}$, calculated from equation 18, are shown side by side at p. 177.

Moment of Rotation of the Flywheel.

Let $\mathfrak{M}$ be the weight of the rim and arms reduced to the circle described by the centre of gravity of flywheel rim, in lbs.



Let $\mathfrak{D}$ be diameter of circle described by the centre of gravity in feet, or diameter of gyration, then the value $\mathfrak{M} \times \mathfrak{D}^2$ is called the moment of rotation, in lbs. per foot².

This expression represents the effect, and therefore the characteristic of a flywheel for a given engine, since according to equation 5 it is proportional to the degree of uniformity $\frac{1}{\delta_0}$, if n be the same. The *moment of rotation* $\mathfrak{M} \times \mathfrak{D}^2$ in lbs. per foot² is always used for calculation in electrical work. If the engine is coupled direct to the dynamo, the moment of rotation $\mathfrak{M} \times \mathfrak{D}^2$ is either referred to the armature only, or part to the armature and the other part to the flywheel.

Approximately, as at p. 39—

$$\mathfrak{M} \times \mathfrak{D}^2 = 1{\cdot}1W_r \times (2 \times R)^2 = 0{\cdot}9 \times W \times D_s^2 \quad . \quad . \quad . \quad . \quad . \quad (19)$$

* The following table shows these determining factors in *modern* rolling mills. The column left blank is to be filled in by the student.

Table VI.—Energy of the Flywheel, and Degree of Irregularity in Rolling Mills.

		Pig iron.	Rails. 72 ft. long.	Rails. 164 ft. long.	Smooth rolled sheet iron.	Boiler plate.	Fine iron wire.	
Brake horse-power	BHP =	800	1,500	3,000	600	1,500	1,000	BHP
Number of revolutions	n =	100	100	90	50	70	100	per min.
Diameter of flywheel †	D_s =	23′ 0″	26′ 8″	27′ 10″	26′ 3″	26′ 3″	21′ 4″	ft. and ins.
Weight "	W_r =	88,000	66,000	141,000	88,000	88,000	44,000	lbs.
Weight of rim	W =	68,200	51,700	109,000	68,200	68,200	34,000	lbs.
Circumferential velocity †	V =	112	128	131	64	39	92	ft. per sec.
Energy of the flywheel per HP, E. ÷ HP	=	16,700	8,780	9,660	7,220	5,600	4,450	ft.-lbs.
Degree of irregularity	δ_0 =	$\frac{1}{525}$	$\frac{1}{270}$	$\frac{1}{300}$	$\frac{1}{111}$	$\frac{1}{125}$	$\frac{1}{175}$	

The last line in the above table gives the "calculated δ_0," but in reality the variations in the speed are about 20 times greater, being 1 in 25 to 1 in 8; that is to say, they amount to from 4 per cent. to 15 per cent.

† See Translator's Note, p. 171.

The last two columns of Table VI. afford a fresh proof that the energy of the flywheel does not stand in a direct ratio to the "calculated" degree of irregularity δ_0.

According to the above table the utmost limit of circumferential speed for rolling mill engines should be—

$$\text{Circumferential velocity } V \leqq 130 \text{ ft. per second}^* \quad . \quad . \quad . \quad (20)$$

Determination of the Weight of the Flywheel.

As already explained under heads 1 to 3 (p. 164), it is always necessary in determining the weight of the flywheel to consider in what way the power developed is utilised; that is to say, whether the degree of uniformity δ_0, or the energy of the flywheel, or both together, are the main points to be studied. For approximate calculations under these heads, see Table IX., p. 177.

(a) Engines for Driving Dynamos.

The method of generating the current must here be taken into account, whether (1) continuous current; (2) three-phase (alternating) current; or (3) single-phase alternating current is employed.

1. *Continuous current* is used for *electric light* and *power transmission over medium distances*, also for installations in which the power required fluctuates much (such as electric cranes and electric railways), and auxiliary batteries are provided to regulate the output. Maximum allowable tension for large plants = about 1000 volts. The motors are started under load.

2. *Alternators*, 2-phase and 3-phase alternating current, are well suited for generating light and power, especially for the transmission of large powers over long distances, 3-phase and 2-phase (asynchronous) current motors start under full load, will bear loads much above the normal, and require little attention. A less degree of irregularity is permissible with them than with continuous-current motors.

3. *Alternators* (single phase) are well suited for electric lighting, but not so well for the transmission of power, because the motors cannot start under load, and if more than their normal output is required they break step. They are also not suitable for lifts if worked intermittently, or for electric railways, iron rolling mills, elevators, etc. The same remarks respecting δ_0 apply to them as to 3-phase and 2-phase current motors.

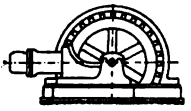
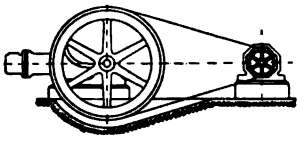
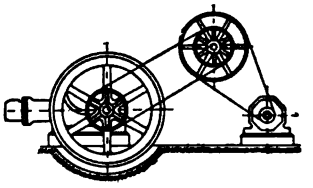
Engines run in Parallel.

If one dynamo does not supply sufficient current for a circuit, two or more dynamos are switched on to the mains in parallel, that is, they all work upon the same mains. Continuous-current dynamos may be driven either through separate power engines (gas or steam) or by one engine driving them all, by means of ropes, belts, or other transmission, the quantity of the current (ampère) is equal to that of all the dynamos together, but the tension does not vary. The tension (volt) must be the same in all the dynamos before they are paralleled. Alternators must be driven independently of each other (otherwise it is impossible to run them in parallel), and must have the same angular velocity.

* TRANSLATOR'S NOTE.—These wheels must be built-up steel wheels; the maximum rim velocity for a built-up cast iron flywheel is 90 ft. per second.

If quivering in the electric light and irregularities in the angular velocity of the dynamo shaft are to be reduced as much as possible, then—

Table VII.—Lowest Allowable Values of δ_0.

	Directly coupled.		Driven by pulley or belt.		Driven through counter shafting.	
						
	Running singly.	Running in parallel.	Running singly.	Running in parallel.	Running singly.	Running in parallel.
Continuous current $\delta_0 =$	$\frac{1}{120}$	$\frac{1}{200}$	$\frac{1}{80}$	$\frac{1}{120}$	$\frac{1}{60}$	$\frac{1}{90}$
3-Phase alternating current, Single-phase alternating current $\delta_0 =$	$\frac{1}{200}$	$\frac{1}{250}$	$\frac{1}{100}$	$\frac{1}{180}$	$\frac{1}{80}$	$\frac{1}{120}$

In *gas engines* a lower value for δ_0 should be taken than in the above table, because even with quantitative or qualitative governing missfires are not infrequent (see Table IV., p. 169). For *steam engines* it is allowable to make δ_0 higher, and as a rule the values assigned to it for "continuous" current may also be used for two or three phase current engines.

In engines coupled direct, the moment of rotation of the flywheel, $\mathfrak{M}\mathfrak{T}^2$, found by equation 5 (p. 166), is referred either to the armature of the dynamo alone, or part to the armature and part to the flywheel.

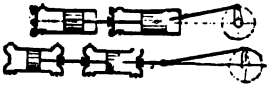
The greater the distance between the engine and the counter shaft, and between the counter shaft and the dynamo, the greater the degree of uniformity, since the masses in rotation which transmit the power, and the elasticity of the pulley belt equalize each other. The values for δ_0 given in Table VII. apply up to a variation of $\frac{1}{4}$ in the load, if the rheostat on the switchboard is very slowly and carefully manipulated, otherwise the light may flicker while the switches are shifted.

If the power required varies considerably, as in cranes and electric railways, *auxiliary or storage batteries* (continuous current) are used, which run in parallel with the dynamo. These batteries form a kind of elastic link between the generators and the motors, and thus help to equalize the load on the dynamo.

If the engine is run under a heavy load, the battery supplies current to the mains; if under a light load it absorbs it.

Example.—The 3-phase current alternator installed in a 500 BHP central power station is to be coupled direct to a 2-cylinder driving engine. Having regard to future possible increase of the plant, it is considered advisable to run the alternator in parallel. According to Table VII. δ_0 should be $= \frac{1}{250}$, but we will take it $= \frac{1}{300}$.

Taking provisionally $n = 100$, i from Table II. (p. 166), $\mathfrak{M}\mathfrak{R}^2$ from equation 5 (p. 166), we get—

Engine.		Coefficient i. From Table II.	$\mathfrak{M}\mathfrak{R}^2$. From equation 5.	$\mathfrak{R}$ being = 15 ft. 9 ins., $\mathfrak{M}$ becomes
	**R 4**, 2 cylinders	10,440	62,640,000	253,000 lbs.
	R D 4, 2 ,,	1,990	11,940,000	48,300 ,,
Steam engine	*Tandem*, 2 ,,	2,120	12,720,000	51,200 ,,
	Compound, 2 ,,	1,375	8,250,000	33,400 ,,

From the above we see that, the number of revolutions being the same, a *compound steam engine* will have the lightest flywheel, while the **R 4** 2-cyl. engine (four-cycle single-acting two cylinder) is quite unsuitable for the proposed object.

(b) Engines for Various Purposes.

In many power generating stations the more or less rapid *changes in the load* play a great part.

If the degree of irregularity δ_0 only were taken as a basis of calculation, the engine might under certain conditions work very badly; in other words, the flywheel might not be able to regulate the variations in the load, without excessive fluctuations of speed. This would greatly affect the efficiency of the engine, and if it were used to generate electric light, a steady light could hardly be obtained.

The difficulty cannot be solved theoretically; approximate results arrived at in actual practice must be used as a basis of calculation. Table IX. shows an attempt in this direction, and the values there given may be corrected and amplified for various working conditions.

As a general rule changes in the load produce much greater variations in the work of a small than of a large engine. Suppose, for instance, in a small carpenter's shop the power required to drive all the machines is 16 HP, of which 8 HP are absorbed by the circular saw.

We may take, as follows :—

Table VIII.—Variations in the Load occurring in Various Industrial Processes.

Plants.	Pumps and stamp mills.	Mechanical carpenter's shop with circular saws and saw mills.	Mechanical workshops.	Grinding mills, cement works, joinery.	Paper and spinning mills.	Iron rolling mills.
	Per cent.	Per cent.	Per cent.	Per cent.	Per cent.	Per cent.
Small . . .	15	50	30	25	15	—
Medium . . .	10	30	20	15	10	—
Large. . . .	5	20	10	10	5	up to 80

Example.—In a moderate-sized carpenter's shop the variations in the power required would, according to the above table, be reckoned at about 30 per cent.

Table IX.—Approximate Values for Determining the Weight of the Flywheel.

Kind of machinery.	Size of plant.	For steady running would suffice if $\frac{1}{\delta_0}$	Taking count of variations in the load (Table VIII.), we should have: E ÷ BHP not less than	Hence $\frac{1}{\delta_0}$	For electric light if the dynamo be driven by a pulley $\frac{1}{\delta_0}$ should not be less than
Pumps, stamp mills . . .	—	20	—	—	—
* Brick kilns	Small 20 HP	30	2180	$1440\frac{n}{i}$	100
	Medium 80 „		1810	$1200\frac{n}{i}$	85
	Large		1450	$960\frac{n}{i}$	60
Mechanical carpenters' shops, with circular saws and saw mills	Small 20 HP	35	2900	$1920\frac{n}{i}$	150
	Medium 120 „		2540	$1680\frac{n}{i}$	90
	Large		2180	$1440\frac{n}{i}$	60
Mechanical workshops .	Small 10 HP	40	1450	$960\frac{n}{i}$	100
	Large		1090	$720\frac{n}{i}$	60
Spinning mills above 300 HP	For low-graded yarn	65	2540	$1680\frac{n}{i}$	—
	„ high- „ „	80	3150	$2090\frac{n}{i}$	—
Paper mills	—	67	2540	$1680\frac{n}{i}$	—

The weight should be calculated from the *highest* of the three above values of δ_0.

Example from Table IX.—Required, a 70 BHP plant in a brick kiln. Let $n = 150$. Diameter of gyration of rim of flywheel D = 10·5 ft.; therefore $V = \frac{10{\cdot}5 \times \pi \times 150}{60} = 82$ ft. per second. It is proposed to make a comparison between the weight of flywheel required, and power developed in an **E 4** single-cylinder gas engine and a *single-cylinder steam engine.*

Gas engine **E 4.** BHP = 70; $n = 150$.

According to Table II.—

$i = 2500$

According to Table IX.—

$\frac{1}{\delta_0}$ should be $> \frac{1200\,n}{i} = \frac{1200 \times 150}{25000} = 7{\cdot}2.$

If the engine is worked without a dynamo, $\frac{1}{\delta_0} = 30$. The higher value being $\frac{1}{\delta_0} = 30$, the weight of the rim of the wheel, according to equation 4, will be: Weight of rim—

$$W = 100 \times 25000 \times \frac{70}{82^2 \times 150 \times \frac{1}{30}} = 5200 \text{ lbs.}$$

If the engine drives a dynamo, $\frac{1}{\delta_0} = 85$. Therefore weight of flywheel rim—

$$W = 100 \times 25{,}000 \times \frac{70}{82^2 \times 150 \times \frac{1}{85}} = 14{,}700 \text{ lbs.}$$

Single-cylinder steam engine BHP = 70; $n = 150$.

According to Table II.—

$i = 2280$

According to Table IX.—

$\frac{1}{\delta_0}$ should be $> \frac{1200 \times n}{i} = \frac{1200 \times 150}{2280} = 79.$

If the engine is worked without a dynamo, $\frac{1}{\delta_0} = 30$. The higher value being $\frac{1}{\delta_0} = 78$, the weight of the rim of the flywheel, according to equation 4, will be: Weight of rim—

$$W = 100 \times 2280 \times \frac{70}{82^2 \times 150 \times \frac{1}{78}} = 1230 \text{ lbs.}$$

If the engine drives a dynamo, $\frac{1}{\delta_0} = 85$. Therefore weight of rim—

$$W = 100 \times 2280 \times \frac{70}{82 \times 150 \times \frac{1}{85}} = 1340 \text{ lbs.}$$

* If the material is very soft, the flywheel may be from 30 per cent. to 50 per cent. lighter.

Thus for the same HP and same speed, the flywheel of the gas engine must be considerably heavier than that of the *steam engine;* namely, four times as heavy if worked without, and eleven times as heavy if worked with a dynamo.

To make the duty of both types of engine equal we should further require the—

Maximum output of the gas engine = *maximum output of the steam engine;*

that is, the normal power developed by the gas engine should be about 25 per cent. to 30 per cent. more than that of the steam engine.

Rules for Constructing the Flywheel.

Method of Transmitting the Power.

According to the type of machine to be driven, either the flywheel itself acts as a driving pulley, or a separate driving pulley is fitted on to the shaft.

1. For high-speed machinery such as dynamos for generating electricity, fans, exhaustors, centrifugal pumps, etc., *the flywheel acts as the pulley*, as shown in Fig. 13.

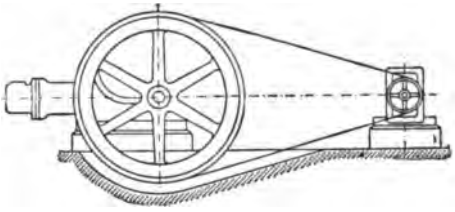


FIG. 13.—10 HP Engine. Scale 1 : 80.

Of the different arrangements of flywheel and driving pulley combined given in Figs. 14–16, the best is Fig. 16.

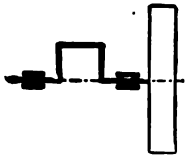


FIG. 14.

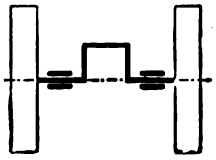


FIG. 15.

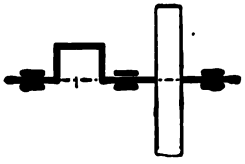


FIG. 16.

2. A *separate pulley* is used for driving counter shafting or machinery running at moderate speeds.

As gas engines must run at relatively high speeds, if the belt is driven off the flywheel, the dimensions of the pulley on the counter shaft (which usually makes 160 to 200 revolutions per minute) will be too large.

The most usual arrangement is that shown at Fig. 20, where the pulley is placed between the flywheel and the out end bearing.

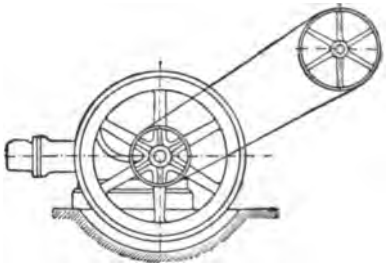


FIG. 17.—With Separate Pulley.

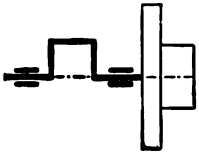


FIG. 18.

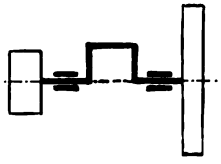


FIG. 19.

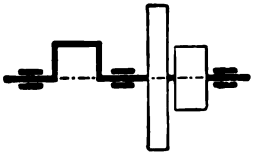


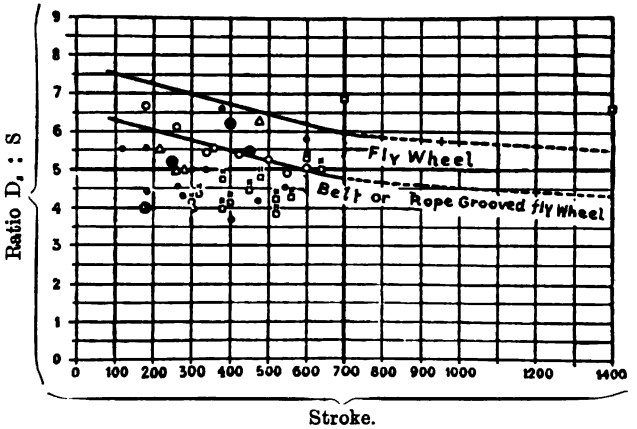
FIG. 20.

Diameter of the Flywheel and of the Driving Pulley respectively.

In gas as in steam engines the ratio adopted is generally—

$$\frac{\text{Diameter of flywheel}}{\text{Stroke}} = 4 \text{ to } 7, \text{ or a mean of } 6 \quad . \quad . \quad . \quad . \quad (21)$$

The diagram (Fig. 21) shows this ratio plotted vertically for a number of working engines.



Diagram, Fig. 21.
Ratio :
$\frac{\text{Diameter of flywheel}}{\text{Stroke of piston}}$

Engines built by the same makers are denoted by similar symbols in Fig. 21.

The values shown on the lower curve may be taken as a basis for calculating the mean for—

Flywheels and Driving Pulleys combined,

and those on the upper curve for—

Separate Driving Pulleys,

or for engines coupled direct, as, for instance, blast furnace gas engines directly coupled to the blowers, etc.

For engines up to 50 HP the flywheel always serves as the driving pulley, and space is provided for a separate pulley, because with this type of engine several of the same size are usually machined at once, and kept in stock as a reserve.

To avoid *too heavy flywheels* their diameter is generally made as large as possible, without regard to the ratio $D_s : S$ (Fig. 21), but the rim speed should be—

For flywheels and driving pulleys combined : circumferential velocity $V < 90$ ft. per second (22)

For solid flywheels " " $V \leqq 100$ ft. per second (23)

The diameter of separate driving pulleys should always be determined by the purpose for which the pulley is intended, but it is desirable to make it as large as possible, because the larger the pulley the less the strain upon the belt.

If $n = 160$ be taken as the usual number of revolutions per minute of the counter shaft, we may write—

$$\text{Diameter of driving pulley on engine} \geqq 2{\cdot}5 \text{ Stroke} \quad . \quad . \quad . \quad . \quad . \quad (24)$$

(In practice the values range from 1·5 to 2·8.)

This rule makes ample allowance for the dimensions of the pulley on the counter shaft.

Width of Flywheels and Pulleys.

1. *Pulley flywheels.* The flywheel acts as the pulley, and the dimensions of the pulley on the counter shaft are taken as unknown.

In Fig. 22, let—
b be the width, δ thickness of belt in inches.
BHP „ power transmitted by the flywheel.
D_s „ diameter of flywheel in feet.
n „ number of revolutions per minute.

Taking into account that with this arrangement the belt is mounted on a very narrow face, we will take the sectional area of the belt—

$$b \times \delta = \frac{64 \times \text{BHP}}{D_s \times n} \text{ in sq. inches} \quad . \quad . \quad . \quad (25)$$

$$\text{Width of rim } B = 1{\cdot}1b \times \tfrac{3}{8} \text{ inches} \quad . \quad . \quad . \quad . \quad . \quad . \quad (25\text{A})$$

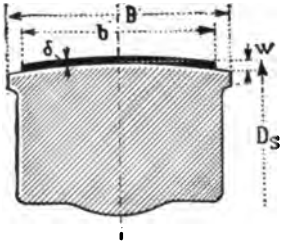


FIG. 22.

Table X.—Approximate Thickness of Belts.

Output HP =	from 2	3–15	16–30	30–50	above 50 HP.
Thickness of belt δ =	$\frac{3}{16}''$	$\frac{7}{32}''$	$\frac{7}{32}''$	$\frac{1}{4}''$	$\frac{9}{32}''$

Example.—A 22 HP engine driven by lighting gas is, according to p. 24, of the following dimensions: $D = 11\frac{3}{4}''$; $S = 15\frac{3}{4}$; $n = 210$ revolutions per minute. Diameter of flywheel $D_s = 7'\ 3''$. What should be the width of the belt?

From equation 25. Sectional area of belt $b \times \delta = \dfrac{64 \times 22}{7{\cdot}25 \times 210} = 0{\cdot}93$ sq. inches.

From Table X. Thickness of belt $\delta = \frac{7}{32}$ inches

Width of belt $b = \dfrac{0{\cdot}93}{\frac{7}{32}''} = 4\frac{1}{4}$ inches.

From equation 25A. *Width of rim* $B = 1{\cdot}1 \times 4\frac{1}{4} + \frac{3}{8} = 5$ inches.

In practice, the rim is usually made wider (see p. 182).

2. *Separate pulley at the side of the flywheel* (Fig. 17, p. 178). The diameter of this pulley on the crank shaft is determined from equation 24.

$$\text{Provisional cross section of belt } b \times \delta = \frac{51 \times \text{HP}}{d \times n} \text{ in square inches} \quad . \quad (26)$$

For the example given above the dimensions would be—

From equation 24. Diameter of pulley $d = 2{\cdot}5 \times 15\frac{3}{4}'' = 3$ ft. 4 inches.

„ „ 26. Sectional area of belt $b \times \delta = \dfrac{51 \times 22}{3{\cdot}28 \times 210} = 1{\cdot}63$ sq. inch.

„ Table X. Thickness of belt $\delta = \frac{7}{32}$ inch.

Therefore width of belt $b = \dfrac{1{\cdot}63}{\frac{7}{32}''} = 7{\cdot}5$ inches.

Thus from equation 25A, *Width of rim* $B = 1{\cdot}1 \times 7\frac{1}{2}'' + \frac{3}{8}'' = 8\frac{5}{8}$ inches.

3. *Double belts.* Single belts are used up to a width of about 12 inches and $3\frac{1}{4}$ square inches cross section; above that double belts are preferable; if possible, the diameter of the pulley should not be less than 100δ.

Double belts do not transmit twice as much power as single belts of the same width, as they are not so flexible. If their width is the same, they will transmit 1·7 times as much power.

4. The size of the belt can only be accurately determined if the position and number of revolutions of the counter shaft are known. A full explanation of this will be found in "Konstruieren und Rechnen," 3rd Edition, pp. 180*b et seq.* Also see table of horse-power of belts and rim speeds of pulleys at the end of the book.

Standard Dimensions of Flywheels up to 10 ft. 10 ins. Diameter (3300 mm.).
(For larger types, see chapter on Large Gas Engines.)

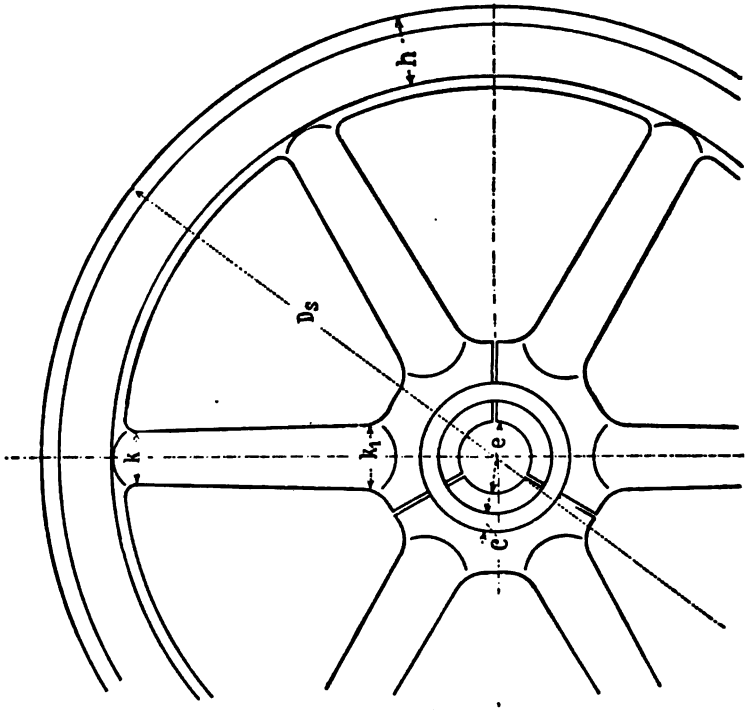


FIG. 24.

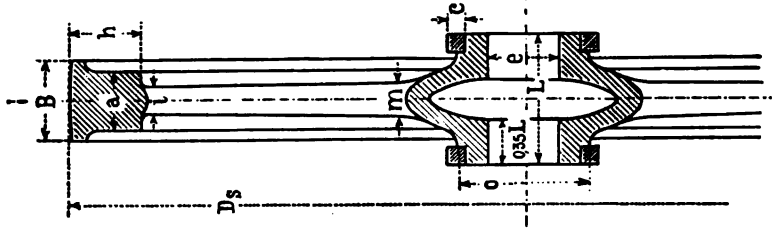


FIG. 23.

Engine.			Weight of rim W in lbs.		D_s	Radius of centre of gravity of rim R.	Cross section of rim A in sq. ins.		Rim.						Nave.				Arms.							
									With $\delta_0 = 1/50$			With $\delta_0 = 1/80$								With $\delta_0 = 1/50$				With $\delta_0 = 1/80$			
HP	S	D	$\delta_0 = 1/50$	$\delta_0 = 1/80$			$\delta_0 = 1/50$	$\delta_0 = 1/80$	B	h	a	B	h	a	c	e*	L*	o*	Number.	i	k	k_1	m	i	k	k_1	m
	ins.	ins.	lbs.	lbs.	ins.	ins.	sq. ins.	sq. ins.	ins.	ins.	ins.	ins.	ins.	ins.	ins.	ins.	ins.	ins.		ins.	ins.	ins.	ins.	ins.	ins.	ins.	ins.
0·6	5	$3\frac{1}{4}$	363	585	$31\frac{1}{2}$	$28\frac{1}{2}$	$15\frac{1}{2}$	$24\frac{3}{4}$	$3\frac{1}{2}$	$4\frac{3}{4}$	$3\frac{1}{8}$	$4\frac{7}{8}$	$5\frac{1}{2}$	$4\frac{1}{2}$	$\frac{7}{8}$	$2\frac{1}{8}$ / $1\frac{5}{8}$	$4\frac{11}{16}$ / $3\frac{15}{16}$	$4\frac{5}{16}$ / $3\frac{5}{16}$	5	1	2	$2\frac{3}{8}$	$1\frac{1}{4}$	$1\frac{1}{4}$	$2\frac{3}{8}$	$2\frac{3}{4}$	$1\frac{3}{8}$
1·2	$6\frac{1}{4}$	4	506	820	$39\frac{1}{2}$	$35\frac{1}{2}$	$17\frac{1}{2}$	28	$3\frac{3}{4}$	$5\frac{3}{16}$	$3\frac{5}{16}$	$5\frac{1}{8}$	$5\frac{7}{8}$	$4\frac{11}{16}$	$\frac{15}{16}$	$2\frac{9}{16}$ / 2	$5\frac{1}{2}$ / $4\frac{5}{16}$	$4\frac{7}{8}$ / $3\frac{15}{16}$	5	$1\frac{1}{4}$	$2\frac{3}{8}$	$2\frac{3}{4}$	$1\frac{3}{8}$	$1\frac{1}{2}$	3	$3\frac{1}{2}$	$1\frac{3}{4}$
2	$7\frac{1}{2}$	5	725	1,150	45	$40\frac{1}{2}$	$21\frac{3}{4}$	34	$4\frac{1}{8}$	$5\frac{9}{16}$	$3\frac{1}{2}$	$5\frac{11}{16}$	$6\frac{3}{8}$	$5\frac{1}{8}$	1	$2\frac{15}{16}$ / $2\frac{5}{16}$	$6\frac{1}{4}$ / $5\frac{1}{8}$	$5\frac{11}{16}$ / $4\frac{11}{16}$	5	$1\frac{1}{2}$	3	$3\frac{3}{8}$	$1\frac{3}{4}$	$1\frac{7}{8}$	$3\frac{3}{4}$	$4\frac{3}{8}$	$2\frac{1}{8}$
3·5	$8\frac{3}{4}$	$5\frac{3}{4}$	970	1,550	51	46	$25\frac{3}{4}$	41	$4\frac{1}{2}$	$5\frac{7}{8}$	$3\frac{15}{16}$	$6\frac{1}{8}$	$6\frac{7}{8}$	$5\frac{1}{2}$	$1\frac{1}{8}$	$3\frac{1}{2}$ / $2\frac{3}{4}$	$7\frac{1}{2}$ / $5\frac{7}{8}$	$6\frac{11}{16}$ / $5\frac{1}{2}$	5	$1\frac{3}{4}$	$3\frac{1}{2}$	4	2	$2\frac{3}{8}$	$4\frac{1}{2}$	$5\frac{1}{8}$	$2\frac{5}{8}$
7	$10\frac{5}{8}$	$7\frac{1}{4}$	1,260	2,020	63	56	27	43	$4\frac{3}{4}$	$6\frac{5}{8}$	4	$6\frac{1}{4}$	$7\frac{5}{8}$	$5\frac{1}{2}$	$1\frac{1}{4}$	$4\frac{3}{4}$ / $3\frac{3}{4}$	$9\frac{7}{8}$ / $7\frac{7}{8}$	9 / $7\frac{1}{2}$	5	$2\frac{3}{8}$	$4\frac{1}{2}$	5	$2\frac{5}{8}$	$2\frac{3}{4}$	$5\frac{1}{2}$	$5\frac{7}{8}$	$3\frac{1}{8}$
11	$12\frac{1}{4}$	$8\frac{3}{4}$	1,650	2,650	$70\frac{3}{4}$	$63\frac{3}{4}$	$31\frac{1}{2}$	$50\frac{1}{2}$	$5\frac{1}{8}$	$7\frac{1}{8}$	$4\frac{3}{8}$	$6\frac{3}{4}$	$8\frac{1}{4}$	$5\frac{7}{8}$	$1\frac{3}{8}$	$5\frac{3}{8}$ / $4\frac{3}{4}$	11 / $9\frac{1}{4}$	$9\frac{7}{8}$ / $8\frac{1}{4}$	6	$2\frac{3}{4}$	$5\frac{3}{8}$	$5\frac{3}{4}$	3	$3\frac{1}{4}$	$6\frac{1}{4}$	$6\frac{7}{8}$	$3\frac{1}{2}$
17	$13\frac{3}{4}$	$10\frac{1}{4}$	2,250	3,700	$78\frac{3}{4}$	71	$38\frac{1}{2}$	$61\frac{1}{2}$	$5\frac{7}{8}$	$7\frac{5}{8}$	$4\frac{7}{8}$	$7\frac{7}{8}$	9	$6\frac{7}{8}$	$1\frac{5}{8}$	$5\frac{7}{8}$ / 5	$11\frac{7}{8}$ / $9\frac{7}{8}$	11 / $9\frac{1}{4}$	6	$3\frac{1}{4}$	$6\frac{1}{4}$	$6\frac{3}{4}$	$3\frac{1}{2}$	$3\frac{3}{4}$	$7\frac{1}{4}$	$7\frac{7}{8}$	$4\frac{1}{8}$
25	$15\frac{3}{4}$	$11\frac{7}{8}$	3,200	5,120	87	78	$49\frac{1}{2}$	80	$6\frac{3}{4}$	$8\frac{1}{4}$	$5\frac{3}{4}$	$9\frac{1}{8}$	$9\frac{3}{4}$	$8\frac{1}{8}$	$1\frac{3}{4}$	$6\frac{7}{8}$ / $5\frac{1}{4}$	13 / $11\frac{3}{8}$	$12\frac{1}{4}$ / $10\frac{1}{4}$	6	$3\frac{3}{4}$	$7\frac{1}{8}$	$7\frac{7}{8}$	$4\frac{1}{8}$	$4\frac{1}{2}$	$8\frac{5}{8}$	$9\frac{1}{8}$	$4\frac{3}{4}$
33	$17\frac{3}{4}$	13	4,150	6,600	$94\frac{1}{2}$	85	59	95	$7\frac{1}{2}$	9	$6\frac{1}{4}$	10	$10\frac{1}{2}$	$8\frac{7}{8}$	2	$7\frac{1}{2}$	$14\frac{1}{8}$	13	6	$4\frac{1}{8}$	$7\frac{7}{8}$	$8\frac{5}{8}$	$4\frac{1}{2}$	$4\frac{7}{8}$	$9\frac{5}{8}$	$10\frac{1}{4}$	$5\frac{3}{8}$
42	$19\frac{3}{4}$	$14\frac{1}{4}$	5,200	8,300	102	92	68	110	$8\frac{7}{8}$	$9\frac{5}{8}$	$6\frac{7}{8}$	$10\frac{7}{8}$	$11\frac{1}{4}$	$9\frac{5}{8}$	2	$7\frac{7}{8}$	15	$13\frac{3}{4}$	6	$4\frac{1}{2}$	$8\frac{5}{8}$	$9\frac{5}{8}$	$4\frac{7}{8}$	$5\frac{1}{4}$	$10\frac{1}{2}$	$11\frac{1}{4}$	$5\frac{7}{8}$
55	$21\frac{7}{8}$	$15\frac{3}{4}$	7,050	11,300	108	$97\frac{1}{4}$	$88\frac{1}{2}$	142	$9\frac{5}{8}$	$10\frac{1}{4}$	$8\frac{1}{4}$	13	12	$11\frac{5}{8}$	$2\frac{1}{8}$	$8\frac{1}{2}$	$15\frac{3}{4}$	$14\frac{5}{8}$	6	$4\frac{7}{8}$	$9\frac{5}{8}$	$10\frac{1}{4}$	$5\frac{3}{8}$	$6\frac{1}{8}$	$11\frac{7}{8}$	$12\frac{5}{8}$	$6\frac{5}{8}$
80	$23\frac{5}{8}$	19	11,100	17,900	114	102	132	210	$13\frac{1}{4}$	$10\frac{7}{8}$	$11\frac{7}{8}$	$17\frac{1}{2}$	13	$16\frac{5}{8}$	$2\frac{1}{4}$	$9\frac{1}{8}$	$17\frac{3}{4}$	$16\frac{5}{8}$	6	$6\frac{1}{8}$	$11\frac{5}{8}$	$12\frac{3}{4}$	$6\frac{3}{4}$	$7\frac{1}{2}$	$14\frac{3}{8}$	$15\frac{5}{16}$	$8\frac{1}{8}$
132	$27\frac{1}{2}$	$23\frac{3}{8}$	18,800	30,000	130	116	194	310	$17\frac{7}{8}$	$12\frac{1}{4}$	$15\frac{1}{2}$	$22\frac{7}{8}$	$14\frac{3}{8}$	$21\frac{1}{4}$	$2\frac{3}{8}$	$11\frac{5}{8}$	$23\frac{1}{4}$	$19\frac{3}{4}$	6	$7\frac{1}{2}$	$14\frac{3}{8}$	$15\frac{3}{8}$	$8\frac{1}{8}$	$9\frac{1}{2}$	$18\frac{3}{8}$	$19\frac{1}{4}$	10

* The measurements for *e*, L, and *o* apply to engines with three bearings, the figures printed in small type to engines with two bearings, and the flywheel overhung at the end of the shaft.

2nd Example from Table X. In a 110 HP engine driven with lighting gas, the main dimensions as given at p. 24 are as follows: D = 24 inches; S = 28 inches; n = 150 revolutions per minute; D_s = 10 feet 10 inches. What should be the width of the belt if the power is transmitted to it from the flywheel?

According to equation 25, cross section of belt $b \times \delta = \frac{64 \times 110}{10{\cdot}8 \times 150}$ = about $4\frac{3}{8}$ sq. inches.

From Table X., if $b \times \delta = 4\frac{3}{8}$ sq. inches, $\delta = \frac{9}{32}$ inch (bare).

Therefore width of the belt $b = \frac{4\frac{3}{8}}{\frac{9}{32}''} = 15\frac{1}{4}$ inches.

If, again, we take the ratio of the driven pulley to the flywheel as 1 : 5, then diameter of the driven pulley = $\frac{130}{5}$ inches = 26 inches, or about 100 × δ, the thickness of δ being = $\frac{9}{32}$ inches (bare).

Table XI. gives dimensions of flywheel for standard engines, type E 4, single cylinder, taken from Table I., p. 24.

The measurements are worked out for two different degrees of irregularity in running, namely, $\delta_0 = \frac{1}{50}$ = 2 per cent. variation, and $\delta_0 = \frac{1}{80}$ = 1·25 per cent. ($1\frac{1}{4}$) variation in the speed. Engines for industrial purposes can have lighter flywheels, with $\delta_0 = \frac{1}{50}$; for driving continuous-current dynamos $\delta_0 = \frac{1}{80}$ is generally sufficient (compare Table VII., p. 175). The engineer must be content with a lesser degree of uniformity in running gas engines than steam engines, otherwise the weight of the flywheel becomes excessive.

For weights of the flywheel corresponding to both degrees of irregularity the same measurements apply for the nave, while those for the rim and arms are different, corresponding to the variations in the weight.

The stress of the arms upon the rim of the flywheel varies from 1400 to 1550 lbs. per sq. inch, calculated from equation 61, p. 196.

Width of the Flywheel.

The width of the belt is determined from the power to be transmitted (see pp. 180 *et seq.*). The width of the face of the rim may (according to "Konstruieren u. Rechnen," 3rd Edition, p. 183) be taken at,—

$$B = 1{\cdot}1b + \tfrac{3}{8} \text{ inch} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (27)$$

To make gas engines run as steadily as possible, it is necessary, as we have said, to have comparatively heavy flywheels, and this requires a large rim cross section. The width of the flywheel rim cannot, however, altogether depend on the width of the belt (see equation 27), but to avoid too great a depth of rim, flywheels must under certain circumstances be made broader than the width given in equation 27. Too deep a rim has not a neat appearance, and if h is unduly large, there will be waste of material.

Let D_s be the outside diameter of the flywheel in feet;

R „ radius of circle described by centre of gravity in feet (radius of gyration);

W „ weight of rim in lbs. from equation 4 (p. 165).

γ „ lbs. per cubic inch of material composing the flywheel;

(B, a, and h, are measured in inches from Fig. 25.)

f = approx. $h \times a$. Cross section of rim in sq. inches.

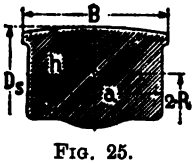


FIG. 25.

Then—

$$\text{Rim section } f = \frac{W}{\gamma \times 2 \times R \times \pi \times 12} \text{ about } 0{\cdot}051 \frac{W}{R} \text{ in sq. inches} \quad . \quad (28)$$

Hence depth of rim $h = \frac{f}{a}$ in inches ; $a = \frac{f}{h}$ in inches . . . (29)

We may take approximately—

$$R \sim 0{\cdot}45D_e\,;\ a = 0{\cdot}9B\,;\ \gamma = 0{\cdot}262 \text{ lbs. per cubic inch} \quad . . . \quad (30)$$

in other words—

$$\left.\begin{array}{l} h \leqq \frac{1}{6}D_e \text{ if } D_e \leqq 2 \text{ ft. 8 ins.} \\ h \leqq \frac{1}{7} \text{ to } \frac{1}{10}\,D_e \text{ if } D_e > 2 \text{ ft. 1 in.} \end{array}\right\} \quad . . . . . \quad (31)$$

The circumference of the rim is made either flat or convex. Where power is transmitted from the driving to the counter pulley, Gehrckens recommends that the former be made flat (not rounded), and the driven pulley convex. It ought to suffice if, B being width of rim in inches (Fig. 22, p. 180), the convexity is—

For the large pulley $w = 0{\cdot}02B$ in inches . . . . . (32)

For the smaller pulley $= (d \div D) \times w + \frac{1}{16}$ in. . . . . (33)

(Compare "Konstruieren u. Rechnen," 3rd Edition, p. 183*l*.)

Calculation of the Tensile Strength of the Flywheel.*

(a) Shape and Dimensions of Rim Cross Section (see Fig. 1, next page).

Let W = weight of the rim in lbs. calculated from equation 4, p. 165;
D_e = external diameter of the rim in feet (or of the pitch circle, if the flywheel is grooved for ropes);
S = centre of gravity of rim section;
f = cross section of rim in sq. inches;
γ = specific weight = about 0·262 lbs. per cubic inch for *cast iron*.

The main dimensions of the cross sections must be considered for: solid flywheels, pulley flywheels (*i.e.* for belts), and flywheels grooved for ropes.

The following *approximate* values may be taken for R:—

For *solid flywheels*	$R = 0{\cdot}45D_e$	. . . (1)
„ *pulley flywheels* (*i.e.* for belts)	$R = 0{\cdot}485D_e$	. . (2)
„ *flywheels grooved for ropes*	$R = 0{\cdot}475D_e$	. . (3)

Therefore weight of rim $W = 2 \times \pi \times R \times 12 \times f \times \gamma$

$$\mathbf{W} = 0{\cdot}262 \times 2\pi \times R \times 12 \times f = \mathbf{19{\cdot}7} \times R \times f \text{ in lbs.} \quad . . \quad (4)$$

Whence approximate rim section $f = \frac{W}{19{\cdot}7 \times R}$ in sq. inches. . . . (5)

R must for the present be found by equations 1–3, in feet, since only D_e is known.

With solid flywheels the depth of the rim is taken provisionally at = 1·2 of the width.

An outline drawing of the rim corresponding to the area f and the above dimensions should be made, and the exact radius of the circle described by the centre of gravity R determined; if this radius differs from the first rough estimate, the area f must be corrected to make them agree.

Distance of the centre of gravity x_0 from the outer edge of the rim: $x_0 = \frac{\Sigma f \times x}{\Sigma f}$ . . . (6)

* In order to work from the usual allowable stresses (or safe working loads), when calculating the flywheel rim fastenings, and at the same time to keep practical conditions of work in view, some engineers do not calculate from the normal number of revolutions, but from speeds, say, 10 per cent. to 15 per cent. higher. As this method complicates the subject, the author has always based his figures on the normal speed n, and has simply taken the allowable stresses at less than the values usually assigned them.

Σ signifies a total

Σf signifies the sum (or total) of all the cross sections

therefore $f_1 + f_6 + f_2 + \ldots$

$\Sigma f \times x$. Sum of the products of the separate cross sections, multiplied by the corresponding distances x of their centres of gravity from the edge of the rim; thus—

$$f_1 \times x_1 + f_2 \times x_2 + f_3 \times x_3 + \ldots$$

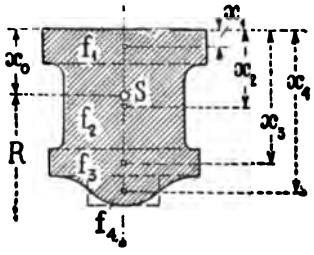


FIG. 1.

(b) Centrifugal Force and Tensile Stress in the Flywheel Rim.

Referring to Fig. 2, let—

V = circumferential speed in circle described by the centre of gravity

$$= \frac{R \times \pi \times n}{30} \text{ in ft. per sec.} \quad \ldots \ldots \ldots \quad (7)$$

S′ = centre of gravity of one half of the wheel.

v = circumferential velocity of centre of gravity S^1 in ft. per sec.

e = distance of centre of gravity S^1 from the centre of the flywheel in feet.

σ_z = tensile stress on the rim in lbs. per sq. inch.

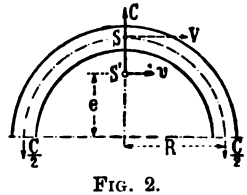


FIG. 2.

The centrifugal force tends to rend the flywheel into two opposite rim sections, therefore half the weight of the rim $\left(\frac{W}{2}\right)$ must be considered. The centrifugal force exerted in one half of the flywheel is thus—

$$C = \frac{W}{2} \times \frac{1}{g} \times \frac{v^2}{e} = \frac{W}{2g} \times \frac{2V^2}{\pi R} = 0{\cdot}195 f V^2 \text{ in lbs.} \quad \ldots \ldots \ldots \quad (8)$$

$$\text{Furthermore } e = \frac{2R}{\pi};\ v = \frac{e}{R} \times V = \frac{2V}{\pi} \quad \ldots \ldots \ldots \quad (9)$$

Taking half the cross section of the rim, we get—

$$0{\cdot}5C = 0{\cdot}098V^2 \times f \text{ in lbs.} \quad \ldots \ldots \ldots \quad (10)$$

From equations 8 and 9 we can determine the

$$\text{Theoretical stress due to tension } \sigma_z = \frac{0{\cdot}5C}{f} = 0{\cdot}098V^2 \text{ in lbs. per sq. inch.} \quad . \quad (11)^*$$

This value is increased by the effect of the arms, and important bending stresses are set up, which hinder the free expansion of the rim (Fig. 3), and increase the stress coming upon it.

Let us take the

$$\text{combined stress} = x \times \sigma_z \text{ in lbs. per sq. inch} \quad \ldots \quad (12)$$

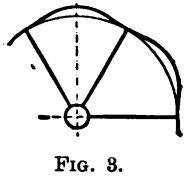


FIG. 3.

* The size and shape of the rim cross section has thus no effect on the theoretical stress due to tension.

Table I.—Approximate Values for Factor x in Equation 12.

$\frac{12 \times (R \text{ in feet})}{h \text{ in inches}}$ R in feet h in inches	Pulley flywheels.		Pulleys for belts and rope-grooved wheels.				Solid flywheels.		
$\frac{12 \times R}{h} =$	65	50	40	30	20	15	10	7	5
$x =$	2·9	2·7	2·4	2·1	1·9	1·7	1·5	1·4	1·3

The values of x given in the above Table apply to carefully turned flywheels divided along the arms (as shown at Fig. 7, see next page).

If the flywheel is divided between the arms (Fig. 6) the stresses should be taken at 20 per cent. higher.

Example.—Let R = 5 ft.; $h = 5\frac{1}{2}$ ins.; V = 74 ft. per sec.; according to equation 11, the theoretical stress due to tension will be $\sigma_z = 0{\cdot}098 \times 74^2 = 540$ lbs. per sq. in. Again, from Table I., $x = 1{\cdot}58$, therefore the combined stress (if the flywheel is divided along the arms) will be $530 \times 1.58 = 835$ lbs. per sq. in. If the flywheel is divided between the arms, the combined stress will be $1{\cdot}2 \times 835 = 1000$ lbs. per sq. in. We may take, therefore, the actual stress between the arms of the flywheel, according to Table I., as approximately *—

For belt pulleys and rope grooved wheels $= 1{\cdot}9\ \sigma_z$ . . . . . (13)

For solid flywheels $= 1{\cdot}4\ \sigma_z$ . . . . . (14)

Therefore, from equations 13 and 14, for wheels divided along the arms (Fig. 7, next page) we get

Table II.—Combined Stress.

Circumferential speed V =	40	60	80	90	100	150	200	270	300 feet per second
Belt pulleys =	300	680	1220	1540	1900	4250	7600	13,800	17,000 lbs. per sq. in.
Solid flywheels † =	225	500	900	1130	1400	3150	5600	10,200	12,600 ,, ,,

Allowable stress with this method of calculation from equations 11 to 14.

For cast iron = 1500 lbs. per sq. inch . . . . . (15)

(Table II. shows that in belt pulleys the allowable stress is attained at a circumferential speed of 90 ft. per sec., that the limit of elasticity is reached at a circumferential speed of 260 ft. per sec., and the breaking load at 320 ft. per sec.)

Thus, within the usual limits of speed (V = 30 to 100 ft. per sec.) there is never a risk of fracture with a solid rim.

But if the rim is grooved, or the symmetry of the section destroyed (as in Figs. 43 and 44, p. 194, for example), the combined stress must be increased in the same ratio as the total rim section bears to the effective rim section.

* These values can be accurately determined for a given engine from Goebel's rule (*Zeitschrift des Vereines deutscher Ingenieure*, 1898, p. 351). The values given in Table I. apply only if the flywheel rotates at a uniform speed. If through a sudden increase in the load (and therefore a rapid fall of the governor) the speed is retarded, the engine will hunt badly, but the amount of retardation cannot be determined beforehand. If the rim fastenings (clamps, shrunk rings, bolts, etc.) are comparatively far from the circle described by the centre of gravity of the rim section, this hunting may under certain conditions set up important bending stresses, the extent of which must be determined by Goebel's rule mentioned above.

† In gas engines only solid flywheels need be considered $\frac{12 \times R}{h} = < 15$.

TRANSLATOR'S NOTE.—Nearly all larger gas engines in England have built-up flywheels.

(c) Rim Joints.

The principal methods of constructing flywheels are shown in Figs. 6 to 8.

The *best way* is to divide it along the arms as shown at Fig. 7, as by this means the fastenings and bolts are exposed to tensile stresses only, the bending stresses being taken by the arms; the tensile stress c can be calculated by equation 11, p. 186.

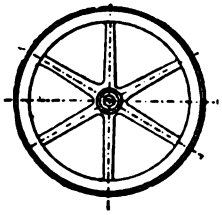


FIG. 5.—Solid Wheel.

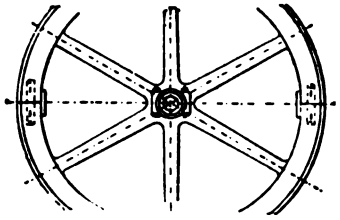


FIG. 6.—Divided between the Arms.

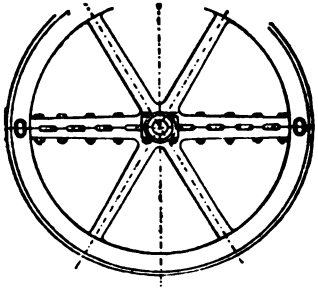


FIG. 7.—Divided along the Arms.

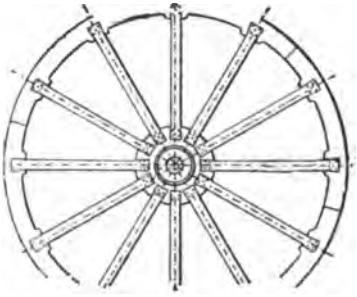


FIG. 8.—In Several Segments with Wrought Iron Arms.

Many engineers are of opinion, however, that if the flywheel is built as shown in Fig. 7 undesirable strains may be set up in the metal, owing to the unequal rate of cooling in the mould after casting.

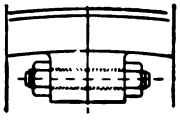
1. Joints and Bolts.*

We must here distinguish between—

Stress on the bolts due to tightening up.

Stress on the bolts when the wheel is in motion.

The stress † to be here considered is that set up when the flywheel is in motion. Care must be taken to see that the joints on the segments are bedded well down on to one another, and the bolts well tightened up.



* For a fuller account, see Haeder's *Zeitschrift*, 1904, p. 132.

† The stress due to tightening up is taken at—

$$s = \frac{0{\cdot}85P}{f_s} \text{ in lbs. per sq. inch where } f_s = \text{the area of the bolts in sq. inches} \quad . \quad . \quad (16)$$

If the belts are tightened up more than is necessary, then if S represents the stress due to tightening up in lbs. per sq. inch, the stresses on the bolts when the flywheel is in motion—

$$= \sigma_2 = \frac{S + 0{\cdot}35P}{f_s} \text{ in lbs. per sq. inch} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (17)$$

But if the bolts are not tightened up to the extent given in equation 16, there will be working at the joints (see Haeder's "Steam Engines," 8th Edition, pp. 282 *et seq.*).

Let—

f be the sectional area of the rim in sq. ins.

V be circumferential speed in circle described by centre of gravity, in ft. per second by equation 7, p. 186.

$P = 0{\cdot}0971V^2 \times f$, total tensile stress on the bolts in lbs.

1·2P denotes the effect due to tightening up the bolt.

f_s Total cross section at the bottom of the thread of all the bolts in sq. ins., then—

Working stresses on the bolts (when in motion) $\sigma_s = \mathbf{1{\cdot}2P} \div f_s$ in lbs. per sq. in. (18)

Corresponding cross section of the bolts at the bottom of the thread $f_s = \mathbf{1{\cdot}2P}\sigma_s$ in sq. ins. (19)

Allowable stress = 4250 – **5700** – 7000 lbs. per sq. in. . . . . (20)*

Example.—Let a flywheel have a rim cross section of F = 57 sq. in. (see Fig. 10). The circumferential speed is 77 ft. per second. The rim is held together by three bolts, each $1\frac{7}{8}$ in.

The stresses will be as follows—

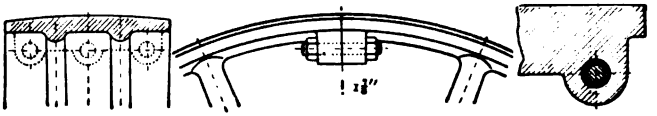


FIG. 10. FIG. 11. FIG. 12.

Total stress due to tension $P = 0{\cdot}0971 \times 77^2 \times 57 = 32{,}800$ lbs.

Total cross section at the bottom of the thread $f_s = 3 \times 1{\cdot}986$ sq. in. = 5·95 sq. in.

According to equation 18, working stresses $\sigma_s = 1{\cdot}2 \times 32{,}800 \div 5{\cdot}95 = 6600$ lbs. per sq. in.

2. Fastening with Cotters.

This method of fastening the rim is shown in detail in Figs. 13, 14.

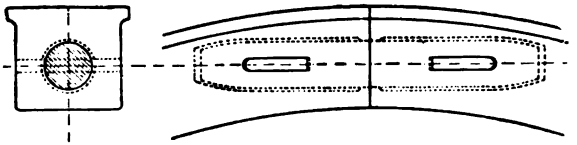


FIG. 13.

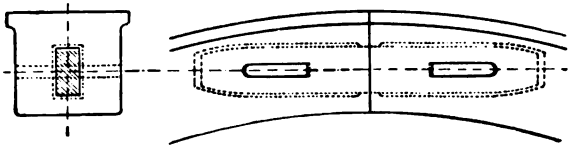


FIG. 14.

The bolts are replaced by the round or square dowel N, and the cotter M, Fig. 15.

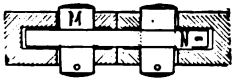


FIG. 15.

As explained in the section on bolt fastenings, p. 188, we have in both cases two separate kinds of stresses to consider, namely, that due to tightening up,† and the working stresses on the fastenings (when in motion).

The same symbols as those for bolts can be used, except that f_s denotes the cross section of the dowel in square inches.

* See Table of Bolts, etc., at end of book.

† The footnote, p. 188, applies to a certain extent to these fastenings.

Therefore—

Working stresses on the dowel (when in motion) $\sigma_z = 1{\cdot}2P \div f_0$ in lbs. per sq. in. (21)

Corresponding cross section of dowel $f_0 = 1{\cdot}2\ P \div \sigma_z$ in sq. ins. . . (22)

Allowable stress = 4250 – 5700 – 7000 lbs. per sq. in. . . . . (23)

The cross section of the dowel is shown by the shaded area in Fig. 16, for which we may take—

For round dowels $f_0 = (\pi \div 4) \times d^2 - d \times b$ in sq. ins. . . . . (24)

„ square dowels $f_0 = a \times c - a \times b$ . . . . . (25)

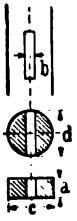


FIG. 16.

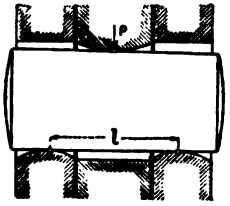


FIG. 17.

The cotters must be calculated for stresses due to bending, as shown in Fig. 17. The three places on which the stresses are assumed to fall are indicated by curved lines, which in any rough design of this kind of fastening would mark them with sufficient accuracy.

The calculation is worked out as follows :—

Bending moment $M_b = 1{\cdot}2\ Pl \div 4$ in inch-lbs. . . (26)

where $P = f_0 \times \sigma_z$ in lbs.*

Modulus of resistance to bending $Z_b = \frac{1}{6}\ bh^2$ in cub. ins. . . . (27)

Actual stress due to bending $\sigma_b = M_b \div Z_b$ in lbs. per sq. in. (28)

Allowable stress for steel, **14,000** *lbs. per sq. in.* . . . (29)

The length l should be approximately :—

For dowels of round section $l = 1{\cdot}7d$ . . . . . (30)

„ „ square section $l = 1{\cdot}5a$ . . . . . (31)

To calculate the rim fastenings we must also consider the surface pressure of the cotter upon the cast-iron surface of the flywheel rim (see Fig. 18)—

$$\sigma = \frac{\sigma_z f_0}{2zb} \text{ in lbs. per sq. in.*} \quad . \quad . \quad . \quad . \quad . \quad . \quad (32)$$

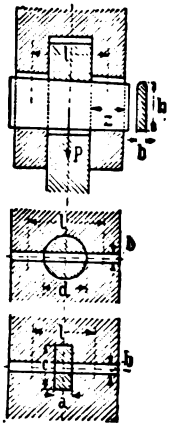


FIG. 18.

Surface pressure of the cotter upon the dowel—

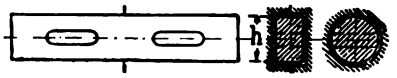
$$\sigma = \frac{\sigma_z f_0}{bd} \quad \text{or} \quad \sigma = \frac{\sigma_z f_0}{ba} \text{ in lbs. per sq. in.*} \quad . \quad . \quad . \quad . \quad (33)$$

Allowable surface pressure of the cotter upon the flywheel rim—
5700 to 8500 lbs. per sq. in.
Allowable surface pressure of the cotter upon the dowel—
11,000 to 14,000 lbs. per sq. in. } (34)

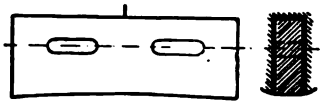
* See footnote, p. 188, which to a certain extent is applicable here.

Dowels are made round, but it is better to have them of rectangular cross section, because the holes in the rim are then easier to machine (Figs. 19–21).

Dowels for heavy flywheels (Figs. 22, 23) are convenient when fitting the parts together. They



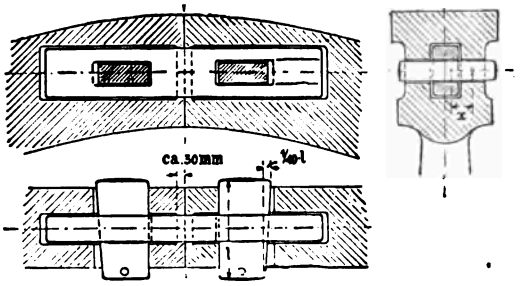
FIGS. 19–21.



FIGS. 22, 23.

are entered from the inner surface of the rim, but cannot always be recommended, because, firstly, they are expensive to manufacture, and, secondly, the core may be wasted away in casting.

Although *round dowels* are much used they are not so good as dowels of rectangular section, because if the holes are cored, they are apt to be out of truth. On the other hand, it is easy to bore out the holes if they are not quite true. Dowels of rectangular section are, however, better, as we have said, because it is easier to machine the holes for them.



FIGS. 24, 25, 26.

To ensure that the two halves, or several *segments of the rim*, shall run true, it is usual to design the flywheel with a tongue piece in one segment fitting into a recess in the next. Another method is to make the dowels accurately fit the holes in the flywheel, which are bored out to a depth of $\frac{3}{8}$ in. to $1\frac{1}{8}$ in. from the face of the joint (Figs. 24–26). The other part of the hole can be carefully cored out, allowing a clearance of $\frac{5}{16}$ in. to $\frac{3}{8}$ in. round the dowel. The faces of the joint where the segments of the rim butt against one another is often cast with chipping-strips, and afterwards finished off with chisel and file; and the hole for the bolt, after trueing up the joint, may be simply and accurately marked off. To cast the segments of *heavy flywheels* with the chipping strips is not considered "good practice," because the edges may easily be damaged in transit, or when erecting the flywheel, and it is always better to plane the surfaces. To avoid unnecessary work when fitting the cotter a clearance of $\frac{1}{16}$ in. to $\frac{1}{8}$ in. should be left at the sides.

FIG. 27.

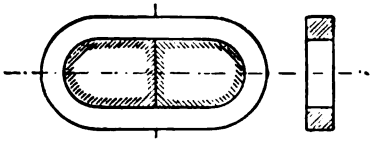
FIG. 28.

Shrunk clamps are also much used, although considerable skill is required to fit them accurately, and gauge the correct amount of shrinkage. The heads must be well recessed. It is easy to file or turn the faces of the clamps flush with the side of the rim afterwards; they must fit exactly. Boiler plate scrap is used to hold them together provisionally while on the lathe, and is knocked off afterwards.

The *dove-tailed fastening* shown at Fig. 28 should on no account be used, as it has a tendency to work out, and acts like a wedge to split the rim (see *Zeitschrift des Vereines deutscher Ingénieure*, 1898, p. 357).

Shrunk rings with a corresponding projection or shoulder on the rim can only be used if fitted to both sides of the rim, in the circle described by the centre of gravity of the cross section, or on both the inner and outer sides of the rim, and not on the inside only. To prevent them from flying at

the outside edge, as sometimes happens while hammering them in, they should be of trapezium-shaped section. The opening for them on the rim should be as short as possible (Fig. 31). When



FIGS. 29, 30.

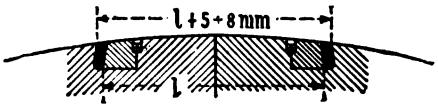


FIG. 31.

the ring has cooled, the recesses are filled in with white metal and caulked, and the ring is also held in position by a couple of grub screws.

3. Calculation of the Shrunk Rings.

It is not intended to repeat here the exhaustive dissertation on shrunk rings in Haeder's "Konstruiren und Rechnen" (3rd Edition), but only to give the results theoretically determined.

Let—

P be the tension in the rim, that is the force which seeks to tear the ring asunder, in lbs.

1·2P is taken as the actual load.

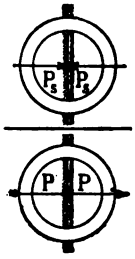
s the shrinkage tension, that is, the tensile stresses set up in the shrunk ring as it cools, in lbs. per sq. in.

f_1 cross section of the shrunk rings of a rim fastening in sq. ins.

P_s pressure on the joints when at rest, that is, the pressure with which the surfaces of the joints are tightened against each other, while stationary, in lbs.

σ_z working pull on the rings, that is, the actual stress coming upon the shrunk ring when in motion, in lbs. per sq. in.

At Rest.



In Motion.

Then we have the tension in the rim $\mathbf{P = 0{\cdot}0971\ V^2 \times f}$ in lbs. . . . (35)

(in which V is the circumferential velocity of the circle described by the centre of gravity from equation 7, p. 186, and f is cross section of rim in sq. ins.).

Cross section required $f_1 = 1{\cdot}2\ P \div k_z$ in sq. ins. . . . . (36)

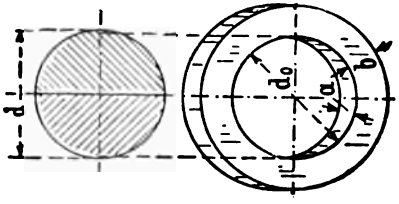
Pressure on the joints when not in motion $P_s = s \times f_1$ in lbs. . . . (37)

" " " in motion $= \sigma_z \times f_1 - P$ in lbs. . (38)

Tension due to shrinkage $s = 0\ 8\ \sigma_z$ in lbs. per sq. in. . . . . (39)

DIMENSIONS OF THE SHOULDER.

(a) Round Shrunk Rings.



To determine these dimensions we must take count of the *strength of the shoulder*. It is exposed to a shearing stress, in consequence of the tension set up during shrinkage, therefore

$$\underbrace{\text{Resistance of one-half of the shoulder}}_{\frac{1}{8}\pi d^2 k_s} = \underbrace{\text{Tensile stress of the shrunk ring}}_{2abk_z} \quad . \ . \ . \ (40)$$

If, to make sure that the shoulder is strong enough, we calculate from $k_s : k_z = 4$,* it works out at—

$$d = 4{\cdot}5\sqrt{a \times b} \text{ in ins.} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (41)$$

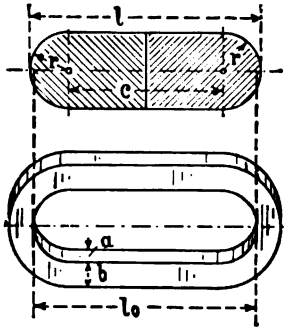
If P be determined from the normal number of revolutions (see footnote on p. 185), then the

Stress on the shrunk ring $k_z = 5700$ lbs. per sq. in. . . . . (42)

Table III.—Examples worked out from Equations 36 and 41 ($k_z = 5700$ lbs. per sq. in.).

(The fastening consists of two shrunk rings of square section.)

P =	22,000	44,000	88,000	132,000	176,000	220,000 lbs.
a =	$1\frac{1}{16}$	$1\frac{1}{2}$	$2\frac{3}{16}$	$2\frac{5}{8}$	$3\frac{1}{16}$	$3\frac{3}{8}$ inches.
d =	$4\frac{3}{4}$	$6\frac{3}{4}$	10	12	$13\frac{3}{4}$	$15\frac{1}{4}$,,



The shrunk rings and hoops forming the fastening of the flywheel rim cannot be machined with absolute accuracy. The necessary allowance for shrinkage is obtained by making the clear width of the ring less than the diameter of the shoulder by about 1 per cent. The less careful the workmanship of the fastening, the greater difference must be allowed between the dimensions of these two parts.

(b) Elliptical Shrunk Rings (Shrunk Hoops).

For these, as for round rings, $rlk_s = 2abk_z$ . . (43)

As regards the permissible stresses k_s and k_z, the remarks under head (a) apply, from whence we get—

$$rl = 8ab \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (44)$$

For instance, if $l = 6r$, then $r = 1{\cdot}2\sqrt{ab}$; $l = 7\sqrt{ab}$. (45)

Table IV.—Examples worked out from Equations 36 and 45 ($k_z = 5700$ lbs. per sq. in.).

(The fastening consists of two shrunk rings of square section.)

P =	22,000	44,000	88,000	132,009	176,000	220,000 lbs.
a =	$1\frac{1}{16}$	$1\frac{1}{2}$	$2\frac{3}{16}$	$2\frac{5}{8}$	$3\frac{1}{16}$	$3\frac{3}{8}$ inches.
b =	$7\frac{1}{2}$	$10\frac{5}{8}$	$15\frac{3}{8}$	$18\frac{1}{2}$	$21\frac{5}{8}$	24 ,,

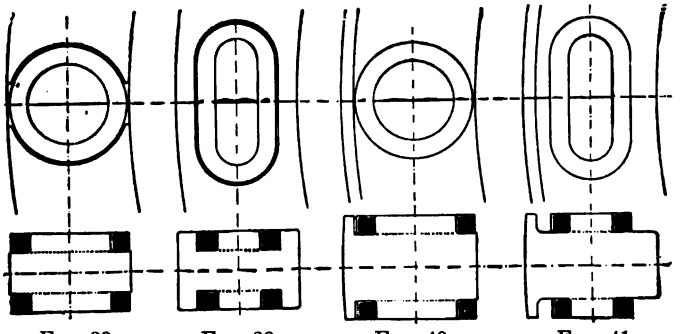


FIG. 38. FIG. 39. FIG. 40. FIG. 41.

For rolling mill engines. Shrunk ring recessed in.

For steam engines, gas engines, etc. Flywheel turned. Shoulder for shrunk ring projecting.

With V = 98·5 ft. per second; f = 234 sq. inches; P = 220,000 lbs.

For the method of allowing for shrinkage, see the remarks at p. 192. The clear width of the ring must be 1 per cent. smaller than the length of the shoulder.

* If space allow, $k_z : k_s = 5$.

For fastening the flywheel, round shrunk rings are sometimes used (Figs. 38 and 40), but elliptical shrunk rings are more general (Figs. 39 and 41).

As the shoulder must be of a certain size to resist the shearing stress on it, a round shoulder takes up more room than one for an elliptical shrunk ring. If Figs. 38 and 40 be compared with Figs. 39 and 41, in all of which the value of P is the same, it will be seen that in the two latter the full depth of the rim is not utilised. If the cross section of the ring were larger, a greater circumferential velocity would be permissible, the size of the shoulder being also naturally increased.

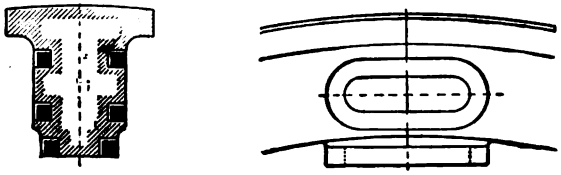


FIG. 42.

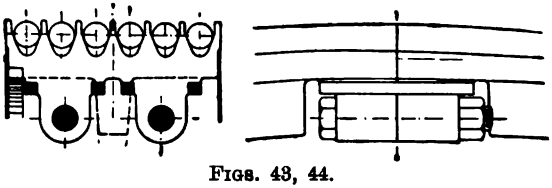
If the circumferential velocity be very high, it is advisable to utilise both the outer and inner sides of the rim for the fastenings, and to have three or four shrunk rings on each joint.

The rings are either of rectangular or square section.

To put a third ring on the inside of the rim only does not seem to be good practice; a corresponding ring should be fitted also to the outside of the rim.

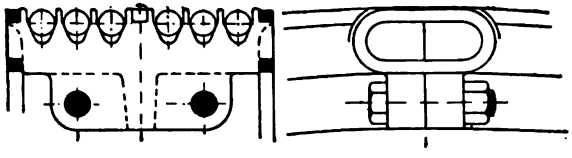
(c) Usual Forms of Rim Fastenings.

The most usual method of *fastening the rim together* is with *screwed bolts*, *dowels*, and *cotters*, or *bolts* only. With flywheels running at high speeds, *shrunk rings* are preferred. Figs. 47, 48 illustrate the usual arrangement for bolts, dowels, and cotters; Figs. 43–46 for bolts and shrunk rings.



FIGS. 43, 44.

Fig. 43 should not be adopted if the rim is deep and the circumferential velocity great, because the bolts and shrunk rings are too far from the circle described by the centre of gravity of the rim.



FIGS. 45, 46.

Calculation * of Rim Fastenings with Screwed Bolts and Dowels.

As far as possible the dowel and cotter fastenings should be in the circle described by the centre of gravity of the rim (Fig. 48A), and the bolts should go through the arms where they intersect the flywheel. As they cannot always be put in with heads, they should have nuts at either end, and lock nuts or other means of preventing any risk of their slacking back whilst running.

* See also footnote on p. 185.

Let P = tension on the fastening in lbs. from equation 35, p. 192 (compare Figs. 47, 48).
s = diameter of the rim bolts at the bottom of the thread in ins.
$a \times c$ = cross section of the dowel in sq. ins.
$b \times h$ = „ „ „ cotter „ „
Then—

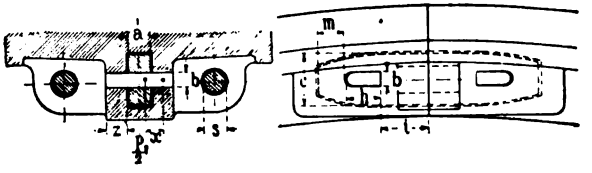
$$\left.\begin{array}{l}\text{Sectional area of the bolts } f' = 2 \times \frac{\pi}{4}s^2 \\ \text{„ \quad „ \quad dowel } f'' = (c - b)a\end{array}\right\} \text{ in sq. ins.} \quad (46)$$

Hence stress due to tension—

$$= \sigma_z = \frac{1{\cdot}2P}{f' + f''} \text{ in lbs. per sq. in.} \quad (47)$$

Allowable stress or safe working load for wrought iron—

$$k_z = 4250\text{–}\mathbf{5700}\text{–}7000 \text{ lbs. per sq. in.} \quad (48)$$



FIGS. 47, 48.—Rim Fastenings with Screwed Bolts and Dowels.

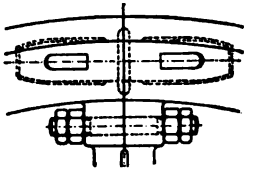


FIG. 48A.

The *strength of the cotter* is taken at—

$$P' = \frac{1{\cdot}2P}{f' + f''} \times f'' \text{ in lbs.} \quad (49)$$

and must further be tested for bending, thus—

$$\text{Bending moment (from Bach's formula)} = M = \frac{P'}{2} \times \left(\frac{a}{4} + \frac{z}{2}\right) \quad (50)$$

$$\text{Modulus of resistance to bending } Z = \frac{b \times h^2}{6}\text{; therefore } \sigma_b = \frac{M}{Z} \text{ lbs. per sq. in.} \quad (51)$$

Allowable stress or safe working load with best mild *steel*—

$$k_b = \mathbf{8500}\text{–}11{,}000 \text{ lbs. per sq. in.} \quad (52)$$

Surface pressure of the cotter on the cast iron surface of the wheel—

$$\sigma = \frac{P'}{f_0} \text{ lbs. per sq. in.; } f_0 = 2z \times b \text{ sq. ins.} \quad (53)$$

$$\text{Safe working pressure } k = \mathbf{7000}\text{–}10{,}000 \text{ lbs. per sq. in.} \quad (54)$$

Shearing the dowel at either end (2 surfaces)—

$$f_0 = 2am \text{ in sq. ins.} \quad (55)$$

$$\text{Therefore } \sigma = \frac{P'}{f_0}\text{; allowable stress } k_s = \mathbf{3500} \text{ to } 4250 \text{ lbs. per sq. in.} \quad (56)$$

(d) Flywheel Arms.

These are usually solid, of oval section (Fig. 52), or circular and hollow (Fig. 53), while the form (Fig. 49) is also often used. For high-speed flywheels the arms are of sharp-ended oval

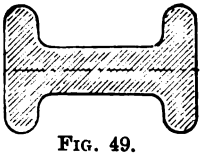


FIG. 49.

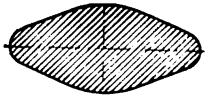


FIG. 50.

section (Fig. 50), to diminish the friction of the air. Very large flywheels, for rolling mill engines, etc., have wrought-iron arms of rectangular cross section.

1. Tensile Stress on the Arms, due to the Centrifugal Force of the Rim.

The centrifugal force of a segment of the rim is calculated by the formula—

$$C^1 = \frac{W}{z} \times \frac{1}{g} \times \frac{V^2}{R} \quad . \quad . \quad . \quad . \quad . \quad (57)$$

where z denotes the number of arms.

$V = \frac{R\pi n}{30}$ circumferential velocity of the centre of gravity of the rim section in feet per second. (The radius of the centre of gravity of the rim segment is taken as approximately equal to the radius of the centre of gravity of the cross section of the whole rim.)

Under the most unfavourable conditions, when the arm is in its lowest position, half the weight of the rim must be added to C′.

$$\text{Therefore } P = C' + \frac{W}{2} \text{ in lbs.} \quad . \quad . \quad . \quad . \quad (58)$$

Here the tensile strength of the rim is neglected.

Cross section of the arms *near the flywheel rim.*

FIG. 52.

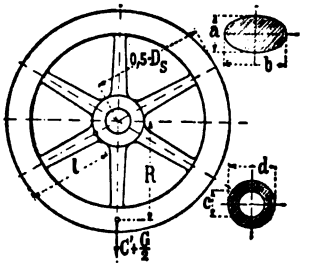


FIG. 51. FIG. 53.

$$\text{For solid oval arms (Fig. 52) } f_1 = ab\,\frac{\pi}{4} \text{ sq. ins.} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (59)$$

$$\text{For hollow arms, of circular section (Fig. 53) } f_1 = \frac{\pi}{4}\,(d^2 - c^2) \text{ sq. ins.} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (60)$$

$$\text{Therefore stress due to tension } \sigma_t = \frac{P}{f_1} \text{ lbs. per sq. in.; } f_1 = \frac{P}{k_t} \text{ sq. ins.*.} \quad . \quad (61)$$

2. Stress due to Tension and Bending at the Nave.

Let B.H.P. = effective horse-power of the engine.
$0{\cdot}5D_s$ = radius of the flywheel rim in ft.
P = tangential effort at the rim at the normal output in lbs.
n = revolutions per minute.

$$\text{Then we have } P = \frac{5250\text{BHP}}{n \times 0{\cdot}5D_s} \text{ in lbs.} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (62)$$

When calculating the maximum moment about the flywheel, P must be taken at *double* (belt pull) the value given by equation 62, therefore—

$$\text{Bending moment of one arm at the nave } \mathbf{M} = \frac{2P}{z} \times l \text{ inch-lbs.} \quad . \quad . \quad . \quad (63)$$

z being = number of arms.

l being = distance from the circumference of the wheel to the arm at the nave (see Fig. 51) in inches.

Modulus of resistance—

$$\text{For arms of solid oval section (Fig. 52), } Z = 0{\cdot}1ab^2 \text{ cub. in.} \quad . \quad . \quad . \quad (64)$$

$$\text{For hollow arms of circular section (Fig. 53), } Z = 0{\cdot}1\left(\frac{d^4 - c^4}{d}\right) \text{ cub. ins.} \quad . \quad (65)$$

$$\text{Therefore stress due to bending } \sigma_b = \frac{M}{Z} \text{ in lbs. per sq. in.} \quad . \quad (66)$$

$$\text{To this must be added the stress due to tension } \sigma_t = \frac{C'}{f'} \text{ lbs. per sq. in.} \quad . \quad . \quad (67)$$

C′ is calculated by equation 57, f' = cross section of the arm near the nave in sq. ins., therefore the combined stress will be—

$$\sigma = \frac{M}{Z} + \frac{C^1}{f^1} \text{ lbs. per sq. in.} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (68)$$

The allowable stress or safe working load is taken at For cast iron at the rim $\Big\}$ k = 2100 to 2550 lbs. per sq. in. . (69)

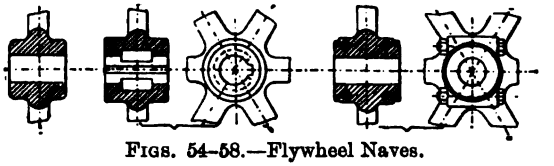
* For allowable stress, see equations 70, 71 on the next page.

The allowable stress or safe working load is taken at For cast iron at the nave } k = 1700 to 2100 lbs. per sq. in. . (70)

The allowable stress or safe working load is taken at For wrought iron } k = 3550 to 5700 lbs. per sq. in. . (71)

(e) Nave Fastenings.

The nave is usually split into several parts to avoid casting strains, or rather strains set up due to the unequal cooling of the flywheel in the mould; it is afterwards secured by shrunk rings or screwed bolts; care must be taken to divide the nave, so as to distribute the weight evenly. Figs. 54 to 58 illustrate the most usual types of nave fastenings.



FIGS. 54–58.—Flywheel Naves.

If the flywheel is split into two or more parts, it requires considerable skill to make a good job with shrunk rings, and even then the tension due to two or more segments of the wheel will never be quite the same. Except therefore for large gas engines, shrunk rings are no lenger used, but the nave is usually split in two parts, and bolts screwed at both ends; the thread on these bolts is usually a fine thread,* and the nuts are prevented from slacking back by lock nuts, or split pins, etc. The bolts should be made of the best mild steel, and can be stressed up to 11,000 lbs. per sq. inch.

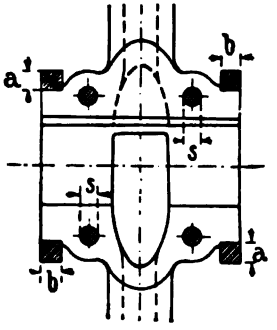


FIG. 59.

To avoid a great mass of metal in one place, the type of nave shown at Fig. 59 should be used for flywheels 11 ft. 6 ins. and upwards in diameter.

According to equation 8, p. 186, the centrifugal force of one-half of the wheel is—

$$C = \frac{W}{2g} \times \frac{2V^2}{\pi R} = 0{\cdot}01 \frac{WV^2}{R} \text{ lbs.} \quad . \quad . \quad . \quad . \quad (72)$$

Combined sectional area of bolts and shrunk rings—

$$f_2 = 4 \times \frac{\pi}{4} s^2 + 4ab \text{ in sq. ins.} \quad . \quad . \quad . \quad . \quad . \quad (73)$$

where s = diameter of the bolts at the bottom of the thread.

Tensile stress on these fastenings $\sigma_z = \frac{C}{f_2}$ in lbs. per sq. in. . . . . (74)

Allowable stress or safe working load for wrought iron—

$$k_z = 3500\text{–}6500 \text{ lbs. per sq. in.} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (75)$$

Bursting of Flywheels.

The cause of these constantly recurring *accidents* with flywheels running at usual circumferential speeds of 3000 to 6000 ft. per minute is not to be attributed to the weakness of the rim. It is nearly always due to—

(f) Defects in the Lugs cast on the Rim for coupling the Wheel together.

With *solid* flywheels, the segments of which are fastened together by dowels and cotters, shrunk rings, or clamps, such accidents hardly ever occur, nor is there any danger of the rim giving way from weakness of cross section. But with pulley flywheels and flywheels grooved for ropes, the risk of fracture of the lugs coupling the segments, or of the rim itself at the junctions, is great, especially if the *rim is grooved for ropes or of weak cross section*. Particular attention should be paid to the design of the lugs, and also to the arms where they join the rim; flywheels having a *high* rim velocity must always be divided along the arms.

* [TRANSLATOR'S NOTE.—It is usual to fit bolts with six threads per inch up to 3½" diameter.]

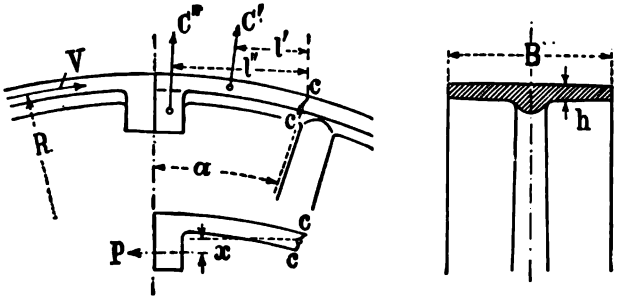
Let W′ = weight of a segment of a rim of length a.
W″ = „ „ lug together with the bolts.

The centrifugal forces $C' = \frac{W'}{g} \times \frac{V^2}{R}$ and $C'' = \frac{W''}{g} \times \frac{V^2}{R}$ in lbs. . . . (76)

exert a bending stress on the cross section cc, Figs. 60–62. To this must be added the bending moment Px through the bolts, etc. The load P is equal to half the centrifugal force of one-half of the wheel, and must therefore be calculated from equation 10, p. 186, the nave fastenings being practically neglected. The total bending moment thus works out at—

$$M = C'l' + C''l'' + 1{\cdot}2\ Px \quad (77)$$

To this must be added the stress due to tension of cross section $f = Bh$, acting on cross section cc, according to equation 11, p. 186.



FIGS. 60–62.—Flywheel divided between the Arms.

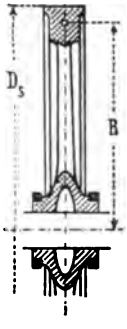
In Haeder's book on the "Steam Engine," 8th Edition, p. 282, the stresses on a pulley flywheel causing it to burst have been calculated.

Example.—Required the dimensions of a pulley wheel for a single cylinder four-cycle gas engine (driving a continuous-current dynamo). Diameter of cylinder 400 mm. = $15\frac{3}{4}$ ins.; stroke 550 mm. (say $21\frac{1}{2}$ ins.); revolutions n = 190 per minute. Diameter of flywheel = 2750 mm. = 9 feet.

1. Determination of the Output.

By equation 3, p. 18, effective piston area $A = \frac{\pi}{4} \times 15\frac{3}{4}''^2 = 195$ sq. inches.

„ 7, p. 20, mean piston speed $v = \frac{2 \times 21\frac{1}{2} \times 190}{12} = 681$ feet per minute.



From Table II., p. 20, mean pressure on piston $p_1 = 20\frac{3}{4}$ lbs. per sq. inch.
By equation 8, p. 20, maximum output or IHP—

$$= \frac{195 \times 681 \times 20\frac{3}{4}}{33{,}000} = 83\frac{1}{4} \text{ IHP.}$$

From Table I., p. 18, mechanical efficiency $\eta = 0{\cdot}84$
By equation 2, p. 18, actual output of engines $= 0{\cdot}84 \times 83\frac{1}{4} = 70$ BHP.

2. Determination of the Weight of the Rim.

From Table II., p. 166, coefficient i 21,000 based on the IHP.
By equation 5, p. 39, radius of centre of gravity of rim section

$$R = 9'\ 0'' \times 0{\cdot}45 = 4{\cdot}05 \text{ feet.}$$

By equation 2, p. 39, circumferential velocity $V = \frac{4{\cdot}05 \times \pi \times 190}{30} = 80\frac{1}{2}$ ft. per second.

From Table VII., p. 175, degree of irregularity in running $\delta_0 = \frac{1}{80}$.

By equation 4, p. 165, weight of rim $W = 100 \times 21000 \times \frac{83\frac{1}{4}}{190 \times 80\frac{1}{2}^2} \times \frac{1}{\frac{1}{80}} = 11{,}350$ lbs. (approx.).

3. Width of Flywheel Rim (approximate estimate).

By equation 28, p. 184, cross section of rim $f = 0{\cdot}051 \times \frac{11350}{4{\cdot}05} = 143$ sq. inches.

„ 31, p. 185, depth of rim $h = \frac{1}{6} \times 9'\ 0'' = 12$ inches.

„ 29, p. 185, dimension $a = \frac{141^2}{12} = 11\frac{3}{4}$ inches (approx.).

„ 30, p. 185, width of rim $B = \frac{11\frac{3}{4}''}{0{\cdot}9} = 13$ inches (approx.).

4. Calculation of the Flywheel.

An outline of the rim corresponding to the dimensions of the cross section given under head 3 is drawn (Fig. 65), the positions of the centres of gravity of the different areas are then determined, and x_0 calculated from them by equation 6, p. 185—

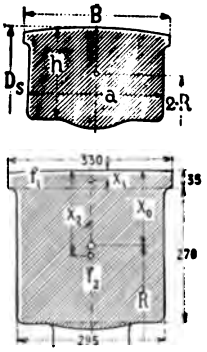


FIG. 65.

$$f_1 = 13'' \times 1\tfrac{3}{8}'' = 18 \text{ sq. inches}$$
$$f_2 = 11\tfrac{5}{8}'' \times 10\tfrac{5}{8}'' = 124 \quad ,,$$
$$f = 142$$

$$f_1 \times x_1 = 18 \times \tfrac{11}{16}'' = 12.4 \text{ inches}^3$$
$$f_2 \times x_2 = 124 \times 6\tfrac{11}{16} = 828 \quad ,,$$
$$\Sigma fx = 840{\cdot}4$$

By equation 6, p. 185, $x_0 = \frac{840}{142} = 6$ inches; R = 4′ 6″ − 6″ = 4 feet.

,, 4, p. 185, weight of rim W = 0·262 × 2 × π × 12 × 4 × 142 = 11,200 lbs. (approx.).

From equation 11, p. 186, theoretical tensile stress in the flywheel rim $\sigma = 0{\cdot}098 \times 80\tfrac{1}{2}^2 = 630$ lbs. per sq. in.

Since $\frac{12R}{h} = \frac{12 \times 4}{12} = 4$

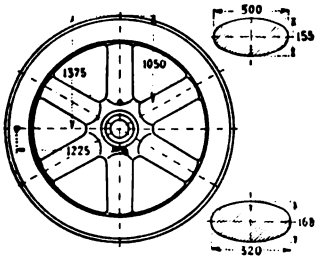
then from Table I., p. 187, $x = 1{\cdot}3$

therefore from equation 12, p. 186—

combined stress = 1·3 × 630 = 820 lbs. per sq. inch.

5. Flywheel Arms.

(*a*) *Tensile Stress on the Rim* (see p. 196).



FIGS. 66, 67.

The centrifugal force in a rim segment by equation 57 is—

$$C' = \frac{11{,}200 \times 80{\cdot}5^2}{6 \times 32 \times 4} = 95{,}500 \text{ lbs.}$$

Combined tensile stress by equation 58, p. 196—

$$P = 95{,}500 + 5650 = 101{,}150 \text{ lbs.}$$

Cross section of the arms at the rim. By equation 59—

$$f_1 = \frac{\pi}{4} \times 6\tfrac{1}{8}'' \text{ (155 mm.)} \times 11\tfrac{11}{16}'' \text{ (300 mm.)} = 57 \text{ sq. ins.}$$

Therefore the tensile stress on the rim (equation 61)—

$$\sigma_z = \frac{101{,}150}{57} = 1780 \text{ lbs. per square inch.}$$

(*b*) *Stresses on the Nave due to Tension and Bending* (see p. 196).

By equation 62, p. 196, circumferential pressure or tangential tension $P = \frac{5250 \times 70}{190 \times 4{\cdot}5}$ = 430 lbs. per sq. in.

,, ,, 63, p. 196, bending moment $M_b = \frac{2 \times 430}{6} \times 41{\cdot}5 = 5950$ inch-lbs.

,, ,, 64, p. 196, modulus of resistance $Z = 0{\cdot}1 \times 6\tfrac{1}{8}'' \times 11\tfrac{13}{16}''^2 = 85$ inches³ (approx.).

,, ,, 66, p. 196, stress due to bending $\sigma_b = \frac{5950}{85} = 70$ lbs. per square inch.

Add to this the stress due to tension.

,, ,, 59, p. 196, cross section of arm where it joins the rim $f_1 = \frac{\pi}{4} \times 6\tfrac{5}{8}'' \times 12\tfrac{5}{8}''$ = 66 square inches.

,, ,, 67, p. 196, stress due to tension $\sigma_z = \frac{95{,}500}{66} = 1450$ lbs. per square inch.

,, ,, 68, p. 196, combined stress due to bending and tension $\sigma = 70 + 1450$ = 1520 lbs. per square inch.

Thus it is the stress due to tension produced by the centrifugal force and weight of the flywheel which is the chief factor in determining the load upon the arms.

6. Width of the Belt.

By equation 25, p. 180, cross section of belt $b\delta = \frac{64 \times 67}{9 \times 180} = 9{\cdot}65$ square inches.

By Table X., p. 180, thickness of belt $\delta = \frac{9}{32}$ inches

therefore the width of the belt $b = \frac{2{\cdot}65}{\frac{9}{32}} = 9\frac{1}{2}$ inches.

Rules for fitting together the Different Parts of the Flywheel.

1. Boring out the Nave.

This must be done carefully because the least inaccuracy of workmanship produces "wobbling."*

The American practice of machining the flywheel is as follows:—

The hole in the nave is bored out to exactly fit the shaft, then a clearance is slotted out as shown to the left hand in Fig. 68; the two keys are then fitted and driven home, and the process pulls the wheel over so that the true part of the original bore at *a* bears on the shaft.

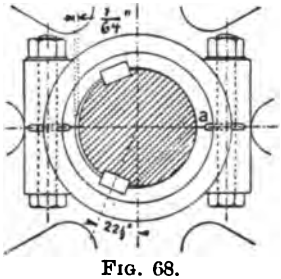


FIG. 68.
Boring out the Nave.

2. Securing the Flywheel to the Shaft.

For small flywheels up to 10 ft. diameter, a single feather or sunk key is generally used, for larger wheels two so-called tangential keys are fitted; to receive the latter two recesses are cut in the wheel, and two keys are bedded in each recess (Fig. 70).

According to Bach:—if d be given in inches, then—

For flat (or sunk) keys $\begin{cases} b = 0{\cdot}9\sqrt{d} \text{ inches} & \ldots\ldots (78) \\ h = 0{\cdot}5\sqrt{d} \quad ,, & \ldots\ldots (79) \end{cases}$

For tangential keys $\begin{cases} b = 0{\cdot}25d \quad ,, & \ldots\ldots (80) \\ h = 0{\cdot}06d \quad ,, & \ldots\ldots (81) \end{cases}$

Angle a generally 120°.

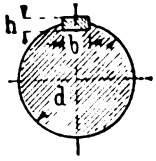


FIG. 69.
Sunk Key.

Heavy flywheels, especially if intended to drive dynamos for electric lighting, are always secured by two keys fitted 90° or 120° apart, and the keyways are in the same plane as the arms. This method is supposed to secure the flywheel more firmly, and to prevent "creeping" along the shaft.

Figs. 69 and 70 show the arrangement of single and double keys.

As a rule the flywheel shaft is of round section, but for very large flywheels like those for rolling mill engines the flywheel is sometimes staked on to a square shaft.

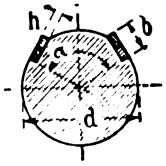


FIG. 70.
Tangential Keys.

3. Nave secured by Shrunk Rings.

Flywheels up to about 10 ft. in diameter are usually cast in one piece (Fig. 5, p. 188); above this size they are always made in two or more parts, because of the difficulties of transport by rail. If the metal is badly distributed, and if the rim or the nave is very heavy, flaws may be developed and serious strains set up in the casting, due to contraction and to unequal cooling in the mould. In order to guard against these dangers it is usual to split the nave into three or more parts, which gives a certain amount of freedom, and relieves the arms from these strains. To part the nave, wrought iron plates coated with graphite are laid in the mould, in positions shown in Fig. 71; these plates are knocked out after the wheel is cast, the spaces thus formed filled in with zinc or white metal, and a ring R is shrunk on either side over the nave.

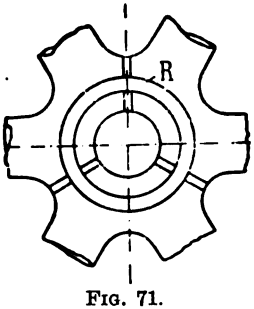


FIG. 71.

* For method of correcting "wobbling" of the flywheel, see Haeder's "Machinenmeister."

4. Keys for securing the Flywheel.

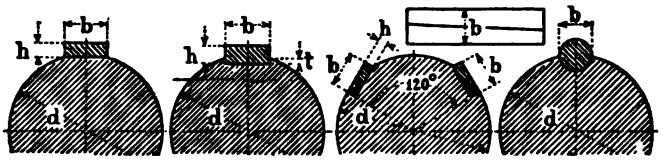
The tables used in different engineering workshops show differences of 50 per cent. in the width of these keys. Their dimensions can hardly be determined theoretically.

The recess t to take the key is usually cut deeper into the shaft than the depth given in Table IV., but a deeper recess increases the twisting strains on the shaft.

In round keys (which are rarely used) b is generally smaller.

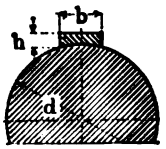
Table IV.—Keys.

(Dimensions in millimetres.)



Flat key. Sunk key. Tangential key. Round key.

Shaft.	Width.	Sunk key.		Tangential key.	Light keys for eccentrics, valve gear and governors, etc.		
d	b	h	t	h	b	h	t
30—39	12	7	2	—	—	—	—
40—49	14	8	3	—	—	—	—
50—59	16	9	3	—	—	—	—
60—69	18	10	3	—	—	—	—
70—79	20	12	4	—	—	—	—
80—89	23	13	4	—	—	—	—
90—99	26	14	4	—	—	—	—
100—119	30	16	5	—	—	—	—
120—139	34	18	6	—	18	10	3
140—159	38	20	6	—	20	12	4
160—179	42	22	7	—	22	13	4
180—199	46	24	8	—	24	13	4
200—219	50	26	8	15	26	14	4
220—239	55	29	9	16	28	14	5
240—259	60	32	10	17	30	15	5
260—279	65	35	11	18	32	16	6
280—299	70	38	12	20	34	17	6
300—324	75	40	13	22	36	18	6
325—349	80	44	14	23	40	20	7
350—374	85	48	15	25	42	21	7
375—399	90	50	17	26	44	22	7
400—449	95	54	19	28	46	23	8
450—500	100	58	21	30	50	25	8



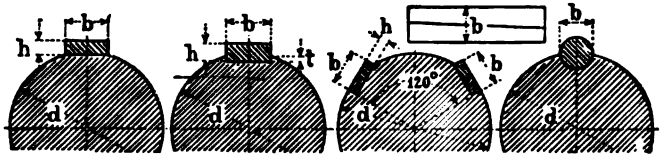
Hollow key.

Hollow keys are only used as temporary fasteners, *i.e.* when setting the valve gear or for small powers.

Feather keys should be bedded into a recess $1{\cdot}3t$ deep, and the depth of the key should be about $0{\cdot}3t$ deeper.

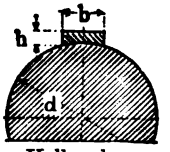
Table IVa.—Keys.

(Dimensions in inches.)



Flat key. Sunk key. Tangential key. Round key.

Shaft *d*.	Width.	Sunk key.		Tangential key.	Light keys for excentrics, valve gear and governors, etc.		
d	*b*	*h*	*t*	*h*	*b*	*h*	*t*
ins.	ins.	ins.	ins.	ins.	ins.	ins.	ins.
$1\frac{3}{16}$ to $1\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{4}$	$\frac{3}{32}$	—	—	—	—
$1\frac{9}{16}$,, $1\frac{7}{8}$	$\frac{9}{16}$	$\frac{5}{16}$	$\frac{1}{8}$	—	—	—	—
2 ,, $2\frac{5}{16}$	$\frac{5}{8}$	$\frac{3}{8}$	$\frac{1}{8}$	—	—	—	—
$2\frac{3}{8}$,, $2\frac{11}{16}$	$\frac{11}{16}$	$\frac{7}{16}$	$\frac{1}{8}$	—	—	—	—
$2\frac{3}{4}$,, 3	$\frac{3}{4}$	$\frac{1}{2}$	$\frac{5}{32}$	—	—	—	—
$3\frac{1}{8}$,, $3\frac{1}{2}$	$\frac{7}{8}$	$\frac{1}{2}$	$\frac{5}{32}$	—	—	—	—
$3\frac{9}{16}$,, $3\frac{7}{8}$	1	$\frac{9}{16}$	$\frac{5}{32}$	—	—	—	—
4 ,, $4\frac{5}{8}$	$1\frac{3}{16}$	$\frac{5}{8}$	$\frac{3}{16}$	—	$\frac{11}{16}$	$\frac{7}{16}$	$\frac{1}{8}$
$4\frac{3}{4}$,, $5\frac{7}{16}$	$1\frac{5}{16}$	$\frac{11}{16}$	$\frac{1}{4}$	—	$\frac{11}{16}$	$\frac{1}{2}$	$\frac{1}{8}$
$5\frac{1}{8}$,, $6\frac{1}{4}$	$1\frac{1}{2}$	$\frac{3}{4}$	$\frac{1}{4}$	—	$\frac{3}{4}$	$\frac{1}{2}$	$\frac{5}{32}$
$6\frac{3}{8}$,, 7	$1\frac{5}{8}$	$\frac{7}{8}$	$\frac{5}{16}$	—	$\frac{7}{8}$	$\frac{1}{2}$	$\frac{5}{32}$
$7\frac{1}{8}$,, $7\frac{7}{8}$	$1\frac{13}{16}$	$\frac{15}{16}$	$\frac{5}{16}$	—	$\frac{13}{16}$	$\frac{1}{2}$	$\frac{5}{32}$
8 ,, $8\frac{5}{8}$	2	1	$\frac{5}{16}$	$\frac{9}{16}$	1	$\frac{9}{16}$	$\frac{5}{32}$
$8\frac{11}{16}$,, $9\frac{7}{16}$	$2\frac{3}{16}$	$1\frac{1}{8}$	$\frac{3}{8}$	$\frac{5}{8}$	$1\frac{1}{16}$	$\frac{9}{16}$	$\frac{3}{16}$
$9\frac{1}{2}$,, $10\frac{3}{16}$	$2\frac{3}{8}$	$1\frac{1}{4}$	$\frac{3}{8}$	$\frac{5}{8}$	$1\frac{3}{16}$	$\frac{5}{8}$	$\frac{3}{16}$
$10\frac{1}{4}$,, 11	$2\frac{9}{16}$	$1\frac{3}{8}$	$\frac{7}{16}$	$\frac{11}{16}$	$1\frac{1}{4}$	$\frac{5}{8}$	$\frac{1}{4}$
$11\frac{1}{2}$,, $11\frac{3}{4}$	$2\frac{3}{4}$	$1\frac{1}{2}$	$\frac{1}{2}$	$\frac{3}{4}$	$1\frac{5}{16}$	$\frac{11}{16}$	$\frac{1}{4}$
$11\frac{7}{8}$,, $12\frac{3}{4}$	3	$1\frac{9}{16}$	$\frac{1}{2}$	$\frac{7}{8}$	$1\frac{7}{16}$	$\frac{11}{16}$	$\frac{1}{4}$
$12\frac{1}{8}$,, $13\frac{3}{4}$	$3\frac{1}{8}$	$1\frac{3}{4}$	$\frac{9}{16}$	$\frac{7}{8}$	$1\frac{9}{16}$	$\frac{13}{16}$	$\frac{9}{16}$
$13\frac{1}{8}$,, $14\frac{3}{4}$	$3\frac{3}{8}$	$1\frac{7}{8}$	$\frac{9}{16}$	1	$1\frac{11}{16}$	$\frac{13}{16}$	$\frac{9}{32}$
$14\frac{1}{2}$,, $15\frac{3}{4}$	$3\frac{1}{2}$	2	$\frac{11}{16}$	1	$1\frac{3}{4}$	$\frac{7}{8}$	$\frac{9}{32}$
$15\frac{1}{2}$,, $17\frac{5}{8}$	$3\frac{3}{4}$	$2\frac{1}{8}$	$\frac{3}{4}$	$1\frac{1}{8}$	$1\frac{13}{16}$	$\frac{15}{16}$	$\frac{5}{16}$
$17\frac{3}{4}$,, $19\frac{3}{4}$	4	$2\frac{1}{4}$	$\frac{7}{8}$	$1\frac{1}{8}$	2	1	$\frac{5}{16}$



Hollow key.

Hollow keys are only used as temporary fasteners, *i.e.* when setting the valve gear or for small powers.

Feather keys should be bedded in a recess 1·3*t* deep, and the depth of the key should be about 0·3*t* deeper.

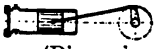
Main Dimensions of a Gas Engine.

Pages 18 *to* 24 *continued.*

The following Tables, I. to VI., will be found useful for *determining* approximately the principal dimensions of gas engines easily and *quickly*. To calculate them in small and medium-sized engines, power gas (suction or pressure) has been taken as the motive agent, with a mean pressure on the piston p_m = 60 to 70 lbs. per sq. inch (p. 22).* In the larger engines the motive power is assumed to be *blast furnace gas*, and the mean pressure p_m = 57 to 68 lbs. per sq. inch. There is no fixed rule for p_m; in practice it will be found to vary by about 25 per cent. Therefore the values given in the following tables must only be taken as *rough* and *approximate estimates*.

For the number of revolutions Table III., p. 147, has been used, and this table also shows for what sizes and types of engine it is necessary to cool the piston with water.

The maximum output is about 1·2 to 1·4 times more than the normal output (HP) given in Tables I. to VI.

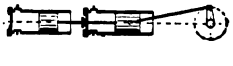
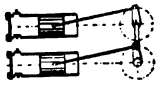
Table I.—  **E 4 Single-cylinder Engine.**

(Dimensions in millimetres.)

Normal output . BHP =	10	15	20	25	30	40	50	60	80	100	125	150	175	200
Cylinder diameter . D =	245	285	325	360	390	435	485	525	590	655	730	800	860	925 mm.
Stroke . . . . . S =	340	400	440	480	520	560	600	630	680	720	800	880	950	1020 ,,
Number of revolutions n =	230	215	200	187	180	173	165	160	153	145	133	120	113	105 per min.

(Dimensions in inches.)

Normal output . BHP =	10	15	20	25	30	40	50	60	80	100	125	150	175	200
Cylinder diameter . D =	9½	11¼	12¾	14¼	15½	17	19	20¾	23¼	25¾	28¾	31½	34	36½ ins.
Stroke . . . . . S =	13½	15¾	17½	19	20½	22	23½	24¾	26¾	28½	31½	34½	37½	40 ,,
Number of revolutions n =	230	215	200	187	180	173	165	160	153	145	133	120	113	105 per min.

Table II.—   **R 4 = Two-cylinder Engines.**
Z 4 = ,, ,,

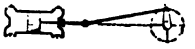
(Dimensions in millimetres.)

Normal output . BHP =	120	150	175	200	225	250	275	300	350	400	450	500
Cylinder diameter . D =	525	580	640	650	690	725	760	800	860	920	975	1030 mm.
Stroke . . . . . S =	630	670	720	720	760	800	840	880	950	1000	1070	1130 ,,
Number of revolutions n =	162	146	133	150	140	134	128	120	113	106	100	94 per min.

(Dimensions in inches.)

Normal output . BHP =	120	150	175	200	225	250	275	300	350	400	450	500
Cylinder diameter . D =	20¾	23	25¼	25½	27¼	28½	30	31½	34	36¼	38½	40½ ins.
Stroke . . . . . S =	24¾	26½	28½	28½	30	31½	33	34½	37½	39½	42	44½ ,,
Number of revolutions n =	162	146	133	150	140	134	128	120	113	106	100	94 per min.

* Translator's Note.—The most modern English gas engines work with mean pressures of 90 to 95 lbs. per sq. inch for lighting gas, and 80 to 85 lbs. per sq. inch for producer gas; therefore the power of these engines can be increased in proportion.

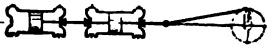
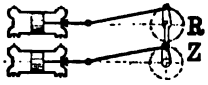
Table III.—  **D 4 Single-cylinder Engine.**

(Dimensions in millimetres.)

Normal output . . BHP =	200	225	250	275	300	350	400	450	500	550	600
Cylinder diameter . . D =	660	690	730	770	800	865	925	980	1030	1085	1130 mm.
Stroke . . . . . S =	790	830	880	920	960	1040	1100	1170	1240	1300	1350 ,,
Number of revolutions n =	130	124	116	110	106	98	92	87	83	78	75 per min.

(Dimensions in inches.)

Normal output . BHP =	200	225	250	275	300	350	400	450	500	550	600
Cylinder diameter . . D =	26	27	28¾	30¼	31½	34	36½	38½	40½	42¾	44½ ins.
Stroke . . . . . S =	31	32¾	34¾	36¼	37¾	41	43½	46	49	51	53 ,,
Number of revolutions n =	130	124	116	110	106	98	92	87	83	78	74 per min.

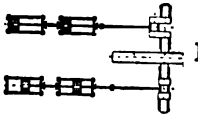
Table IV.—   **R D 4, Two-cylinder, Z D 4, Two-cylinder Engine.**

(Dimensions in millimetres.)

Normal output . BHP =	600	650	700	750	800	900	1000	1100	1200	1300
Cylinder diameter . . D =	790	820	850	880	910	965	1020	1070	1120	1160 mm.
Stroke . . . . . S =	950	980	1020	1060	1100	1160	1230	1280	1350	1400 ,,
Number of revolutions n =	108	104	100	97	93	88	83	80	76	73 per min.

(Dimensions in inches.)

Normal output . BHP =	600	650	700	750	800	900	1000	1100	1200	1300
Cylinder diameter . . D =	31	32¼	33½	34½	36	38	40	42	44	46 ins.
Stroke . . . . . S =	37½	38¾	40	41¾	43¼	45¾	48½	50	53	55 ,,
Number of revolutions n =	108	104	100	97	93	88	83	80	76	73 per min.

Table V.—  **R Z D 4 Four-cylinder Engine.**

(Dimensions in millimetres.)

Normal output BHP =	1200	1300	1400	1500	1600	1700	1800	1900	2000	2100	2200	2300	2400
Cylinder diameter . D =	770	800	840	865	900	920	950	975	1000	1025	1050	1075	1100 mm.
Stroke . . . . . S =	920	960	1000	1040	1080	1100	1140	1170	1200	1230	1260	1300	1320 ,,
Number of revolutions n =	110	106	100	98	95	92	90	87	85	83	81	80	77 per min.

Table V.—*continued.*

(Dimensions in inches.)

Normal output BHP =	1200	1300	1400	1500	1600	1700	1800	1900	2000	2100	2200	2300	2400
Cylinder diameter . D =	30½	31½	33	34	35½	36¼	37½	38½	39½	40½	41½	42½	43½ ins.
Stroke . . . . . S =	36	37¾	39½	41	42½	43½	45	46	47	48	50	51	52 ,,
Number of revolutions n =	110	106	100	98	95	92	90	87	85	83	81	80	77 per min.

Table VI.—Double-acting, Two-cycle, Single-cylinder Engine (D 2 1 Cyl.).

(Dimensions in millimetres.)

Normal output . BHP =	200	250	300	350	400	450	500	550	600	700	800	900	1000
Cylinder diameter . D =	480	540	590	635	680	725	760	795	835	900	965	1020	1075 mm.
Stroke . . . . . S =	820	920	1000	1080	1160	1230	1300	1350	1420	1530	1640	1730	1820 ,,
Number of revolutions n =	108	96	88	82	77	72	68	66	62	57	54	52	48 per min.

(Dimensions in inches.)

Normal output BHP =	200	250	300	350	400	450	500	550	600	700	800	900	1000
Cylinder diameter . D =	19	21¼	23¼	25	26¾	28½	30	31¼	33	35½	38	40	42 ins.
Stroke . . . . . S =	32¼	36	39	42½	45½	48½	51	53	56	60	64½	68	71½ ,,
Number of revolutions n =	108	96	88	82	77	72	68	66	62	57	54	52	48 per min.

These engines, *i.e.* **D 2** with **two cylinders**, if of the same dimensions and arranged side by side or twin, will give twice as much power as stated above.

If the above tables be used to determine the approximate dimensions of a large gas engine, the values must be expressed in round numbers.

Example I.—Required to determine the approximate dimensions of a 250 BHP **R 4** 2 cyl. engine (four-cycle tandem), and of a **D 2** single-cyl. engine (double acting, two-cycle).

For an **R 4** 2-cyl. engine, Table II. gives: D = 28½″; S = 31½″; n = 134.

For a **D 2** single-cyl. engine, Table VI. gives: D = 21¼″; S = 36″; n = 96.

Example II.—A blast furnace gas engine, driving a blowing engine, is required to develop 2400 BHP (normal output). What size of engine would be best for the purpose? The most suitable would, according to Table V., be an **R Z D 4** 2-cyl. engine, with—

D = 43½″; S = 52″; n = 77.

Example III.—According to Table IV., a 1000 HP **R D 4** 2-cyl. engine (tandem, double-acting, four-cycle) should have D = 40″; S = 48½″; n = 83. If the engine runs at the lower speed of 72 revolutions per minute, what will be the HP developed?

Since the output is in proportion to the number of revolutions, if n = 72, then—

$$\text{Output} = \frac{1000 \times 72}{83} = \text{say } \mathbf{870\ HP}.$$

To make the subject clearer, and to afford means of drawing a better comparison, we subjoin tables giving the power developed by steam engines, and their corresponding dimensions.

As already mentioned at p. 30, it is necessary to run gas engines at a higher speed than steam engines of the same power. The number of revolutions given in the following Tables, VII. to IX., correspond to those in present practice :— *

Table VII.—Single-cylinder Condensing Engine. Boiler Pressure 100 lbs. per sq. inch.

p_m taken at = 33 lbs. per sq. inch; $\eta = 0{\cdot}88$; $S = 1{\cdot}7D$; hence normal output $BHP = \frac{D^3 \times n}{5100}$

$$D = 17{\cdot}2 \times \sqrt[3]{\frac{BHP}{n}}$$

(Increased output 1·25 times the normal, maximum output 1·5 times the normal output.)

(Dimensions in millimetres.)

Normal output BHP =	10	15	20	25	30	40	50	60	80	100	125	150	175	200	250	300	400	500	600	700
Cylinder diameter D =	170	195	220	240	250	280	310	340	390	430	470	510	540	570	630	680	750	830	870	930 mm.
Stroke . . . . S =	290	330	380	410	440	490	540	580	660	730	800	870	920	970	1070	1150	1300	1400	1500	1600 ,,
No. of revolutions n =	175	168	163	158	153	143	135	127	114	104	98	95	92	89	84	80	77	75	74	72 per min.

(Dimensions in inches.)

Normal output BHP =	10	15	20	25	30	40	50	60	80	100	125	150	175	200	250	300	400	500	600	700
Cylinder diameter D =	6¾	7¾	8¾	9½	10	11	12½	13½	15½	17	18½	20	21¼	22½	25	26¾	29½	32¾	34¼	36½ ins.
Stroke . . . . . S =	11½	13	15	16¼	17½	19¼	21¼	23	26	28¾	31½	34½	36¼	38¼	42	45¼	51¼	55¼	59	63 ,,
No. of revolutions n =	175	168	163	158	153	143	135	127	114	104	98	95	92	89	84	80	77	75	74	72 per min.

Table VIII.—Tandem Condensing Engine. Boiler Pressure 130 lbs. per sq. inch.

p_m taken at = 33 lbs. per sq. inch; $\eta = 0{\cdot}88$; $S = 1{\cdot}15D$; hence normal output $HP = \frac{D^3 \times n}{7550}$

$$D = 19{\cdot}6\sqrt[3]{\frac{HP}{n}}$$

(Increased output 1·2 times the normal, maximum output 1·4 times the normal output.)

(Dimensions in millimetres.)

Normal output BHP =	40	50	75	100	150	200	250	300	400	500	600	700	800	900	1000	1200	1400	1500	1750	2000
Diam. of HP cyl. d =	190	210	245	270	325	370	410	440	490	540	585	620	650	680	720	760	800	830	875	920 mm.
Diam. LP cyl. D =	315	340	400	450	530	605	670	720	810	890	960	1020	1080	1125	1180	1250	1325	1360	1440	1510 ,,
Stroke . . . S =	360	400	460	510	610	700	770	830	930	1020	1105	1170	1240	1300	1360	1440	1520	1560	1660	1740 ,,
No. of revs. . n =	158	154	145	137	122	111	102	98	92	87	84	82	80	78	76	75	74	73	72	71 per min.

* The number of revolutions in Tables VII. to IX. correspond to those in the Author's book on "Valve Gear," Edition 1903, p. 368.

Table VIII.—*continued.*

(Dimensions in inches.)

Normal output BHP . . =	40	50	75	100	150	200	250	300	400	500	600	700	800	900	1000	1200	1400	1500	1750	2000
Diam. of HP cyl. d =	7½	8¼	9¾	10¾	12¾	14½	16¼	17¼	19¼	21¼	23	24½	25½	26¾	28½	30	31½	32½	34½	36½ ins.
,, LP cyl. D =	12½	13½	15¾	17⅝	20⅞	24	26½	28½	32	35	38	40	42½	44½	46½	49½	52¼	53½	57	59½ ,,
Stroke . . . S =	14¼	15¾	18	20	24	27½	30½	32¾	36¾	40	43½	46	48¾	51¼	53½	56¾	60	61½	65½	68½ ,,
Number of revs. n =	158	154	145	137	122	111	102	98	92	87	84	82	80	78	76	75	74	73	72	71 per min.

Table IX.—Compound Condensing Engine. Boiler Pressure 130 lbs. per sq. inch.

p_m taken at = 33 lbs. per sq. inch; $\eta = 0{\cdot}88$; S = 1·15D; hence normal output HP $= \frac{D^3 \times n}{7550}$

$$D = 19{\cdot}6\sqrt[3]{\frac{BHP}{n}}$$

(Increased output 1·2 times the normal, maximum output 1·4 times the normal output.)

(Dimensions in millimetres.)

Normal output . BHP =	40	50	75	100	150	200	250	300	400	500	600	700	800	900	1000	1200	1400
Diameter of HP cyl. . d =	200	215	255	285	340	390	430	460	520	570	615	660	690	720	750	810	850 mm.
,, LP ,, . D =	325	360	420	470	560	630	700	755	850	940	1010	1080	1130	1180	1230	1320	1400 ,,
Stroke. . . . . S =	375	410	490	540	645	730	810	870	980	1080	1160	1240	1300	1360	1420	1520	1610 ,,
Number of revolutions n =	143	136	126	117	105	97	89	86	80	75	72	69	68	67	66	64	63 per min.

(Dimensions in inches.)

Normal output . BHP =	40	50	75	100	150	200	250	300	400	500	600	700	800	900	1000	1200	1400
Diameter of HP cyl.. d =	8	8½	10	11¼	13½	15½	17	18¼	20½	22½	24¼	26	27¼	28½	29½	32	33½ ins.
,, LP ,, . D =	12¾	14½	16½	18½	22	24¾	27½	29¾	33½	37	39¾	42½	44½	46½	48½	52	55 ,,
Stroke . . . . . S =	14¾	16¼	19¼	21¼	25½	28¾	32	34¼	38½	42½	45¾	49	51¼	53½	56	60	63½ ,,
Number of revolutions n =	143	136	126	117	105	97	89	86	80	75	72	69	68	67	66	64	63 per min.

Example I.—Required to determine the approximate dimensions of a 250 BHP single-cylinder condensing steam engine, and of a compound condensing steam engine.

For the single-cylinder engine Table VII. gives D = 25″, S = 42″, n = 84. For the compound engine Table IX. gives d = 17″, D = 27½″, S = 32″, n = 89.

Example II.—A steam engine driving a rolling mill develops at normal load 1500 BHP; what size of engine should be chosen?

According to Table VIII., the best size will be a tandem engine having d = 32½″, D = 53½″, S = (in round numbers) 62″, n = 73.

Example III.—According to Table IX., in a 1200 HP engine $d = 32''$, $D = 52''$, $S = 60''$, $n = 64$. What HP will be developed if this engine runs at 52 revolutions per minute?

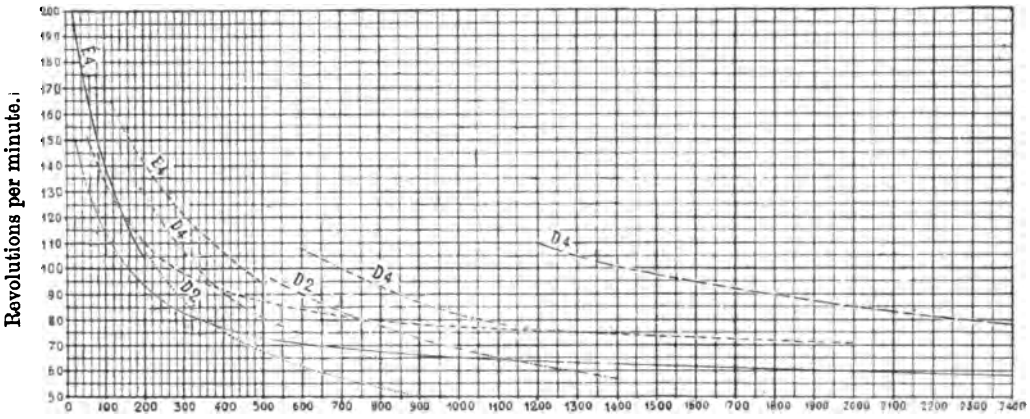
As the output is in proportion to the speed, if $n = 52$, we shall have—

$$\text{Output} = \frac{1200 \times 52}{64} = 975 \text{ BHP.}$$

The values given for the stroke in Tables VII. to IX. must be expressed in round numbers in the same way as those of gas engines, and made to agree with the standard dimensions in Haeder's book on "Steam Engines."

Speed in Gas and Steam Engines.

The diagram, Fig. 1, shows the number of revolutions at which gas and steam engines are at present usually run. In the larger gas engines blast furnace gas is taken as the motive power, as suction and power gas plants are not made for higher powers.



Normal Output in Brake HP.

FIG. 1.—Number of Revolutions per minute for a given HP developed.

The lines in the above diagram denote—

— ————	**E 4 single cyl.**	Single-acting,	**four**-cycle	engine,	one cylinder.
. . . . .	**E 4 two cyl.**	,,	,,	,,	two ,,
— · · — · · — · ·	**D 4 single cyl.**	Double-acting	,,	,,	one ,,
— · · — — · · —	**D 4 two cyl.**	,,	,,	,,	two ,,
— — — — —	**D 4 four cyl.**	,,	,,	,,	four ,,
. . . . .	**D 2 single cyl.**	,,	**two**-cycle	engine,	one ,,
— · — · — · — ·	**D 2 two cyl.**	,,	,,	,,	two ,,

— — — — — — Single cylinder, compound and triple expansion steam engines.*

— — — — — Tandem steam engine.*

Example.—What would be the number of revolutions in a **D 4** two-cylinder (double-acting, four-cycle) blast furnace gas engine, having a normal output of 950 BHP?

The diagram, Fig. 1, at this power, gives $n = 84$ revolutions per minute.

* The speed is often much higher in these engines, especially if vertical.

Section VII.* Costs of Working Gas and Steam Engines.

In estimating the relative cost of working different kinds of engines, a wide margin may be allowed in comparing the economies realised. The steam engineer manipulates his figures in favour of steam plants, and the gas engine builder does the same for gas engines.

Working Costs.

To separate the cost of the fuel from the cost of maintenance, we will make the

Working costs = subsidiary costs + cost of fuel.

(a) Cost of the Plant.

In determining these costs we must assume a proportionally higher output for the gas engine, because in order to have the *same reserve of power* in both cases the

Maximum output of the steam engine should be = *maximum output of the gas engine.*

We must therefore put the cost of the gas engine and producer higher than is sometimes done, because specifications are often based on the maximum output, or on 0·85 × maximum output, whereas in steam engines it is always understood that 0·7 of the maximum output is taken as the basis of calculation (in Germany).

Table I.—Cost of Installation, including Buildings, etc. (therefore of Plant complete) per HP at Normal Load.

Normal load = 0·7 of maximum load.

Maximum output =		14		42		70		100		140	210		420		700	1400 BHP
Normal output =		10		30		50		70		100	150		300		500	1000 ,,
		£	£	s.	£	s.	£	s.	£	£	s.	£	s.	£	£	s.
Cost of complete plant per HP at normal load	Gas engine =	36	23	10	18	10	15	15	14	12	5	10	15	10	9	5
	Steam engine =	36	24	0	21	0	18	0	16	14	0	12	0	11	9	10

Example.—A plant developing 100 HP at normal power costs, if worked with power gas, 100 × 14 = £1400; if worked with steam, 100 × 16 = £1600. Under specially favourable circumstances the costs of construction and erection may be taken at 25 % less.

(b) Depreciation.

As explained at p. 59 *et seq.*, the *forces acting on the cross head*, etc., and the *inertia of the moving parts* are greater per HP in gas than in steam engines. To keep the dimensions of the crank shaft, for instance, within moderate limits, it is usual to allow for *greater stresses* in the former engines. The following Table II. gives the comparative values of the stresses on the crank shafts of various engines.

* This section is added to form a guide for the student in estimating the first cost and running expenses, etc., of a gas engine installation, but the figures do not clearly show the enormous saving to be effected by replacing steam plants by gas engines, as they relate to gas engines worked with producers stoked with anthracite or lignite briquettes (both most expensive fuels), whereas the steam engines are supplied with steam generated by boilers under which cheap coal is burnt. As it is now possible to generate gas in a producer from any kind of fuel or even refuse (some classes of refuse which are converted into gas in producers cannot be burnt under boilers at all), in order to make a fair comparison between the costs of the two systems, it is necessary to use the same fuel in both cases. Again, the cost of attendance is taken as the same in both cases, whereas the attendance in a power gas plant has been reduced to a minimum.

Table II.—Stresses on Crank Shafts.

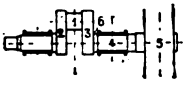


FIG. 1.—With two bearings.

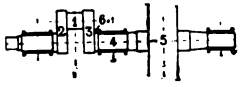


FIG. 2.—With three bearings.

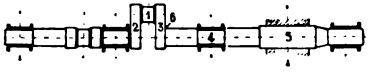


FIG. 3.—Two-throw crank shaft.

Section.		1	2	3	4	5	6	Mean.
Crank shaft, Fig. 1, *gas engine* developing .	5 HP	12,200	11,400	12,000	7,800	—	10,000	10,700
" " 2, " " .	35 "	11,700	10,000	10,000	7,000	2,800	8,900	8,500
" " 2, " " .	110 "	14,000	10,000	10,000	8,300	4,350	11,300	9,800
" " 2, " " .	100 "	11,500	7,300	7,300	6,000	5,700	Not determined	7,600
By Ensslin's method, Fig. 3, *gas engine* developing	100 "	9,000	5,500	7,500	10,000	5,700		7,600
Crank shaft, Fig. 3, *steam engine* developing .	650 "	5,500	3,500	4,500	3,300	3,300		4,000
By Ensslin's method, Fig. 3, *steam engine* developing	650 "	5,500	4,000	3,900	5,600	3,100		4,500

To compare the stresses in gas and in steam engines, it is necessary to take those which have been calculated by the same method. Thus, from the above table, if we use the method described at p. 83 *et seq.*—

Allowing for the inertia of the moving parts $\left\{ \dfrac{\text{mean stresses on a 100 HP gas engine} = 7600 \text{ lbs. per sq. in.}}{\text{mean stresses on a 650 HP steam engine} = 4000 \text{ " "}} \right. = \mathbf{1{\cdot}9}$

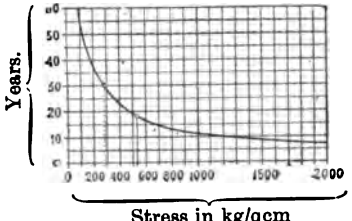
By Ensslin's method . . $\left\{ \dfrac{\text{mean stresses on a 100 HP gas engine} = 7600 \text{ lbs. per sq. in.}}{\text{mean stresses on a 650 HP steam engine} = 4500 \text{ " "}} \right. = \mathbf{1{\cdot}7}$

Both methods show that the average stresses on the crank shaft of a gas engine are about $1\frac{3}{4}$ times greater than the stresses on a steam engine shaft.

Neglecting the various changes in the stresses and taking the average as given above, we get—

Stresses in a gas engine = 7600 *lbs. per sq. in.* *Stresses in a steam engine* = 4000 *lbs. per sq. in.* (Table II.).

Taking again the life of a steam engine crank shaft = 30 years, and the breaking load = 70,000 lbs. per sq. in. (say 31 tons per sq. in.), the diagram plotted at the side shows that the life of a gas engine crank shaft under the same conditions = 18 years.



The same rule applies to the stresses in the other parts of the engine. For instance, the effects of the high temperatures on the cylinder and valves, to which the frequent cracking of badly designed breech ends, and distortion of the exhaust valve bear witness, are considerably greater in gas than in steam engines.

The allowance for *depreciation*, therefore, should after ten years be three-quarters the original value of a gas engine, and half the original value of a steam engine. In other words, for depreciation and interest on capital—

9 % of the value, when new, of a steam engine, and
11 % " " " of a gas engine, should be written off.

(c) Subsidiary Costs.

These are made up as follows: Interest on capital, depreciation of engine and buildings, maintenance and repairs, lubrication and cleaning, attendance, lighting, insurance, cleaning the boilers, and general overhauling.

To determine these subsidiary costs accurately, they should be separately calculated for each case. Only the results of approximate values and rough averages are given in Table III. below. Rather more, as has been explained, is written off for gas than for steam engines, but the consumption of lubricating oil, attendance, maintenance, and so forth, are taken as the same for both.*

Table III.—Subsidiary Costs per HP hour, for Gas and Steam Engines, exclusive of Cost of Fuel.

Normal output . . .	10	30	50	70	100	150	300	500	1000 BHP
Cost for gas engines .	0·53	0·375	0·28	0·235	0·222	0·188	0·177	0·17	0·15 penny
„ steam engines	0·53	0·38	0·30	0·25	0·22	0·19	0·17	0·15	0·14 „

These figures are calculated on a basis of 3000 working hours in the year. For night and day shifts they vary slightly, because the interest on capital is spread over almost twice as much time.

(d) Price of Coal and Steam.

Anthracite in pieces of from $\frac{3}{4}$ to $1\frac{1}{8}$ inch is used for suction gas producers, also lignite briquettes, $2\frac{1}{2}'' \times 2\frac{1}{2}'' \times 1\frac{1}{2}''$; and bituminous coal, lignite, peat, wood chippings and shavings, or any kind of fuel, or even refuse.

It is possible to determine approximately the prices, in various districts, of German and English anthracite, and of lignite briquettes,† for driving gas engines. Allowing for the unavoidable waste in screening the coal before use, and in transport, we may take the

Price delivered at the engine house = 1·1 × *price of the anthracite at the pit's mouth, plus* 1·1 × *cost of transit by rail.*‡

If the coal is required for small industries, in quantities of less than 10 tons, 15% to 25% must be added to the prices. For steam engines the different kinds of coal used must be considered. Even in the same district, and from the same pit, the characteristics of the coal (its heating value, clinker ash, etc.) will be found to vary greatly. To accustom the student to calculate examples, a few average values for various coals are given in Table IV. (and in greater detail in Haeder's "Dampfkessel," 4th Edition, p. 407).

Since in generating steam many different kinds of fuel are used (poor coal, cheap coal, wood refuse, etc.), it is hardly possible to take the consumption of coal per HP hour as the unit of comparison. Better and more reliable results will be obtained if the calculations for steam engine consumption are based on the

Cost of the quantity of coal required to generate 10 *lbs. of steam.*

This can always be easily determined. Table V. gives approximate values of the cost of fuel on this basis for various districts.

According to the type of boiler plant, whether more or less efficient, it is possible to generate from 8 *to* 12 *lbs. of steam*, say a mean, easily attained, of 10 lbs. from coal containing 16,200 *thermal units* per lb.

* Translator's Note.—The cost of attendance for a gas engine plant in England is from 30% to 75% less than for a steam engine plant.

† Very little used in England.

‡ The further the coal has to travel, and the more often it is transhipped, the greater will be the waste in screening.

Table IV.—Residual Products, Heating Value, Evaporative Power (mean) and Price of Coal and Steam in Various Colliery Districts.

	Residual products in %. Mean %.	Heating values. B.Th.U. per lbs.	1 lb. coal yields, of steam, lbs.	Price in neighbourhood of colliery of	
				1 lb. coal. Pence.	10 lbs. steam.* Pence.
1. *Anthracite.*					
Westphalian	3	14,300	—	0·01	—
Belgian	2·5	14,300	—	0·01	—
English	1·6	14,700	—	—	—
2. *Hard or Bituminous Coal.*					
Ruhr	10	13,150	8·5	0·048	0·059
Saar	12	11,350	7·8	0.053	0·07
Upper Silesia	5	12,600	8·0	0·037	0·059
Northern Silesia †	9	11,150	7·3	0·043	0·08
Saxony †	7·5	11,150	6·3	0·048	0·089
Aix-la-Chapelle	19	11,250	7·2	0·048	0·07
3. *Lignite or Brown Coal.*					
Bohemia	10	9,000	5·6	0·037	0·07
Province of Saxony †	10	5,950	2·7	0·022	0·059
Southern Bavaria	17	7,850	5·1	0·037	0·096
Cologne and Aix-la-Chapelle	10	9,000	5·6	0·043	0·07

To determine the average values, the freight charges by rail or water, and the cost of transport of the coal to its destination, must be taken into account.

Tables V. and X. give approximate values of the cost of fuel, which should be checked and additions made.

To determine the prices of authracite and steam in various *widely separated districts*, the author has procured information through inquiries in Haeder's *Zeitschrift*, No. 1, 1904, and other sources, and also from Reports issued by the Imperial Ministry of the Interior (the latter, however, only gives prices for house anthracite). Several of the places mentioned below show considerable differences in price.

Table V.—Comparative Values of the Cost of Fuel.

	Cost, including all expenses, of—	Duisburg.		Königsberg.		Kattowitz in Silesia.		Bitterfeld, Saxony.				
Normal output	1 lb. anthracite	0·112		0·2		0·21		0·19				Pence
	10 lbs. steam *	0·095		0·19		0·088		0·082				,,
	Consumption per BHP:	Gas.	Steam.	Gas.	Steam.	Gas.	Steam.	Gas.	Steam.	Gas.	Steam.	
50 BHP	1 lb. anthracite	0·12		0·21		0·22		0·2				Pence‡
	20 lbs. steam		0·176		0·28		0·16		0·15			,,
300 BHP	0·9 lb. anthracite	0·1		0·18		0·2		0·17				Pence‡
	14·5 lbs. steam		0·126		0·2		0·116		0·108			,,

* By cost of steam must be understood only the cost of the coal required to generate it. The other subsidiary costs will be found under Working Costs, head (c).

† In these districts lignite and hard coal are used.

‡ If lignite briquettes are used, the price will be 0·01*d.* less, if there are lignite mines in the neighbourhood as, for instance, at Bitterfeld.

The cost of fuel in Table V. has been worked out from Tables VIII. and IX. These tables show that the question of the economy to be effected in fuel by using gas engines in place of steam engines can only be determined separately for each case.

For instance, Table V. shows that

For engines developing 50 BHP at Königsberg, it is 25 % cheaper to drive them with gas than with steam.

" " " 300 BHP " " 9·3 % cheaper to drive them with gas than with steam,

when the producers are stoked with anthracite (a most expensive fuel) and the steam boilers stoked with cheaper fuel.

If the same fuel is used in both cases, the saving in favour of gas engines will be still greater.

(e) Consumption of Fuel.

Gas engine builders generally give the consumption of fuel only at maximum load, although they do not draw attention to the fact. In practice this maximum load is seldom attained; it is of more importance to determine the consumption at lesser loads, because, as already explained at pp. 209, 210, it is necessary at the outset to select an engine giving higher powers than those usually required of it.

Gas Engines.

The best consumption of fuel (*i.e.* the most economical) is realised when the engine is running at maximum load.

Consumption of fuel in gas engines (mean values).

Output =	1	0·9	0·8	0·7	0·6	0·5	0·4 of full load.
Ratio of consumption =	1	1·05	1·10	1·20	1·30	1·50	1·7

Consumption when running light = 0·45 of consumption at full load.

Example.—A suction gas plant in which the consumption at maximum load is 1½ lb. of lignite per HP hour, is to run under 0·4 of the maximum load. Therefore we may calculate consumption = 1·5 × 1·7 = 2·55 lbs. lignite per HP hour.

In oil engines in which the oil is injected into the combustion chamber (as in the Diesel, Trinkler, etc.), the relative consumption does not increase with diminishing load to the same extent as in gas engines.

Consumption of fuel in oil injection engines.

Output =	1	0·9	0·8	0·7	0·6	0·5	0·4
Ratio of consumption =	1	1·01	1·04	1·09	1·14	1·22	1·34

Consumption running light } = 0·3 of the consumption at full load.

Example.—A Diesel engine consumes at maximum load 0·44 lb. of crude oil per HP hour; at one-half the maximum load, therefore, the consumption will be—

0·44 × 1·22 = 0·54 lb. per HP hour.

Steam Engines.

In steam engines the normal output or load is taken to mean the output at which the cost of working, including interest on capital, sinking fund, and repairs to engine and boiler, are reduced to a minimum.

The most economical consumption of fuel is obtained when the engine is running at about 0·85 of the maximum load. Mean experimental values show that the consumption of fuel in steam engines is (normal load being = 1)—

Output =	1·4	1·2	1	0·85	0·6	0·4
Ratio of consumption =	1·10	1·05	1	0·96	1·02	1·07

Consumption when running light = 0·12 of consumption at full load.

Example.—A steam plant at normal output consumes 2·64 lbs. coal per HP hour. At 0·4 of the normal load the consumption will be

= 2·64 × 1·07 = 2·8 lbs. per HP hour.

To make an *accurate comparison** between a steam and a gas engine, we must allow for the consumption of fuel in the same way as for the cost of the plant (see p. 210), namely—

Normal load = 0·7 of the maximum load.

* Translator's Note.—To make an accurate comparison the same fuel should be burnt under the boilers as that used in the producer.

If, for instance, 100 HP are required to drive a factory, the choice lies between a steam engine and a gas engine with a maximum of 140 HP; thus, in both types of engine we shall have the same reserve of power. Under normal conditions 70 % of the maximum power will be utilised, and the calculation of the working costs must be based on this percentage of output.

Table VII.—Comparative Values of the Cost of Consumption of Fuel in Proportion to the Percentage of Maximum Power developed.

Output		Full load (maximum).	0·9	0·8	**0·7**	0·6	0·5	0·4	0·3	0·2	0·1 of max.
Gas engines	Hit-and-miss governing	1	1·05	1·10	**1·17**	1·26	1·40	1·6	1·9	2·5	4·5
	Quantity or quality governing	1	1·06	1·15	**1·25**	1·40	1·6	1·85	2·4	3·4	6·4
Steam engines		1·1	1·05	1·02	1	0·96	1·02	1·07	1·15	1·3	1·6

In Table VIII. the figures given for the consumption of anthracite in gas engine plants at maximum load are based on actual practice, and have been re-calculated for lesser loads with the help of Table VII.

In Table IX. the consumption of steam experimentally determined is given for the normal output, or about 0·7 of the maximum.

The figures printed in *black type* in Table VIII., giving the consumption at normal load = 0·7 of the maximum load for gas engines, may be compared with the similar figures in Table IX. for steam engines.

In a gas engine the smaller the load, the less favourable (*i.e.* the less economical) is the consumption, as a comparison of the figures in Table VII. will show. The higher the load in these engines, the more economical the consumption.

Table VIII.—Consumption of Anthracite in a Gas Engine Plant in lbs. per BHP hour.

(Hit-and-miss governing.)

Maximum output	$HP_{max.}$ =	14	40	70	100	140	210	420	700	1400	BHP
Normal output	HP =	10	30	50	70	100	150	300	500	1000	,,
Consumption of anthracite in lbs. per BHP hour	at maximum HP =	1·2	0·95	0.9	0·88	0·86	0·82	0·77	0·75	0·7	lbs.
	,, 0·7 ,, ,, =	**1·4**	**1·11**	**1·05**	**1·03**	**1·00**	**0·96**	**0·9**	**0·88**	**0·84**	,,
	,, 0·5 ,, ,, =	1·68	1·33	1·26	1·23	1·20	1·20	1·08	1·05	0·98	,,

If lignite briquettes are used, the values in the above table must be multiplied by 1·65.

Table IX.—Consumption of Steam in a Modern Steam Engine Plant, per BHP hour, taking the Normal Output = 0·7 of the Maximum Output.

Maximum output	=	14	40	70	100	140	210	420	700	1400	BHP
Normal output	=	10	30	50	70	100	150	300	500	1000	,,
Consumption of steam: lbs. per BHP hour	Saturated steam =	33·1	27·6	22·5	21·2	19·4	17·5	16·3	15·5	14·8	lbs.
	Superheated steam =	**28·7**	**25**	**19·9**	**18·5**	**17·2**	**15·5**	**14·6**	**14·1**	**13·5**	,,
	Highly ,, ,, =	24·3	22	17·6	16·3	15	13·7	12·8	12·6	11·9	,,

Example from Tables VIII. and IX.—A 100 HP engine is required for a factory at Cologne. We take the maximum output = 100 : 0·7 = 140 HP.

100 HP Suction Gas Plant.

From Table VIII. Consumption of anthracite * = 1·0 lb. per HP hour.

" " X. Price of authracite per lb. = 0·13*d.* per lb.*

Therefore cost of fuel = 0·13 × 1·0 = 0·130*d.*

From Table III. Subsidiary costs = 0·222*d.*

At 0·7 of maximum output—

Total working costs per HP hour = 0·352*d.*

At 0·5 (half) maximum load—

$$1{\cdot}2 \times 0{\cdot}13 + 0{\cdot}22 = 0{\cdot}376d.$$

At maximum load—

$$0{\cdot}86 \times 0{\cdot}13 + 0{\cdot}22 = 0{\cdot}332d.$$

100 HP Steam Engine Plant with Superheated Steam.

From Table IX. Consumption of steam = 17·2 lbs. per HP hour.

" " X. Cost of 10 lbs. = 0·1*d.*

Therefore cost of fuel = $\frac{17{\cdot}2 \times 0{\cdot}1}{10} = 0{\cdot}17d.$

From Table III. Subsidiary costs = 0·22*d.* per BHP hour.

Total working costs per BHP hour = 0·39*d.*

$$1{\cdot}02\frac{17{\cdot}2 \times 0{\cdot}1}{10} + 0{\cdot}22d. = 0{\cdot}40d.$$

$$1{\cdot}1\frac{17{\cdot}2 \times 0{\cdot}1}{10} + 0{\cdot}22d. = 0{\cdot}41d.$$

According to these figures the working costs at Cologne of a gas engine and producer using the most expensive fuel (anthracite), and a steam engine working with cheap coal, show a considerable saving in favour of the gas engine.

The following Table X. shows the results calculated for different towns in Germany. Places in which coal is exceptionally cheap, as in Table V., are omitted. The calculations are worked out for the normal load, viz. 0·7 of the maximum.

Table X.—Working Costs and Costs of Fuel in Pence per Brake HP hour.

(3000 working hours in the year. The steam is assumed to be moderately superheated. The figures to the right in each column show the total cost of working.)

	Subsidiary costs.	Essen.		Cologne.		Hanover.		Magdeburg.		Dresden.		Frankfort.		Breslau.		Berlin.		
Cost including freight charges—																		
Westphalian anthracite per lb.	—	0·11		0·13		0·15		0·17		0·2		0·165		0·225		0·18		Pence
Coal to evaporate 10 lbs. of steam	—	0·85		0·1		0·11		0·12		0·16		0·11		0·11		0·11		"
Normal output 10 HP Gas	0·53	0·15	0·68	0·17	0·71	0·21	0·74	0·24	0·76	0·28	0·81	0·21	0·74	0·32	0·85	0·26	0·79	Pence
Normal output 10 HP Steam	0·53	0·15	0·79	0·3	0·79	0·32	0·85	0·35	0·89	0·46	1·0	0·31	0·84	0·32	0·85	0·32	0·85	"
Normal output 50 HP Gas	0·28	0·11	0·41	0·13	0·41	0·15	0·46	0·18	0·46	0·21	0·5	0·16	0·45	0·24	0·52	0·19	0·52	"
Normal output 50 HP Steam	0·3	0·165	0·5	0·2	0·5	0·23	0·54	0·25	0·54	0·32	0·61	0·21	0·51	0·23	0·52	0·23	0·52	"
Normal output 300 HP Gas	0·18	0·106	0·3	0·12	0·3	0·14	0·32	0·15	0·33	0·19	0·36	0·14	0·32	0·2	0·38	0·16	0·34	"
Normal output 300 HP Steam	0·17	0·13	0·32	0·15	0·32	0·17	0·33	0·18	0·33	0·24	0·39	0·15	0·32	0·165	0·33	0·16	0·33	"

If the gas generator is stoked with lignite briquettes, see remarks at Table IX., p. 214.

Example from Table X.—For a 300 HP plant at Dresden the working costs would be: with suction gas, 0·36*d.*; with steam, 0·39*d.* per HP hour.

The above figures are only available for the *normal output* of an engine = 0·7 of the maximum load. The cost of driving it at maximum load can also be worked out from Tables VII. to IX. It will be seen that the cost of fuel is in that case 20 % less for gas engines $\frac{1{\cdot}25 - 1}{1{\cdot}25} \times 100 = 20\,\%$, and 10 % higher for steam engines.

On the other hand, at half load, the cost of the fuel in a gas engine is 27 % higher, and in a steam engine 5 % higher; for instance—

At Cologne	Output = 420 HP maximum.	300 HP normal.	210 HP half load.	
Normal	Gas = 0·108	0·12	0·15	Pence per
Output 300 HP	Steam = 0·13	0·12	0·125	HP hour.

* Lignite works out at 0·19*d.*

Besides the special *local conditions*, therefore, the *load* at which an engine will usually be run must, in comparing gas and steam, be also carefully kept in view.

The gas engines show the greatest superiority over steam engines in cases where the load is intermittent, or where the engine is only run for a few hours a day.

If fuel in the form of gas, such as blast furnace gas or coke oven gas, is available, it is better to drive an engine direct with gas, instead of burning the gas under boilers, since gas engines are now quite reliable and run continuously day and night for long periods.

Heating Value of Exhaust Steam in Steam Engines up to 30 HP.

If the exhaust in non-condensing engines can be completely utilised for industrial purposes, the cost of fuel may be halved; if it is used in winter only, for heating, the cost may be reduced by 25 %. The exhaust gases in gas engines contain only from $\frac{1}{3}$ to $\frac{1}{8}$ the heat units carried off in the exhaust of steam engines. According to the size of the plant, the

Exhaust in non-condensing steam engines	contains about	24,000 to 30,000	B.Th.U.	per	HP	hour.
,, gas engines	,, ,,	4,750 to 3,000	,,	,,	,,	,,

To utilise the exhaust gases in a gas engine does not produce so great an economy as in a steam engine. It is also well known that as steam is a more perfect conductor of heat, it is better suited to heating purposes than the class of permanent gases to which the exhaust in a gas engine belongs.

Nevertheless the heat in the exhaust of gas engines is always made use of wherever possible.

In making a comparison between gas and steam, it is also necessary to study the *degree of irregularity in running, weight of the flywheel, and inertia of the moving parts.*

For further details on these subjects, see pp. 159 *et seq.* and 53 *et seq.*

Many of the phenomena in a gas engine are still imperfectly understood. For instance, neither the pressure of explosion nor the mean pressure on the piston can at present be determined beforehand with any degree of accuracy in a new type of engine, and a correct theory of the processes taking place in the cylinder has not been formulated.

EXERCISES.

A GOOD knowledge of steam-engine construction and practice in calculating steam plants are assumed on the part of the learner.

The following exercises are intended to afford the beginner an incentive to diligent study. The numbers on the right-hand side show the page in the book giving the explanation of the question.

		PAGE
1.	What do we understand by a *four-cycle* engine?	1
2.	„ „ „ *two-cycle* engine?	1
3.	How many revolutions does a four-cycle engine make during one working cycle?	1
4.	How many revolutions does a two-cycle engine make during one working cycle?	1
5.	How many strokes does a steam engine make during one cycle of work?	1
6.	Explain the meaning of the term *single*-acting?	1
7.	„ „ „ *double*-acting?	1
8.	Can a *double*-acting four-cycle engine be called a *two-cycle* engine?	2
9.	Which are the best-known types of two-cycle engines?	2
10.	How can it be known, by looking at the outside of an engine, whether it is a two- or a four-cycle engine?	2
11.	Name the most usual types of single-acting four-cycle engines	3
	„ „ „ double-acting „ „	3
	„ „ „ „ two-cycle engines	3
12.	After studying Table I., draw the different types from memory, and put to each the shortened symbols denoting them	3
13.	What is meant by combustion chamber?	4
14.	„ „ explosion pressure?	5
15.	„ „ compression pressure?	5
16.	What effect has the size of the combustion chamber on the final pressure of compression?	5
17.	Give a free-hand tracing of the diagram of the **E 4** engine shown at Fig. 1, p. 2	5
18.	Insert from memory the meaning of the lines forming the diagram, as given in Fig. 16	7
19.	Pages 8–15 to be carefully studied	8–15
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21.	What is meant by the mechanical *efficiency* of an engine?	18
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25. What do we understand by "mean pressure on the piston," and what is the symbol used to denote it? 19
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27. What does the term p_i mean? 20
28. What is meant by *hit-and-miss* governing? 19
29. „ „ *quality* or *quantity* governing? 19
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34. What will be the output of a **D 4 two-cyl.** engine worked with blast furnace gas, in which D = 2·6 feet, S = 3 feet, $n = 90$? 22
35. Required to determine the leading dimensions of an engine of a given HP; what are the principal points to be kept in view? 23
36. What is meant by "*inertia of the moving parts*"? 25
37. What parts of the engine transmit the load on the piston to the crank shaft? . 25
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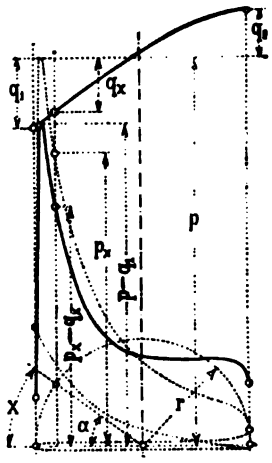
47. What is meant by *tangential* pressure? 34
48. If $a = 40°$, and the thrust on the connecting rod = 4400 lbs., determine the tangential effort graphically—
    (*a*) For a connecting rod having a length = 5 × radius of the crank . . 35
    (*b*) „ „ „ = ∞ 35
49. What is meant by the *diagram of work?* 35
    „ „ „ *inertia diagram?* 35
    „ „ „ *diagram of energy?* 36
50. If an **R D 4** engine has a sectional area of the cylinder A = 496 sq. inches, S = 32″, circumferential velocity in circle described by centre of gravity of rim section of flywheel V = 79 feet per second, what should be the *weight* of the *flywheel rim*, if $\delta_0 = \frac{1}{150}$? 50
51. Is there a pressure on the cap or keep of the *main bearing* of a four-cycle gas engine, or could the engine (in theory) work if the cap were taken off? 58
52. Draw the different crank positions given at p. 60 for an **E 4** engine, and work out the corresponding equations for the forces acting on the cross head and crankpin, also the thrust on the connecting rod, using the data given on p. 59 59–60
53. Do the same for the **D 4** engine, make drawings as at p. 61, and utilise the data given at p. 59 for the equations 59–61
54. Explain the term *change of pressure in the rods* 63
55. How many changes of pressure are there in a double-acting steam engine? . . 63
56. „ „ „ „ an **E 4** gas engine? 63
57. How many times greater is the calculated *pressure on the rods* in a **D 4** single-cylinder engine than in a compound steam engine giving the same power? 64
58. Give the usual *names* of the numbered parts of the gas engine as shown at p. 67 67
59. What is the *temperature* at the explosion end of a gas engine? 68
60. „ „ in the clearance space of a steam engine? 68
61. „ „ of the exhaust gases in a gas engine? 68
62. „ „ „ exhaust in a steam engine? 68
63. What do we understand by a *centrifugal lubricator* on the crank pin? 75
64. What is the object of the *balance weight* on the cheeks of a cranked shaft? 79
65. Which two *positions of the crank* must be taken into account in calculating the stresses on the crank shaft? 83
66. How is the position of the crank at the *maximum turning moment determined?* 84
67. What is approximately the angle of the crank for the maximum turning moment in gas engines? 84

68. *Addition to* p. 83.

At pp. 83 *et seq.*, and also in the examples given at pp. 90 *et seq.*, the forces acting on the crank pin are determined, without taking count of the inertia of the moving parts ; therefore the calculated stresses are valid only when starting the engine.

To know the stresses set up in the crank shaft when the engine is running, the inertia force required to set the reciprocating parts in motion must be subtracted from the pressure of the piston.



In the figure at the side (comp. pp. 25 *et seq.*), let—

q_1 be the inertia force in lbs. per sq. inch of piston area at the dead point;

q_2 be the inertia force in lbs. per sq. inch of piston area at crank position x; and referring to pp. 83 and 84, we get—

(1) For position of crank at the dead point. Thrust on the connecting rod—

$$P = A \times (p - q_1).$$

(2) For position of the crank at the maximum turning moment. Thrust on the connecting rod—

$$P' = A \times (p_x - q_x).$$

If $a = 34°$—

$$p_x = 0{\cdot}75p; \quad q_x = 0{\cdot}78q_1, \text{ approximately.}$$

These calculations apply only to single-acting engines; with double-acting engines both faces of the piston must be taken into account, the higher value thus obtained for $p_x - q_x$ determined, and the calculations worked out from it.

The ratio $p_x : p$, $q_x : q$, and the angle a must be determined graphically.

If account be taken of the inertia of the moving parts in making the calculations at pp. 83 *et seq.*, some of the results, as worked out on pp. 90 *et seq.*, will yield the values shown in the following table:—

Dimensions of engine. Diam./stroke= From Table I., p. 24, n =	3 ins. × 5 ins. 240		10¼ ins. × 13¾ ins. 220		14 ins. × 20 ins. 185		24 ins. × 28 ins. 150		
	at dead point.	at a = 34°	Dead point.	$a = 34°$	Dead point.	$a = 34°$	Dead point.	$a = 34°$	
From Equation 5, p. 26 ϕ_1 =	158	122	360	282	360	282	331	260	ft. per second²
From Equations 7 and 9, p. 26 . . . . . q_1 =	13·8	10·8	54	42	65·5	51·2	66·2	51·3	lbs. per sq. in.
$p - q_1$, $p_x - p_x$ respectively =	355	265	315	234	305	221	305	226	,, ,,
Without inertia of moving parts P and P_1 respectively . . . . . =	2600	1950	30,500	22,800	57,000	42,200	167,000	124,000	lbs.
With inertia of moving parts P and P_1 respectively . . . . . =	2500	1870	26,000	19,300	46,900	34,500	138,000	102,000	,,
Maximum stress on crank pin . . . . . σ =	—	—	—	—	—	—	—		lbs. per sq. in.

Calculating in this way (with allowance for the inertia of the moving parts) the permissible values of σ (allowable stress or safe working load) given at pp. 86 to 89 must be correspondingly reduced. These stresses apply to actual crank shafts, and are *only* valid if the calculations are worked out as at p. 89. With regard to the inertia of the moving parts, the following corrections may be made:—

In equations	73	79	85	91	100	107	114	121	128	137	145	152
k(lbs. per. sq. in.) up to	12,800	11,300	11,300	4200	11,300	8500	3500	7800	5000	7100	4250	11,300

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69. What conditions affect the *overheating* of the bearings? 95
70. What method must be adopted to calculate the tendency of the bearings to overheating? 97
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72. How many oiling rings are required in self-lubricating bearings? 100
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    " " " two-cycle engine? 143
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    (*a*) Fresh water is used? 148
    (*b*) Circulating water is used? 148
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86. What is the object of an air vessel on the pipe conveying the cooling water to the piston? 149
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88. An $\mathbf{E}$ 4 engine works with a piston load of 70,000 lbs. What should be the approximate dimensions of the connecting rod, main bearing, crank pin, and piston pin 163
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90. What does $\mathfrak{M} \times \mathfrak{D}^2$ represent? 166

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91. How many times heavier must the flywheel of a single-acting four-cycle gas engine (E 4 1 cyl.) be than that of a single-cylinder steam engine, other working conditions being the same? 167
92. What do we mean by *energy* of the flywheel? and what equation is used to denote it? 170
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95. What expression is used in electrical work for calculating the size of the flywheel? 173
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97. What system of generating the electric current requires a greater degree of uniformity in the driving engine; in other words, less irregularity in running? 175
98. Two dynamos (directly coupled to the driving engines) are to be run in parallel. For what degree of irregularity in running must the masses in rotation be calculated—
    (*a*) for continuous current? (*b*) for alternating current? 175
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100. Upon what variation in the load (in percentage of the normal output) must the calculation of a medium-sized grinding mill be based? 176
101. What degree of irregularity should be allowed for a large mechanical workshop, and what should be the energy of the flywheel per HP—
    (*a*) If the dynamo does not supply electric light? 177
    (*b*) „ „ supply electric light? 177
102. Required a 25 HP engine to drive a mechanical carpenter's shop with circular saw. To what degree of uniformity in running and of energy must the flywheel correspond? 177
103. What should be the circumferential velocity of the flywheel in gas engines? . 179
104. At what loads, below 100 HP, do small single-acting gas engines have a separate pulley, and at what loads does the flywheel act as the driving pulley? 179
105. Draw the pulley flywheel for a 35 HP gas engine to scale 182
106. For the above flywheel (Exercise 105) determine the degree of irregularity δ_0, the energy of the flywheel, and the stresses in the arms, at a speed of $n = 185$ revolutions per minute.

APPENDIX I.

DESCRIPTION OF THE HUMPHREY PUMP.

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Description of the Humphrey Pump.*

In the simplest case of Humphrey pump, as illustrated in Fig. 1, water has to be raised from supply tank ST to an elevated tank ET. C is the combustion chamber, W the water valve box with valve V opening inwards, and D the discharge pipe. In the top of the combustion chamber are fitted an inlet valve A and an exhaust valve E. A simple interlocking gear is arranged between these two valves, so that when valve A opens and closes it locks itself shut and releases the valve E, and when valve E opens and closes it locks itself shut and releases valve A. Consequently each time that suction occurs in the chamber these valves open in turn.

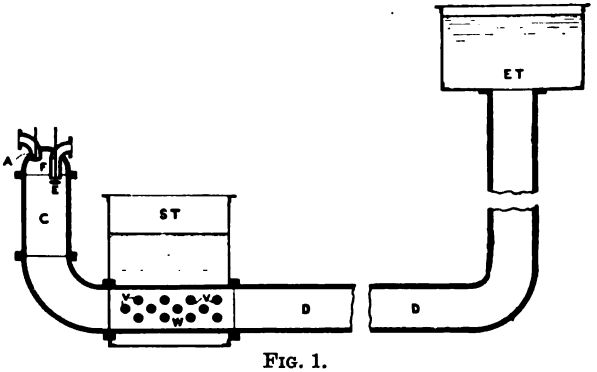


Fig. 1.

Imagine a charge of gas and air to be compressed in the top of chamber C, and fired by a sparking plug which projects through the top casting. All the valves are shut when explosion occurs, and the increase in pressure drives the water downwards in the pump and sets the whole column of water in the discharge pipe in motion. The column of water attains kinetic energy while work is being done upon it by the expanding gases, so that when these gases reach atmospheric pressure the column of water may be moving with considerable velocity. The motion of this column of water cannot be suddenly arrested, hence the pressure in C tends to fall below that of the atmosphere, the exhaust valve E opens and also the water valve V. Water rushes in through the water valves mostly to follow the moving column in pipe D, but partly to rise in chamber C in an effort to reach the same level inside the chamber as exists in the suction tank.

When the kinetic energy of the moving column has expended itself by forcing water into the high-level tank, it comes to rest, and there being nothing to prevent a return flow, the column starts to move back towards the pump, and gains velocity until the water reaches the level of the exhaust valve, which it shuts by impact. A certain quantity of burnt products is now imprisoned in the cushion space F, and the energy of the moving column is expended in compressing this gas cushion to a greater pressure than that due to the static head of the water in tank ET. Hence a second outward movement of the column results, and when the water reaches the level of valve E the pressure of the space F is again atmospheric, and further movement of the water opens valve A against a light spring and draws in a fresh charge of gas and air. If there were no friction the water would fall to the same level as that from which the last upward motion started, but the amount of combustible charge drawn in is slightly less than this movement would represent. Once more the column of water returns under the elevated tank pressure and compresses the charge of gas and air, which is then ignited to start a fresh cycle of operations.

* Reprinted, by permission of the author, from a paper read by Mr. H. A. Humphreys, M.I.C.E., before the Manchester Association of Engineers on Nov. 12, 1910.

The action of the pump is not altered if, instead of delivering into an elevated tank, it discharges into an air vessel, as shown in Fig. 2, or into an open top stand pipe or tower, as shown in Fig. 3, and both these arrangements are useful if a continuous flow from the outlet O is desired. In many cases the pump can be placed directly in a concrete pit, which serves also as a foundation, and the discharge can be by an inclined pipe, as shown in Fig. 4, where the water valve box is shown vertical. This arrangement is very satisfactory, and is economical in material.

In the simple form of pump the degree of compression of the combustible charge prior to ignition depends on the height to which the water is raised, and exceeds the static equivalent of the head. This can be made clear by reference to Fig. 5, where water returning under a pressure AB, and moving through a stroke AD, does work in compressing the gas equal to ABCD. The gas is compressed from A to E, and the work of compression is AED, consequently the shaded area above the line C B must be equal to the shaded area below it, and E, the maximum compression pressure, is much above C. This result is obvious if one remembers that the kinetic energy acquired by the liquid column on its return flow is utilized in compressing the combustible gas, while the compression brings the column to rest.

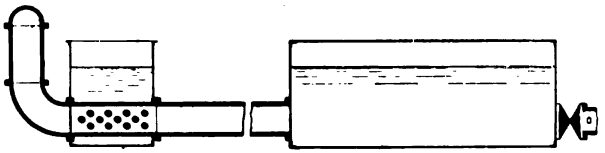


Fig. 2.

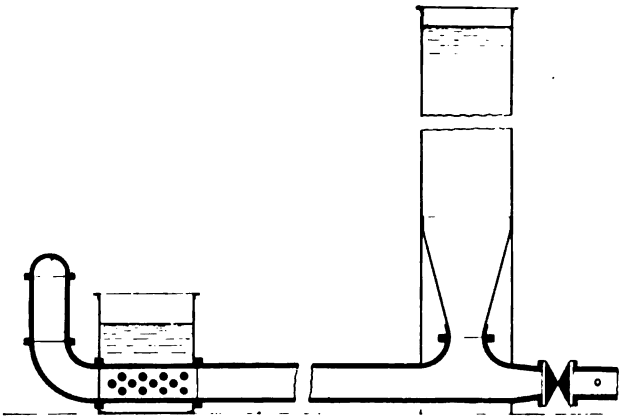


Fig. 3.

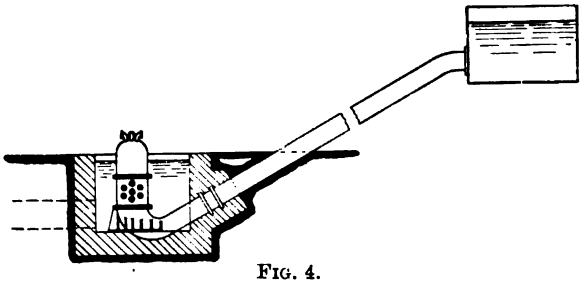


Fig. 4.

The same considerations enter into the question of the cushion pressure attained, but here we are dealing with the compression of a volume of gaseous fluid which occupies the clearance space only, and the stroke of the water column is greater in proportion, because for the first part of the stroke exhaust products are being expelled and no compression occurs. Without going into the mathematics of the subject, it is sufficient to say that the cushion pressure rises rapidly as the height to which the water is lifted increases, and at the maximum lift of about 40 ft. to which the simplest type of pump is limited, it may exceed the explosion pressure when using producer gas, or may approximately equal the explosion pressure when working with town gas or petrol.

Without much sacrifice of simplicity the cushion space may be kept separate from the combustion chamber, and can then be made relatively large because the products or air in the cushion do not then mix with the new charge. This forms one way of keeping the cushion pressure low, and is explained later. Again if air scavenging is employed, as is now usual, the residual burnt products left in the clearance space are mixed with air and the latter may constitute the greater part of the mixture; a rather excessive clearance is then permissible as compared with an ordinary gas engine, and the simple pump will work throughout its whole range of lift with very moderate cushion pressures.

No simple means of indicating the work done by the expanding gases has been found, so that the indicated horse-power of a Humphrey pump cannot be directly ascertained, but indicator cards

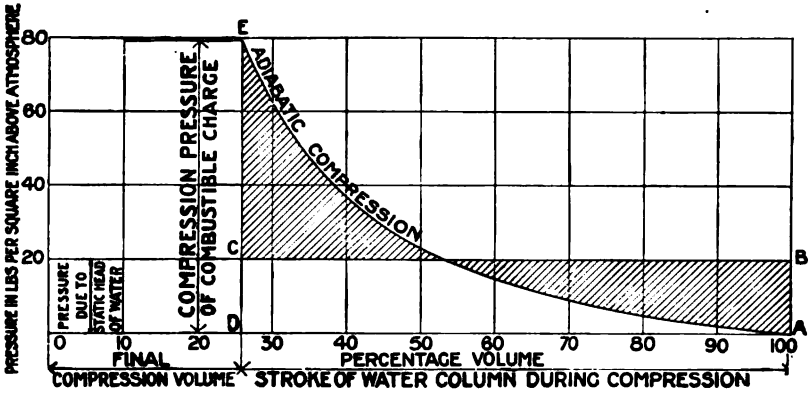


FIG. 5.

showing pressures on a time basis are interesting. Such an indicator card is given in Fig. 6, and is taken from a 45 water HP four-stroke pump, lifting to 39·4 ft. head, and working on town gas. The figures relating to this card are :—

Compression pressure at ignition, 56·5 lbs. per square inch.
Explosion pressure . . . . 163 lbs. ,, ,,
Cushion pressure . . . . 185 lbs. ,, ,,
The pump was delivering 306 gallons per working stroke.

The card may be explained as follows :—

Compression of charge from *a* to *b*.
Explosion from *b* to *c*.
Expansion from *c* to *d*.
Taking in water and scavenging air and partly exhausting, *d* to *e*.
Return stroke completing exhaust till valve shuts, *e* to *f*.
Cushioning, *f* to *g*.
Cushion expansion, *g* to *h*.
Intake of new charge, *h* to *j*.
Next compression of charge, *j* to *k*.

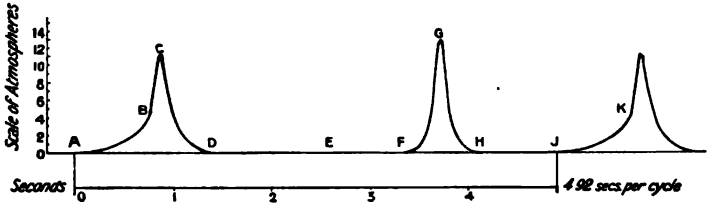


FIG. 6.

The stiff spring used does not permit the suction to be shown on this card, but the partial vacuum at the end of cushion expansion may momentarily be as much as 4 lbs. per square inch, and the water suction may at first be equal to two or three feet of water, both dying down to atmospheric pressure during the respective strokes.

If one thinks of the reciprocating column of water as representing the pendulum of a clock, it is easy to appreciate the fact that the length of the column governs the period of the swing; and if one assumes other things to be equal, and the column to be of uniform section, then the period is almost

proportional to the square root of the length. Mr. H. M. Martin, of *Engineering*, has worked out a number of examples illustrating the effect of the length of the discharge pipe under a given set of conditions, and his results are here reproduced from *Engineering* of 11th February, 1910 :—

Diameter of play pipe, 2 feet.
Maximum velocity of flow permitted, 14 feet per second.

Length of pipe, feet	25	50	100	200
Cycles per minute	61	31	14·3	7·0
Discharge, cubic feet per minute	527	486	447	417
Efficiency $\frac{PHP}{IHP}$	0·98	0·95	0·91	0·82

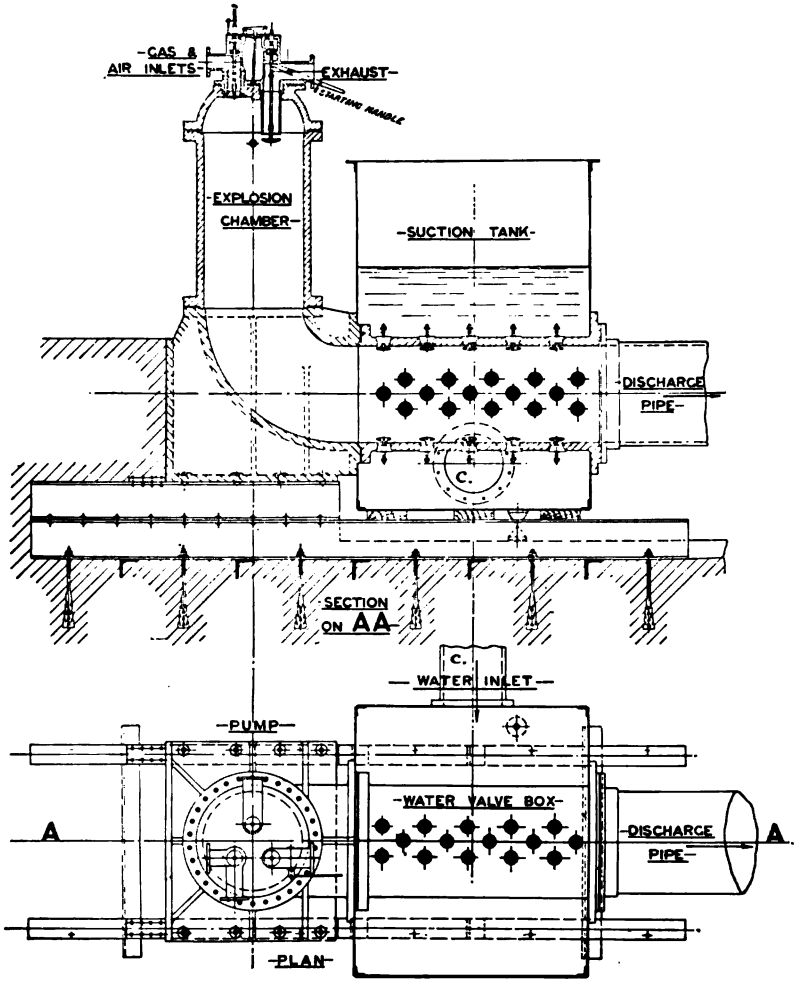


FIG. 7.

Evidently, for a simple type of pump taking in its charge at atmospheric pressure, the first and last columns do not represent practical conditions, the cycles being too rapid in the first and too

slow in the last, the two middle columns are possible cases, and a length of 60 to 80 ft. is generally found advisable. The maximum velocity chosen is merely an example and may be exceeded.

The four-cycle pump has been dealt with at some length because its construction is exceedingly simple, and it is the key to all the others. Moreover, its reliability has been amply proved, and its elasticity in meeting changes of lift without altering the mixture of gas and air or the clearance volume has been demonstrated. The four unequal strokes which result from the free oscillations of the water column give rise to a cycle which is superior to the Otto cycle, and is in fact the theoretically perfect heat cycle described in text-books. On the subject of heat efficiency more will be said later, but we must now pay a little attention to mechanical details.

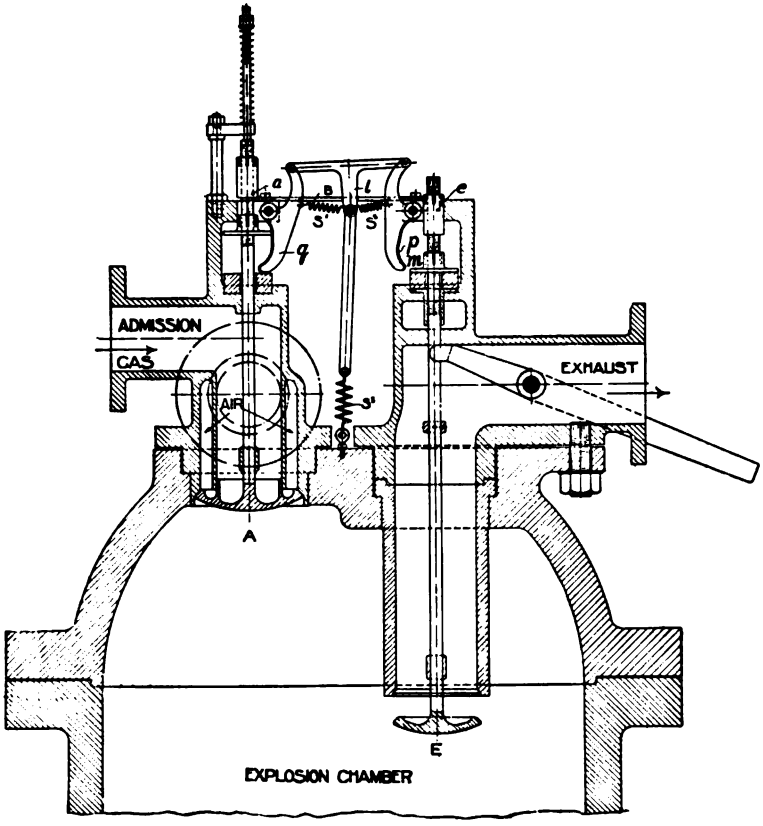


FIG. 8.

Fig. 7 is a sectional view of the Humphrey pump exhibited in operation at the Brussels International Exhibition, and which obtained two "highest possible" awards, namely, a Grand Prix in the class for gas engines, and a Grand Prix in the class for pumps. Its construction can be seen at a glance, and explanation is not needed till we come to the valve gear shown in Figs. 8 and 9. It will be observed that a bolt B sliding horizontally must lock either the admission valve A or the exhaust valve E by engaging under collars *a* or *e*, which are fixed on the stems of their respective valves. Now the bolt is urged right or left, according to whether spring s_1 or s_2 is pulling the hardest, and this again depends on whether the link *l* to which the springs are attached has been shifted to the right or left. Suppose the exhaust valve opened last, then its washer *m* engaging against cam arm *p* moved the system *plq* so that it leans to the right, in which position it is retained

by the tension of spring s_3. This puts tension on spring s_1 and loosens spring s_2, bolt B, therefore, tries to move to the right, but until the exhaust valve shuts it can only press upon collar e. However, when valve E comes on its seat the bolt instantly locks under e, and the same motion which holds valve E shut has released valve A, so that next time a suction occurs in the combustion chamber A only can open. Precisely the same kind of action occurs when A shuts and is locked and E is released again. Thus valves A and E are automatically allowed to act alternately, the difference between them being that while E remains open till shut by the rising water, A shuts under the action of its supporting spring so soon as the suction in the chamber permits the spring to lift the valve to its seat.

The scavenging valve V is shown in the plan of the combustion head, Fig. 9, and as it operates at the end of each expansion stroke its locking and release periods correspond with those of the

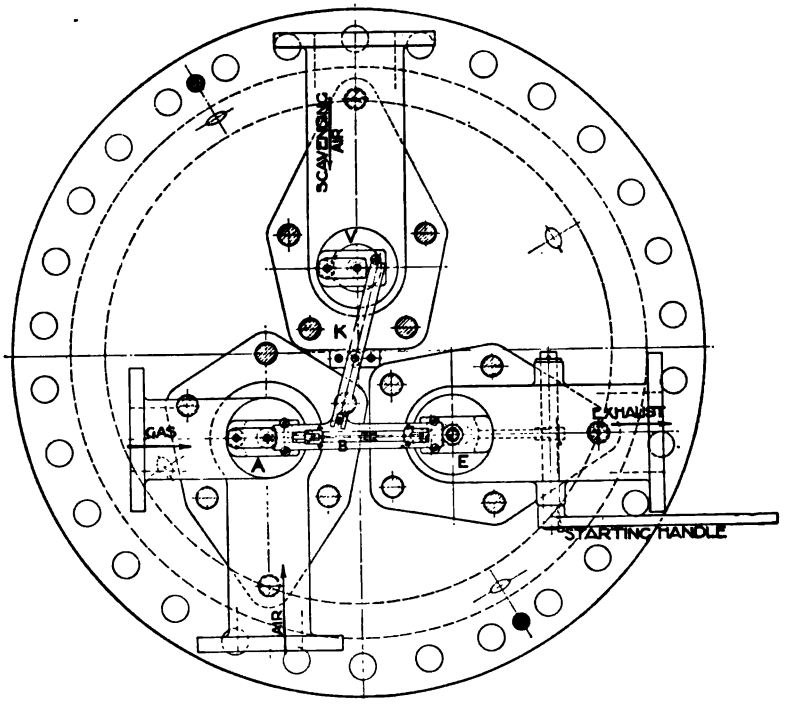


FIG. 9.

exhaust valve, and are made simultaneous by a lever pivoted at K, and operated by a pinion on the bolt B. If the water could rush in fast enough when the pressure falls to atmosphere there would be no scavenging action, but the incoming water has to be accelerated, and that just gives rise to a sufficient suction to effect the desired scavenging. In the exhaust outlet there is a light non-return valve, to prevent burnt products being drawn back into the chamber.

When an internal bell is to be used to provide a separate cushion of burnt products a slight modification of the gear is required to control the three valves. Instead of a scavenging valve we have a valve S opening into the top of the internal bell, as shown in Fig. 10, and on its spindle is a loose non-return valve Q. The valve S is released to fall at the same time as the admission valve opens, but the valve Q prevents any inlet into the top of the bell. The action is as follows. When explosion occurs the bell is full of water and all valves are shut. Expansion below the open bottom of the bell allows the water to escape from the bell, which becomes filled with burnt products. On the first instroke of the water the exhaust valve E is found open as usual and allows the products to

escape, but valve S is shut, so that the burnt products in the bell are trapped, and suffer compression by the rising water, which clears away all the other burnt products outside the bell. The compressed cushion in the bell now expands, and the second outstroke occurs, during which the suction opens the admission valve A to let in the fresh charge as usual, and in addition valve S opens.

The second outstroke being shorter than the first does not lower the water to the bottom of the bell, and the pure combustible charge outside the bell does not mix with the products inside. Lastly, the second instroke occurs, and the water rising in the bell forces the exhaust gases out past the non-return valve Q until valve S is reached. The water then shuts valve S, and the return column of water having gained velocity, depending on the volume of gases discharged from the bell, compresses the pure combustible charge outside the bell. Since the volume of the bell is not limited, the water column may attain a high velocity, and so give a high compression pressure, thus ensuring good efficiency.

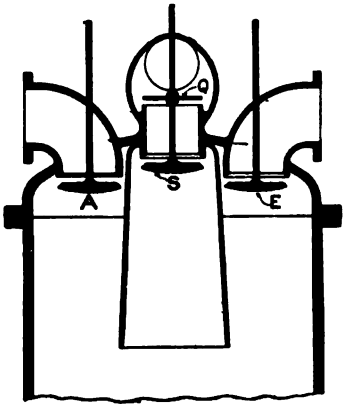


FIG. 10.

One other important modification of the single-barrel pump depends for its successful operation upon the fact that in the Humphrey pump the burnt gases are so well cooled at the end of the long expansion stroke that they will not ignite an incoming combustible charge. It therefore becomes possible to have a true two-cycle pump, and the action will be understood by reference to Figs. 10A and 10B, which show alternative arrangements of the top of the combustion chamber. Dealing first with Fig. 10A, the combustion chamber has to be specially shaped so that the incoming charge, which may be preceded by pure air, displaces the burnt products and mixes as little as possible with them.

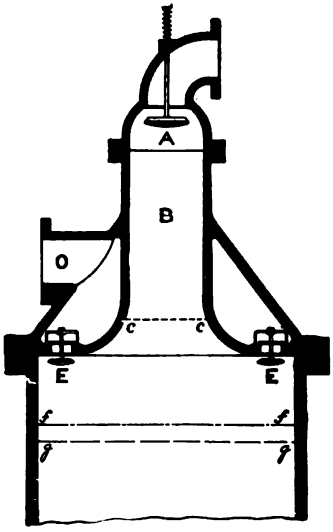


FIG. 10A.

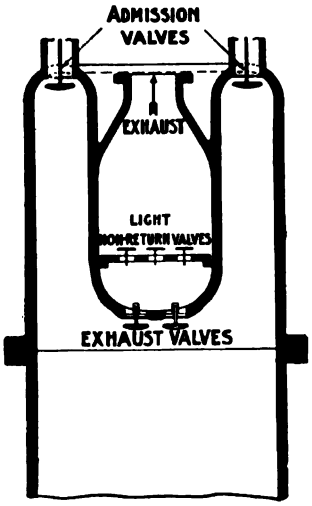


FIG. 10B.

Thus, in the figure, A is the admission valve at the top of a tall but narrow part of the chamber B, in which the full charge volume extends down to the level *cc*. A number of exhaust valves E lead to a common exhaust outlet O, which may be fitted with a non-return valve, or each exhaust valve may carry a light non-return valve on its spindle as shown. The level at which

expansion reaches atmospheric pressures is say *ff*, but this level having been reached by the water, its further movement draws in fresh combustible mixture till it occupies the space down to *cc*, and the liquid level has fallen to *gg*. The column of liquid now returns and drives the exhaust products through the valves E (which opened by their own weight) until these valves are shut by the water. The kinetic energy acquired by the column is now spent in compressing the fresh charge, which is ignited to start a new cycle. Thus each outstroke is a working stroke, and no locking gear is required on the valves.

The same cycle applies to Fig. 10B, but in this case there are a number of admission valves placed in a ring so as to allow the mixture to enter with a low velocity, in order to prevent eddies and mixing with the exhaust products.

The advantages of the cycle will appeal to the reader at once, as a higher compression pressure is obtained than with the simple pump, and consequently higher efficiencies with the same lift.

Before leaving the simple pump it is necessary to call attention to the fact that if a pump when working is stopped by switching off the ignition current, it will continue to complete its cycle up to the point of compressing the new charge, and will pull up, after a few oscillations of the water column, with the fresh charge remaining ready for ignition, to again start the pump when the current is switched on. In starting up from "all cold" a single charge of air is introduced against the head of water by a hand pump or small compressor, and the exhaust valve is opened by a hand lever to allow the air to escape. This starts the oscillation of the column which automatically shuts the exhaust valve, cushions, draws in a new charge, compresses and fires it, and so the pump gets away under load.

Coming now to the double-barrel pump, diagrammatically illustrated in Fig. 11, we have two combustion chambers, A and B, in which explosion occurs alternately. In each chamber we have the usual cycle of compression, explosion, expansion, exhaust, and the taking in of the new charge, thus again involving four strokes of unequal length. In this case, as in that of Fig. 10A, the water column has only one outward and one inward movement per cycle, but the expression "two-stroke" would here be misleading, unless one remembers that it applies only to the water column, and does not refer to what occurs in the combustion chambers.

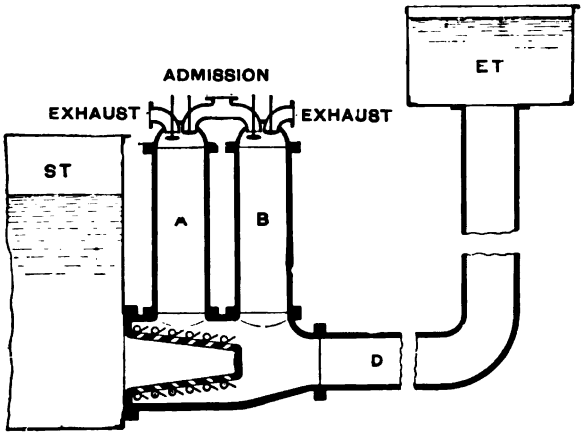


FIG. 11.

Let us follow through the cycle in this case, and we will assume that a compressed combustible charge exists in the top of chamber A, and that B is full of water. Explosion and expansion in A cause the outward propulsion of the water column, and when atmospheric pressure is reached the exhaust valve of A opens, water flows in to follow the column and to rise in A expelling burnt products, and there being insufficient pressure to maintain the water in B the level therein falls, causing an intake into B of a fresh combustible charge. All this takes place during the outward movement of the column of water, and then the column coming to rest commences to return towards the chambers. It finds that the remaining burnt products in A can be readily sent out through the still open exhaust valve, whereas the new charge in B cannot escape as the admission valve which admitted the charge is now shut under the action of its spring. Consequently the column rises at first in A until the exhaust valve is shut, and then in B compressing the new charge. Ignition in B starts a fresh cycle with the functions of A and B reversed.

There are important differences to be noted between the action of this pump and of the single-barrel pump first described. To begin with, the new charge is taken in merely by water falling

under the action of gravity, and does not depend upon the compression and expansion of an elastic cushion. The clearance spaces at the tops of the combustion chambers may therefore be reduced to the very small volume needed to bring easily to rest that portion only of the column which rises in the chamber; the main portion of the column being brought to rest by the compression of the new charge. Again, for a given height of lift the compression pressure attained is higher in the double-barrel pump, because the returning column gains speed while exhausting the last part of the burnt products, and has thus obtained a store of kinetic energy before starting the compression stroke proper.

The advantages of the two-barrel over the simplest of the single-barrel pumps may be briefly stated :—

1. Higher explosion pressure for the same water lift, due to (*a*) reduced clearance resulting in less contamination of the new charge by residual burnt products, and (*b*) higher compression pressure.
2. More cycles possible in unit time.
3. Greater output for a given total weight of pump and play pipe.

There are ways and means of making a single-barrel pump operate on a four-stroke gas and a two-stroke water cycle with many of the advantages pertaining to the two-barrel pump, and although interesting experiments have been made by running a single-barrel pump in such manner, yet it is impossible to deal with them in this paper, and indeed, it is necessary to omit all reference to very many other types of pumps which the author has designed.

So far we have considered only those pumps which have a drowned suction, and a limited lift of 40 ft. We will now take a brief glance at a few of the types of Humphrey pumps capable of working with a suction lift, and then we will come to high-lift pumps.

Fig. 12 is a diagram of one kind of suction lift pump, and the cycle is as follows :—

1. Explosion of compressed combustible charge in A driving water to high-level tank K, continuation of expansion stroke below atmosphere owing to the existence of non-return valve in exhaust pipe. Consequent lowering of pressure till water is raised from low-level tank H, past valve G, until water comes to rest and valve G shuts.
2. Return movement of water column towards chamber A giving exhaust and cushion stroke.
3. Cushion expansion, and inlet of a measured combustible charge into A, and expansion of this charge below atmosphere.
4. Second return of water column towards A giving compression stroke.

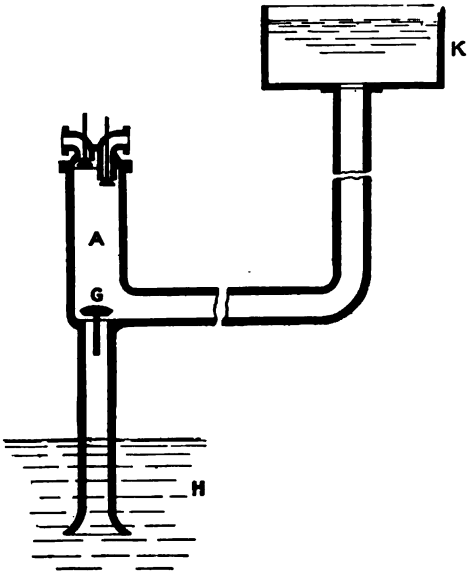


FIG. 12.

The measured charge involved in step 3 of the cycle may be obtained in many ways, one of the simplest being by the use of gas bags, the diaphragms of which are attached to valves, and on being drawn in actuate the valves to close the outlet. The use of a measuring device can, however, be obviated by employing the "rejected charge" device to be described in connection with air compressors.

Another suction pump is illustrated by diagram in Fig. 13. The combustion chambers of A and B of a two-barrel pump are connected to the play pipe D, but the water suction pipe is at the opposite end of the play pipe, and in addition there is an air vessel C, and valves N and M, as shown. The cycle will be readily surmised. On explosion in A the water column moves outwardly from the chambers and compresses the air in C until the pressure is sufficient to lift valve

N and pass water into the high-level tank. On the column coming to rest valve N shuts and the compressed air in C expands again, imparting velocity to the column, and enabling the expansion to pass below atmospere, and so lift water from the low-level tank at the same time that exhaust in A and compression of the new charge in B take place. The cycle is then repeated.

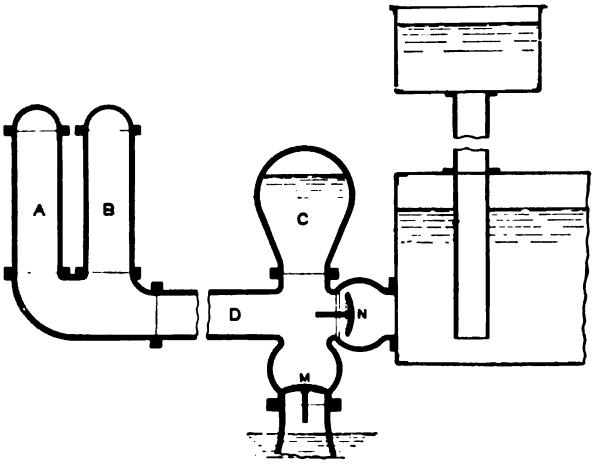


FIG. 13.

A single-barrel pump will operate in a similar manner, but the arrangement of Fig. 14 necessitates two extra strokes, and the cycle is as follows:—

(*a*) refers to what occurs at pump.
(*b*) refers to what occurs at air vessel.

1st Outstroke (caused by explosion in 1).
(*a*) Explosion in 1 followed by expansion to atmospheric pressure, then continued movement taking in scavenging air till column comes to rest.
(*b*) Compression of air in 2 until delivery pressure is reached, then water is delivered through valve 9 until column comes to rest.

1st Instroke (caused by expansion in 2).
(*a*) Exhaust until valve 6 is shut by the water, then compression of cushion which remains.
(*b*) Expansion of air in 2 giving velocity to water column. When partial vacuum equal to suction lift is attained, water is sucked up past valve 10 to follow moving water.

2nd Outstroke (caused by expansion of cushion in 1).
(*a*) Expansion of cushion in 1 and intake of combustible mixture in excess, the first portion being taken from 8 through valve 7, and the remainder through admission valve 5.
(*b*) Compression of air in 2 but no delivery of water.

2nd Instroke (caused by expansion in 2).
(*a*) Rejection of surplus charge past valve 7 until water shuts this valve. Compression of charge which remains.
(*b*) Expansion of air in 2.

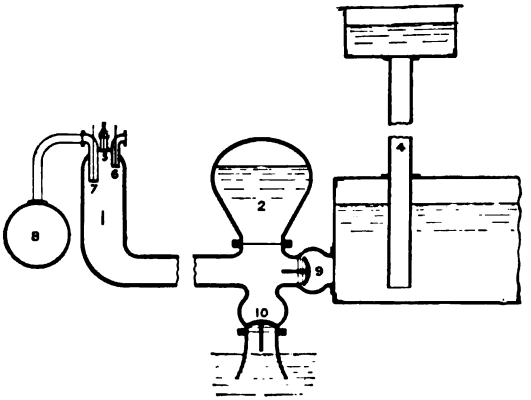


FIG. 14.

Note.—For clearness the usual scavenging valve on pump is not shown. Exhaust and scavenging valves are released (or locked shut) together. Admission and rejected-charge valves are released (or locked shut) together.

Yet another suction pump is shown in Fig. 15, the arrangement being suitable for pumping from a well. Explosion in A drives the water down one leg of the U pipe and up the other leg into the delivery tank. Expansion below atmosphere causes an intake of water through the suction valves in the well, then the first return movement occurs giving the exhaust and cushion stroke, followed

by the cushion expansion and gas suction stroke, and lastly the compression stroke. Many other varieties will probably suggest themselves to the reader, but it is only necessary to describe one more type, which is illustrated by Fig. 16, and is interesting as giving the same high efficiency with a low lift as can be obtained with a high lift.

In this case the energy of water falling to a lower level is used to cause the intake of fresh water, and the pump barrel is placed at a low level to render this possible. Thus in the figure we have a single combustion chamber A, and upon explosion water is forced outwardly, the whole column from the pump to the high-level tank partaking of the motion. The two parts of the column *a*, from pump to valve V, and *b* from valve V to high-level tank ET are so proportioned that the *a* portion loses it kinetic energy first, and is ready to return towards the pump before the *b* portion has come to rest. When the *a* portion flows back downhill fresh water is taken in at valve V between the two portions of the column, and the exhaust and cushion strokes occur in A. Expansion of the cushion in A gives the usual outstroke taking in gas and air, and the cycle is completed by a second instroke which compresses the combustible charge in A. In this last stroke either the *a* portion or the whole column may take part, depending on the proportions of *a* and *b* and the differences of levels. The chief point to observe is that while water is only raised from

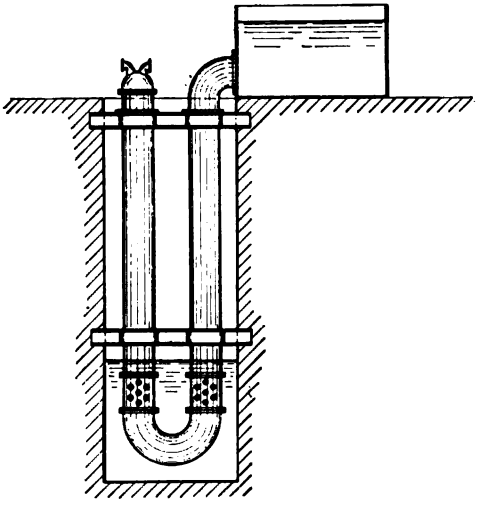


FIG. 15.

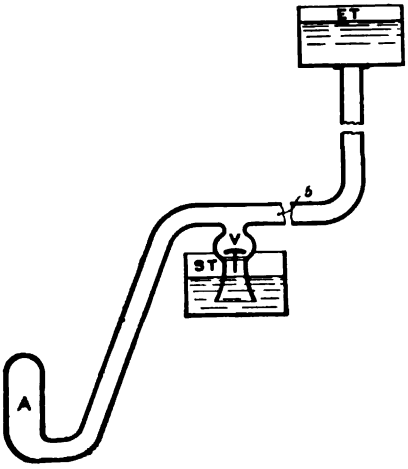


FIG. 16.

tank ST to tank ET, yet the fall to the pump may be sufficient to give a good compression pressure, and thus secure high efficiency.

Coming now to high-lift pumps, any Humphrey pump, whether single or double barrel, may be converted into a high-lift pump by means of an air vessel fitted with valves and called an "intensifier." The idea is to first allow the water column to gain velocity and then to utilize its kinetic energy to (*a*) compress an elastic fluid, (*b*) deliver water under the pressure to which the elastic fluid has been compressed. Fig. 17 will serve to make the cycle clear.

A and B are the barrels of a two-barrel pump, and at the end of the play pipe D there are two air vessels E and F, the latter being large enough to give a continuous flow at outlet O, and to maintain a practically uniform pressure. The smaller air vessel E is fitted with a downwardly projecting pipe K, open to the atmosphere at the top and carrying a valve L at its lower extremity, arranged to close under the action of the rising water. The cycle starts with explosion, all valves except L being shut and the water level as shown. While the water level in E is rising to L, air is merely being discharged into the atmosphere, and as no work is being done by the column of water it gains speed until valve L is shut by impact. Imprisoned in E there is now a definite quantity of air, which suffers compression until its pressure reaches that at which the high-pressure water valves W can open and allow the remaining kinetic energy of the column to force water into F. Valves

W close when the column comes to rest, but there remains enough energy in the compressed air in E to give, by expansion, the return flow, which causes exhaust in A and compression of the fresh charge in B to start a fresh cycle. When the water level falls below valve L, this valve opens and air is admitted into E for the rest of the return stroke.

Now, it is easy to see that if the pipe K is made vertically adjustable with regard to E, the point of the cycle at which L shuts can be varied, and more or less air entrapped in E at will. But the amount of energy stored in this air will also vary with its quantity, for we assume that the degree of compression remains constant, and is indeed fixed by the pressure maintained in F. Consequently the ratio of the total energy of the working stroke to the energy stored in the compressed air in E can be made anything desired, or, in other words, we can obtain any compression pressure of the new charge in B which we like, and this independent of the water lift. The advantage is obvious for compression pressures equal to those in modern gas engines can be employed with a corresponding increase in thermal efficiency. Further, by manipulating the position of pipe K a given pump can be made to meet any conditions as to height of lift, for if the lift increases K can be raised so that the energy stored in the air in E remains the same, there being now less air but at a higher pressure.

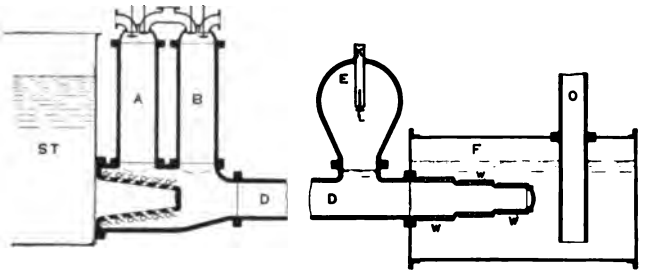


FIG. 17.

A short digression may now be made to describe an important development of the arrangement shown in Fig. 17, and the reader's notice is directed to the fact that at each cycle air is drawn into and rejected from the vessel E. Let us suppose K is connected to a supply of combustible mixture instead of opening into the atmosphere, we shall then have an automatic pump for taking in mixture and discharging it under pressure. If the discharge is into a reservoir from which combustion chambers A and B can be supplied, we have at once a means of quickening the cycles and greatly increasing the output of a given size apparatus. It is convenient to replace vessel E by two vessels, one for air and one for gas, so as to maintain the combustible constituents separate until they enter the combustion chambers. If the first portion of the outstroke of the water column is allowed to reject the surplus air and gas back to the sources of supply, then the action throughout the cycle is precisely that described when using the single vessel E, except that a larger proportion of the total energy is absorbed in the compression of air and gas, but the excess is given out again during the expansion of the pre-compressed charge in either A or B. The chief advantage arises from the more rapid working, as there is no longer any need to wait for the water level in A or B to fall under the action of gravity when the charge is being taken in. In fact, the apparatus becomes practically independent of the water level on the supply side. The 1000 HP Humphrey pump now under construction in Germany will operate in the manner just described, the result being that the dimensions are very moderate, and the pump itself occupies no more space than a 1000 HP tandem gas engine.

The limits of this Appendix preclude any reference to single-barrel high-lift pumps, other than that shown in Fig. 14, or to other modifications of the double-barrel type to suit special conditions, as there still remains the important subject of air compressors to be considered.

If the column of water oscillating in the play pipe of a Humphrey pump is used as a water piston, and caused to rise and fall in an air vessel fitted with suitable valves for the inlet and outlet of air, the combination constitutes an air compressor of a very efficient type and promising many advantages. Let us take the case of a single-barrel pump and a single air vessel, as illustrated in Fig. 18. A is the ordinary pump chamber and C the air compressor chamber, and we may call them for convenience the "pump" and the "compressor" respectively. The inlet valve *i*, for gas and air, and the exhaust valve *e*, for burnt products, exist as before, but the exhaust pipe and valve are adjustable vertically so as to vary the cushion space in the top of A. There is another pipe *p*, fitted with a valve *r*, and also adjustable vertically. A reservoir X, such as an old boiler shell, is connected by a flexible pipe to *p*. Corresponding parts are fitted to the compressor; thus *f* is an air inlet valve, *g* an outlet valve for air which is shut by the water, and *h* a non-return valve for the compressed air. The dip pipe *q*, with its valve *s*, gives communication between the atmosphere

and the compressor. The outlet pipe and the dip pipe are vertically adjustable as in the pump. Lastly, suppose the play pipe D filled with water, then we have all the elements required for working under the most varied conditions.

The cycle can be explained by assuming the water level is well up in A and low in C, as shown in the figure, and that a compressed charge is ignited in A. Expansion occurs, driving down the water in A and up in C. At first air escapes from C through valve *s*, but when the water reaches this valve it is shut and the remaining air is trapped, and suffers compression until the pressure of discharge is reached, when *h* opens and compressed air is delivered. Next the water reaches valve *g* and shuts it so that no more air can escape, and the water column is brought to rest by the continued compression of the remaining air, which forms a cushion. Meanwhile in chamber A expansion to atmosphere allows the exhaust valve *e* to open, and the continued stroke draws in scavenging air (the scavenging valve being omitted from the figure for the sake of simplicity), but valves *i* and *r* remain locked. Now the column begins to return, due to the expansion of the compressed cushion in C, and this causes exhaust and cushion in A, and at the same time there is a fall of pressure in C to atmosphere, then valve *f* is opened against a light spring and a fresh supply of air enters.

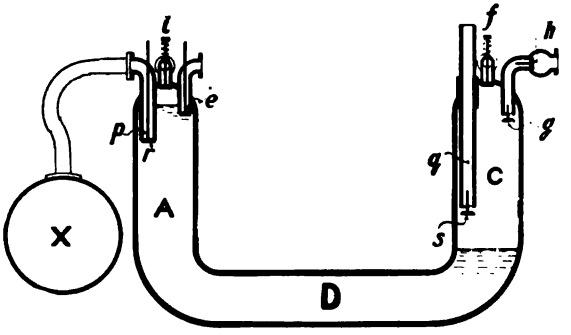


FIG. 18.

So far we have had one outward and one return stroke, but the expansion of the cushion in A starts a second outstroke which results in drawing in a new combustible charge in A through valves *i* and *r*, which were released on the shutting of the exhaust valve *e*. As the second outstroke is nearly as long as the first, it follows that too much combustible mixture is drawn in, but the surplus is got rid of on the second return stroke by allowing it to escape into the receiver X, and to slightly raise the pressure therein. Valve *i* was shut by its spring, but *r* waits to be shut by the action of the water, and when this occurs chamber A contains a definite volume of combustible mixture, the size of charge being fixed by the height of *r*. Finally we have only to observe that the second outstroke towards C merely compressed the air in C without causing delivery to take place, but storing in the air the energy given out by the expansion of the cushion in A. The air in C now expanding gives the compression stroke in A, and the combustible charge is ready to be ignited to start the next cycle.

Perhaps the whole cycle, which may take two seconds to accomplish, sounds a little complicated, but if so, it is merely due to the length of the description. It may be easier to grasp the action if it is set out to show the simultaneous steps in A and C, thus:—

	A.	C.
1st Outstroke.	Expansion to atmosphere. Intake of scavenging air.	Expulsion of air till water shuts valve *s*. Compression and discharge of compressed air till water shuts valve *g*. Cushion till water comes to rest.
1st Instroke.	Exhaust till water shuts valve *e*. Cushion till water comes to rest.	Expansion of cushion to atmosphere. Intake of fresh air.
2nd Outstroke.	Expansion of cushion to atmosphere. Intake of combustible charge in excess.	Compression of air, but not sufficient for further delivery.
2nd Instroke.	Rejection of surplus charge till water shuts valve *r*. Compression of charge till water comes to rest.	Expansion of compressed air.

It will be evident that in the next cycle when admission of combustible charge occurs, the surplus last rejected into reservoir X will want to enter first, because it is under slight pressure, whereas inlet valve *i* has to be sucked open against its spring. Consequently X will empty to atmospheric pressure each time, and only the additional mixture required will be taken in through valve *i*. A slight modification may be made by transferring valve *i* from the pump to the reservoir X.

The flexibility of the air compressor can now be studied. To begin with the pump side, the level of the inlet valve *e*, and the rejected charge valve *r*, are assumed to be variable although Fig. 18, being merely a diagram, does not show how the pipes carrying these valves are moved vertically. As the level of these two valves controls the amount of charge ignited at each cycle, and the amount of the cushion space, their regulation is all that is required to increase or diminish the energy developed per working stroke. On the compressor side the position of the valves *g* and *s* controls the cycle of operations on this side of the apparatus, and renders it possible to compress a large volume of air to a low pressure, or a smaller volume of air to a high pressure, or to make any intermediate changes which may be desired.

Thus all the conditions of output, up to the full limit of the compressor, may be governed at will, and for all ranges the compression pressure of the new charge may be kept up to the required degree, so that the apparatus works at its maximum efficiency throughout the whole range. The amount of water which oscillates between the chambers should theoretically be altered along with the total capacity per working cycle, but the reason for this is merely to prevent the last portion of each down stroke from being wasted by taking in surplus combustible mixture in one chamber, or surplus air in the other chamber, to an undue extent. If the surplus of the combustible mixture is unnecessarily large, the extra amount rejected will increase the pressure in the reservoir X, and this increase of pressure may be made to automatically govern a water supply, and so bring up the total volume of reciprocating water, or to allow part of the water already in the apparatus to escape, so as just to keep a small amount of excess charge for each cycle, no matter what may be the output of the pump.

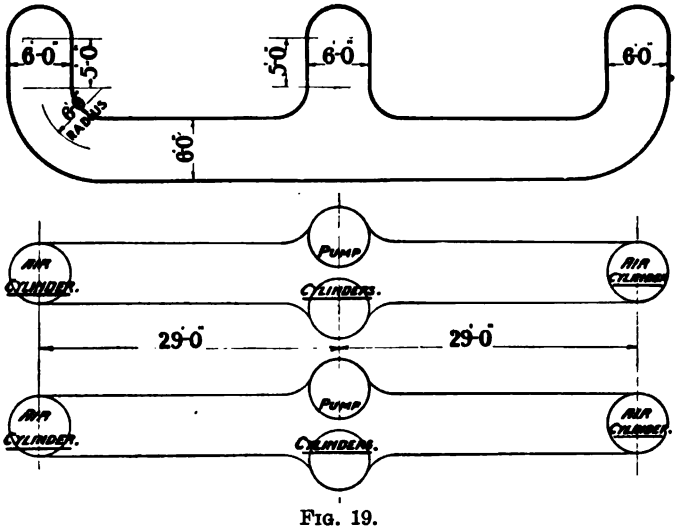


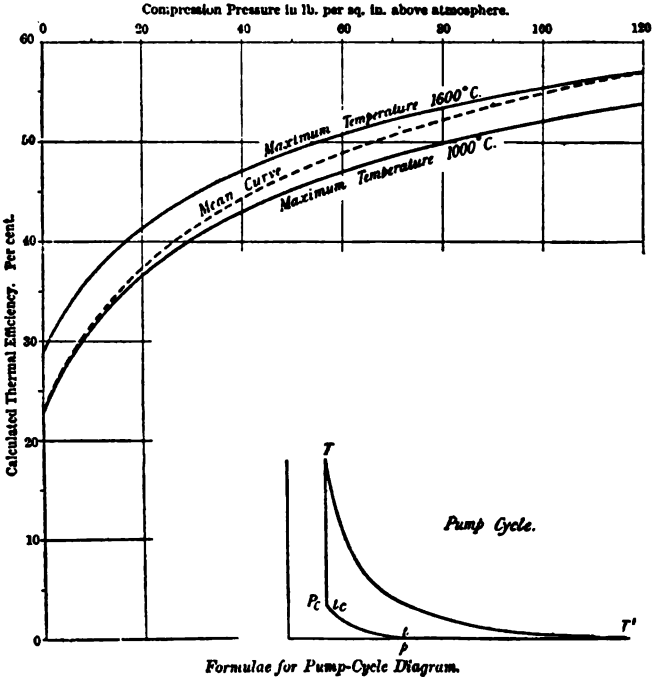
FIG. 19.

To illustrate the importance of the air compressor, and at the same time bring to the reader's notice the use of the double-barrel pumps for this purpose, we may take a definite example, and Fig. 19 shows a compressor capable of dealing with 25,000 cubic feet of free air per minute, and compressing it to 25 lbs. per square inch. It is composed of two units, as shown in elevation and plan, and the dimensions are given. The horse-power in the air delivered is about 1500. As the pressure of delivery increases the forces acting upon the water also increase, and the accelerations become greater. Consequently for a higher pressure of delivery the size of the apparatus becomes smaller for a given output.

Usually compressed air is required to be delivered at as low a temperature as possible, so as to free it from moisture, and it may be noted that where the maximum cooling is required vertical sheets of wire gauze may be fixed in the compressor, so that the gauze takes up water on each stroke and forms a very effective cooling medium in direct contact with the air, thus permitting the compression to be more nearly isothermal than in any other kind of compressor. The reciprocating water itself can be cooled either by cooling the play pipe externally, or by allowing water to be taken in and rejected at each cycle. The latter arrangement has been worked out, but for lack of space it cannot now be described.

One other type of compressor may, however, be briefly referred to, because it differs from the other air compressors in that the pump and the compressor alternate in their functions at each cycle. It has the advantage of giving a greater horse-power output for a given size of apparatus, but it has the disadvantage that the air and the exhaust products are mixed together and delivered under pressure. For power purposes, however, this is not a drawback, and in such a case there is some advantage in delivering the compressed mixture rather hot, so that during its expansion, while performing work in some kind of air motor, there is no chance of its freezing the moisture contained, as frequently happens when the air enters the motor at normal temperatures. The thermal efficiency of such an arrangement is exceedingly high, provided that the air can be utilized close to the compressor so as to make use of its sensible heat.

Theoretical Efficiency Curves, Pump Cycle, Air Standard.



Formulae for Pump-Cycle Diagram.

$$\text{Efficiency} = 1 - y\,\frac{T^1 - t}{T - t_c}$$

$$T^1 = t\left(\frac{T}{t_c}\right)^{\frac{1}{y}}$$

$$t_c = t\left(\frac{p_c}{p}\right)^{\frac{y-1}{y}}$$

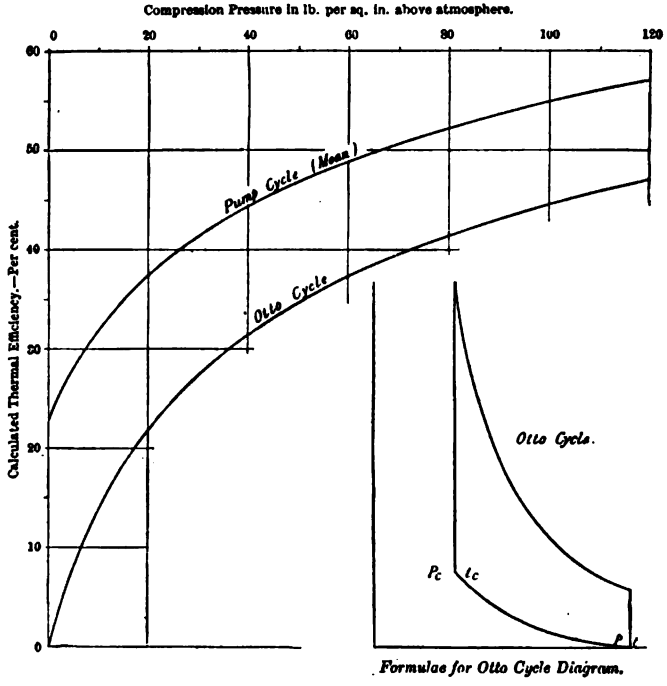
T = (for curve at 1000° C.) = 273° + 1000° C. = 1273° C. Abs.
T = (for curve at 1600° C.) = 273° + 1600° C. = 1873° C. Abs.
$y = 1{\cdot}508$.
t = Suction temperature = 17° C. = 290° C. Abs.
Adiabatic compression and expansion.

FIG. 20.

Turning now to the question of thermal efficiency, the curves shown in Figs. 20, 21, and 22 show the relative efficiency as between the Humphrey pump cycle and the ordinary Otto cycle

calculated upon the usual formula, and upon the assumptions stated on the diagrams. These calculations are based on the assumption of the constant specific heat, but Professor Dr. Eugen Meyer, of Charlottenburg, has made the very laborious calculations involved when taking the true specific heat of each constituent of the gas mixture, and allowing for the variations of the temperature involved at each point on the curve. The absolute results are affected, but the relative results remain approximately the same, and for present purposes it is sufficient to take the curves as here given.

Theoretical Thermal Efficiency Curves—Air Standard.



Formulae for Otto Cycle Diagram.

$$\text{Efficiency} = 1 - \frac{t}{t_c}$$

$$t = 273° + 17° \text{ C.} = 290° \text{ C. Abs.}$$

$$t_c = t\left(\frac{p_c}{p}\right)^{\frac{y-1}{y}}$$

$$y = 1{\cdot}408.$$

FIG. 21.

It may be mentioned, however, that with a compression pressure of eleven atmospheres absolute the theoretical thermal efficiency of the Humphrey cycle is 52·5 %, whereas that of the Otto cycle is only 40 % when all corrections for varying specific heats are allowed for. Such compression pressures have not yet been adopted by the author, but they will be employed in the 1000 HP German pump already referred to. In the meantime, when using only very moderate compression pressures of under 50 lbs. per square inch, an actual thermal efficiency of over 23 % has been obtained on a four-cycle Humphrey pump. This corresponds to 0·95 lb. of anthracite coal per water HP hour, and was obtained on a lift of only 35 ft. It may be expected therefore that as the compression pressure increases the efficiency will also increase, and there is every reason to believe that when the compression pressure reaches that employed in ordinary gas engines, the best

efficiencies attained in gas engines will be reached and probably surpassed by the Humphrey pump.

A large scale experiment is to be tried in which a Humphrey pump will be used to drive a water turbine coupled to an electric generator. The advantages of such an arrangement over the ordinary gas engine arise chiefly from the simplicity of the working parts, the absence of all troubles arising from contraction and expansion due to heating and cooling, the freedom from cyclic irregularity, thus enabling alternators to be driven in parallel, and finally, the high efficiency and the low cost of upkeep. Also, the apparatus may be started and stopped immediately, and can indeed be entirely controlled from a switchboard. One has to face the loss due to conversion of water power into mechanical energy, but the efficiency of well-designed turbines can be guaranteed at over 80 % at full load, and in a central station there is no need to run any of the pumps at less than full load, since they can be so easily started and stopped. Moreover, if blast furnace gas or producer gas derived from cheap fuel is used, the loss of 20 % of the energy is not important, as compared with the advantages gained by the adoption of the gas-hydro-electric system, and the absence of the need for any lubrication would alone more than compensate.

In conclusion, a brief reference may be made to the application of the Humphrey pump for the purposes of marine propulsion. The theoretical efficiency of the jet propeller is well known and

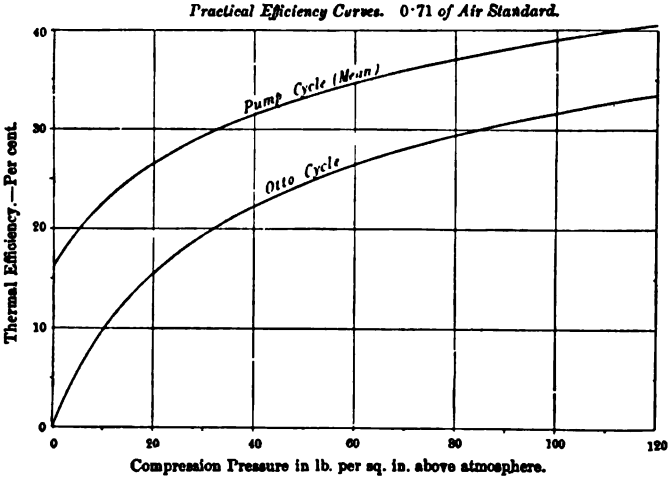


FIG. 22.

admitted. The trouble has been to find a pump which would be at once pump and engine combined, and which would handle large quantities of water so that the area of the jet at the point of exit might be comparable with the area of the water acted upon by an ordinary screw propeller. The Humphrey pump fulfils the required conditions, and the diagram of Fig. 23 shows one possible arrangement in skeleton form. The double-barrel pump with cylinders A, B discharges simultaneously in two directions through equal lengths of pipe C, D leading to air vessels E, F. The suction occurs through the side of the ship and passages G, H as shown. The water from one air vessel E is brought back to air vessel F through a pipe J, and the common discharge is then through the pipe K which opens into the surrounding water at the stern of the ship. Means for diverting the jet as it leaves the vessel either to the right or to the left, or through a complete turn of 180° so as to drive the ship astern, are not shown, as they have been worked out by many experimenters.

The arrangement indicated in the figure prevents all vibration, since the stresses, due to the moving water, are equal and opposite, the pump being situated in the centre, between the air vessels. Also, the air vessels keep the discharge continuous, so that there are no jerks, the result being a system in which full power, either ahead or astern, is available instantly, and the ship so fitted has a radius of action, owing to the fuel economy, equal to at least twice that which can be obtained by any other system. Since jet propellers become more wasteful with increasing velocities of

propulsion, it may be found advisable to limit the application of the system now proposed to cargo steamers of moderate speed, but for such vessels requiring, say, 2000 to 3000 HP the scheme contains all the elements of success. It may eventually prove that a pump having pre-compression of its gas and air will be more suitable for the larger sizes, and, when one remembers the small space occupied by two or three 1000 HP pumps of this type, the outlook is distinctly promising.

The Metropolitan Water Board have to instal a very large pumping plant at their new reservoir at Chingford, and the chief engineer and a committee of the board have for some time past been studying most carefully several schemes submitted to them with the corresponding tenders for

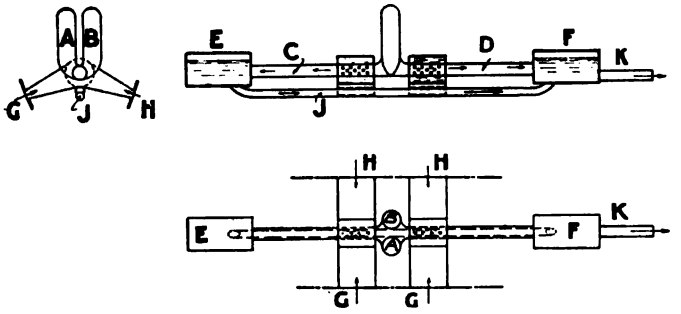


FIG. 23.

different types of plant. As the result of a most thorough investigation the committee unanimously reported in favour of the adoption of the Humphrey pump, and at the meeting of the board on November 4, 1910, the committee's recommendation was approved. The report showed for these pumps a saving in first cost of £19,000 on the complete installation, including buildings and foundations, as compared with triple expansion steam engines running at 350 revolutions per minute and coupled to centrifugal pumps. Moreover, there is a great saving in fuel consumption and running expenses, since the coal consumed is only about half that of the steam plant. Gas engines and oil engines driving centrifugal pumps were also considered.

The quantity of water to be pumped is 180 million gallons per 24 hours, and the lift is 25 to 30 feet. For this duty there will be erected five Humphrey pumps, four of which will each have a capacity of 40,000,000 and one 20,000,000 gallons per day.

APPENDIX II.

USEFUL TABLES.

TABLE I.

STRENGTH AND WEIGHT OF METALS.

Metal.	Tearing Force in Lbs. per Sq. In.	Crushing Force in Lbs. per Sq. In.	Shearing Force in Lbs. per Sq. In.	Elastic Strength in Lbs. per Sq. In.	Weight of a Cubic Inch in Lbs.	Weight of a Cubic Foot in Lbs.
Cast iron	18,000 to 20,000	90,000	10,000	10,500	0·26	450
Steel castings	67,000	—	—	35,000	0·284	490
„ forgings	63,000	—	—	—	0·282	487
Iron „	45,000 to 50,000	—	—	—	0·28	485
Wrought iron bars, BBI.	58,000	50,000	40,000	29,000	0·28	485
Nickel steel (3 per cent. nickel)	112,000	—	—	62,700	—	—
Gun metal	27,000	—	—	6,500	0·315	545
Phosphor bronze (cast)	35,000	—	—	19,000	—	—
Manganese bronze (cast)	55,000	—	—	30,000	0·31	535

TABLE II.

BOLTS AND NUTS.

TABLE OF BRITISH STANDARD HEXAGONAL BRIGHT AND BLACK NUTS, AND BOLT-HEADS.*

(*Abbreviated.*)

Diameter of Bolt.	Nuts.					Lock Nuts.		Bolt-Heads.	
	Width across Flats.		Width across Corners.	Thickness.		Thickness.		Thickness.	
	Max.	Min.	Approximate Max.†	Max.	Min.	Max.	Min.	Max.	Min.
Ins.	Ins.	Ins.	Ins.	Ins.	Ins.	Ins.	Ins.	Ins.	Ins.
¼ (0·25)	0·525	0·520	0·61	0·26 *0·27*	0·25	0·18 *0·19*	0·17	0·23 *0·24*	0·22
⅜ (0·375)	0·710	0·705	0·82	0·39 *0·40*	0·38	0·26 *0·27*	0·25	0·34 *0·35*	0·33
½ (0·5)	0·920	0·915	1·06	0·51 *0·52*	0·50	0·34 *0·35*	0·33	0·45 *0·46*	0·44
⅝ (0·625)	1·100	1·092	1·27	0·64 *0·65*	0·63	0·43 *0·44*	0·42	0·56 *0·57*	0·55
¾ (0·75)	1·300	1·292	1·50	0·76 *0·77*	0·75	0·51 *0·52*	0·50	0·67 *0·68*	0·66
⅞ (0·875)	1·480	1·472	1·71	0·89 *0·90*	0·88	0·59 *0·60*	0·58	0·78 *0·79*	0·77
1	1·670	1·662	1·93	1·01 *1·02*	1·00	0·68 *0·69*	0·67	0·89 *0·90*	0·88
1¼ (1·25)	2·050	2·040	2·37	1·27 *1·28*	1·25	0·85 *0·86*	0·83	1·11 *1·12*	1·09
1½ (1·5)	2·410	2·400	2·78	1·52 *1·53*	1·50	1·02 *1·03*	1·00	1·33 *1·34*	1·31
1¾ (1·75)	2·760	2·750	3·19	1·77 *1·78*	1·75	1·19 *1·20*	1·17	1·55 *1·56*	1·53
2	3·150	3·140	3·64	2·02 *2·03*	2·00	1·35 *1·36*	1·33	1·77 *1·78*	1·75
2¼ (2·25)	3·550	3·535	4·10	2·27 *2·28*	2·25	1·52 *1·53*	1·50	1·99 *2·00*	1·97
2½ (2·5)	3·890	3·875	4·49	2·52 *2·53*	2·50	1·69 *1·70*	1·67	2·21 *2·22*	2·19
2¾ (2·75)	4·180	4·165	4·83	2·77 *2·78*	2·75	1·85 *1·86*	1·83	2·43 *2·44*	2·41
3	4·530	4·515	5·23	3·02 *3·03*	3·00	2·02 *2·03*	2·00	2·65 *2·66*	2·63
3¼ (3·25)	4·850	4·830	5·60	3·27 *3·28*	3·25	2·19 *2·20*	2·17	2·86 *2·86*	2·84
3½ (3·5)	5·180	5·160	5·98	3·52 *3·53*	3·50	2·35 *2·36*	2·33	3·08 *3·09*	3·06
3¾ (3·75)	5·550	5·530	6·41	3·77 *3·78*	3·75	2·52 *2·53*	2·50	3·30 *3·31*	3·28
4	5·950	5·930	6·87	4·02 *4·03*	4·00	2·69 *2·70*	2·67	3·52 *3·53*	3·50
4½ (4·5)	6·820	6·795	7·88	4·53 *4·54*	4·50	3·03 *3·03*	3·00	3·97 *3·98*	3·94
5	7·800	7·775	9·01	5·03 *5·04*	5·00	3·36 *3·37*	3·33	4·41 *4·43*	4·38
5½ (5·5)	8·850	8·820	10·22	5·53 *5·54*	5·50	3·70 *3·71*	3·67	4·84 *4·84*	4·81
6	10·000	9·970	11·55	6·03 *6·04*	6·00	4·03 *4·04*	4·00	5·28 *5·29*	5·25

† The figures in this column represent the maximum width across the corners correct to two decimal places. The figures in italics are those for *black* nuts.

A chamfer of 30° from the face should be adopted on all Bright Nuts and Bolt-heads. The interior diameter of the chamfer circle should not be appreciably less than the width across the flats.

In the common locking arrangement, in which a thick and thin nut are employed, the thin nut should have a thickness approximately equal to two-thirds that of the standard nut.

Castle Nuts should be provided with pin slots as shown in Fig. A.

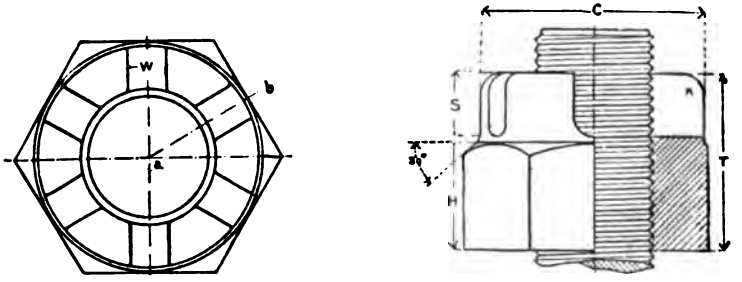


FIG. A.

T = 1·25D; H = ·75D; S = 0·4375D; C = width across flats − $\frac{1}{16}$ in.; where D = full diameter of thread on bolt.

* Abstracted by permission of the Engineering Standards Committee from their Publication [No. 28, Revised November, 1908] Report on British Standard Nuts, Bolt-heads and Spanners.

TABLE III.

Whitworth Standard Bolts and Nuts. Safe Load in Lbs.

Diameter of Bolt.	Threads per Inch.	Diam. at Bottom of Thread.	Area at Bottom of Thread.	Approximate Safe Load in Lbs.					
				At 4000 lbs. per Sq. In.	At 5000 lbs. per Sq. In.	At 6000 lbs. per Sq. In.	At 7000 lbs. per Sq. In.	At 8000 lbs. per Sq. In.	At 9000 lbs. per Sq. In.
Ins.		Ins.	Sq. Ins.	Lbs.	Lbs.	Lbs.	Lbs.	Lbs.	Lbs.
3/16	24	0·134	0·014	56	70	84	98	112	126
1/4	20	0·186	0·027	108	135	162	189	216	243
5/16	18	0·241	0·046	184	230	276	322	378	414
3/8	16	0·295	0·068	272	340	408	478	546	614
7/16	14	0·346	0·094	376	470	504	628	752	846
1/2	12	0·398	0·121	484	605	726	850	970	1,090
9/16	12	0·455	0·155	620	775	930	1,085	1,240	1,395
5/8	11	0·508	0·204	816	1,020	1,224	1,418	1,620	1,824
11/16	11	0·571	0·256	1,024	1,280	1,536	1,792	2,048	2,304
3/4	10	0·622	0·304	1,216	1,520	1,824	2,126	2,430	2,734
13/16	10	0·684	0·367	1,468	1,835	2,202	2,569	2,936	3,303
7/8	9	0·733	0·422	1,688	2,110	2,532	2,954	3,376	3,800
15/16	9	0·795	0·496	1,984	2,480	2,976	3,472	3,968	4,464
1	8	0·840	0·554	**2,216**	2,770	3,324	3,880	4,430	4,990
1 1/8	7	0·942	0·697	2,788	**3,485**	4,182	4,880	5,575	6,270
1 1/4	7	1·067	0·894	3,576	**4,470**	5,364	6,250	7,145	8,035
1 3/8	6	1·162	1·058	4,232	**5,290**	6,348	7,400	8,455	9,510
1 1/2	6	1·287	1·299	5,196	**6,495**	7,794	9,080	10,320	11,610
1 5/8	5	1·369	1·472	5,888	7,360	**8,832**	10,290	11,760	13,230
1 3/4	5	1·494	1·753	7,012	8,765	**10,518**	12,210	13,950	15,700
1 7/8	4 1/2	1·590	1·986	7,944	9,930	11,916	**13,900**	15,890	17,875
2	4 1/2	1·720	2·311	9,244	11,555	13,866	**16,170**	18,480	20,790
2 1/4	4	1·930	2·926	11,704	14,630	17,556	**20,475**	23,400	26,325
2 1/2	4	2·180	3·733	14,932	18,665	22,398	**26,125**	29,855	33,590
2 3/4	3 1/2	2·384	4·464	17,856	22,320	26,784	**31,220**	35,680	40,140
3	3 1/2	2·634	5·450	21,800	27,250	32,700	**38,080**	43,520	48,960
3 1/4	3 1/4	2·856	6·402	25,608	32,010	38,412	**44,814**	51,216	57,618
3 1/2	3 1/4	3·105	7·563	30,252	37,815	45,378	**57,940**	60,504	68,065
3 3/4	3	3·320	8·673	34,692	43,365	52,038	**60,710**	69,384	78,057
4	3	3·573	10·027	40,108	50,135	60,162	**70,190**	80,216	90,240
4 1/4	2 7/8	3·804	11·365	45,480	56,825	68,190	**79,575**	90,960	102,305
4 1/2	2 7/8	4·054	12·908	51,632	64,540	77,448	**90,356**	103,264	116,172
4 3/4	2 3/4	4·284	14·404	57,616	72,020	86,424	**100,830**	115,232	129,636
5	2 3/4	4·534	16·146	64,584	80,730	96,876	**113,020**	129,168	145,314
5 1/2	2 5/8	5·012	19·720	78,880	98,600	118,320	**138,040**	157,760	177,480
6	2 1/2	5·487	23·640	94,560	118,200	141,840	**165,200**	189,120	212,760

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Note.—In order to allow for the twisting stress to which the bolt is subjected due to screwing up (which is, of course, in addition to the tension stress), the loads printed in heavy type should not be exceeded. Table IV. gives loads which may be safely applied to bolts for most parts of gas engines, but for the bolts coupling the flanges of breech ends and water jackets and frame the stress should never exceed 5000 lbs. per sq. in. of the area at the bottom of the thread.

TABLE IIIA.

BOLTS AND NUTS.

WHITWORTH STANDARD HEXAGON BOLTS AND NUTS.

According to the Whitworth system of standard sizes of bolts and nuts, the thickness of the bolt-head is seven-eighths of the diameter, and the nut is equal to the diameter.

Dimensions in Inches.

Size of Bolt and Thickness of Nut.	Nut across Flats.	Nut across Corners.	Tapping Hole.	Thickness of Bolt-head.	Size of Bolt and Thickness of Nut.	Nut across Flats.	Nut across Corners.	Tapping Hole.	Thickness of Bolt-head.
$\frac{1}{8}$	·338	·390	·093	·1093	$2\frac{3}{8}$	4·18	4·82	2·39	2·4062
$\frac{3}{16}$	·448	·517	·134	·164	$2\frac{7}{8}$	4·34	5·02	2·5	2·5156
$\frac{1}{4}$	·525	·606	·186	·2187	3	4·53	5·23	2·63	2·625
$\frac{5}{16}$	·601	·694	·241	·2734	$3\frac{1}{8}$	4·69	5·41	2·75	2·7343
$\frac{3}{8}$	·709	·819	·295	·3281	$3\frac{1}{4}$	4·85	5·60	2·85	2·8256
$\frac{7}{16}$	·820	·947	·346	·3828	$3\frac{3}{8}$	5·01	5·78	2·98	2·9531
$\frac{1}{2}$	·920	1·06	·393	·4375	$3\frac{1}{2}$	5·17	5·98	3·1	3·0624
$\frac{9}{16}$	1·01	1·16	·455	·4921	$3\frac{5}{8}$	5·36	6·19	3·23	3·1718
$\frac{5}{8}$	1·1	1·27	·508	·5468	$3\frac{3}{4}$	5·55	6·41	3·32	3·2812
$\frac{11}{16}$	1·2	1·38	·571	·6016	$3\frac{7}{8}$	5·75	6·64	3·45	3·3906
$\frac{3}{4}$	1·3	1·5	·622	·6562	4	5·95	6·87	3·57	3·5
$\frac{13}{16}$	1·39	1·6	·684	·7064	$4\frac{1}{8}$	6·16	7·11	3·69	3·6094
$\frac{7}{8}$	1·48	1·7	·732	·7656	$4\frac{1}{4}$	6·37	7·36	3·8	3·7046
$\frac{15}{16}$	1·57	1·82	·795	·8203	$4\frac{3}{8}$	6·60	7·62	3·93	3·8271
1	1·67	1·95	·84	·875	$4\frac{1}{2}$	6·82	7·88	4·05	3·9374
$1\frac{1}{8}$	1·86	2·15	·942	·9843	$4\frac{5}{8}$	7·06	8·15	4·18	4·0469
$1\frac{1}{4}$	2·05	2·36	1·06	1·0937	$4\frac{3}{4}$	7·3	8·43	4·28	4·1562
$1\frac{3}{8}$	2·21	2·55	1·16	1·2031	$4\frac{7}{8}$	7·55	8·72	4·4	4·2656
$1\frac{1}{2}$	2·41	2·78	1·28	1·3125	5	7·8	9·01	4·53	4·375
$1\frac{5}{8}$	2·57	2·97	1·37	1·4128	$5\frac{1}{8}$	8·06	9·31	4·66	4·4844
$1\frac{3}{4}$	2·75	3·18	1·49	1·5312	$5\frac{1}{4}$	8·35	9·64	4·76	4·5936
$1\frac{7}{8}$	3·02	3·48	1·59	1·6406	$5\frac{3}{8}$	8·6	9·93	4·88	4·7031
2	3·15	3·63	1·71	1·75	$5\frac{1}{2}$	8·85	10·22	5·01	4·8124
$2\frac{1}{8}$	3·34	3·85	1·84	1·8523	$5\frac{5}{8}$	9·15	10·57	5·13	4·9218
$2\frac{1}{4}$	3·54	4·09	1·93	1·9687	$5\frac{3}{4}$	9·45	10·91	5·24	5·0312
$2\frac{3}{8}$	3·75	4·33	2·05	2·0781	$5\frac{7}{8}$	9·75	11·26	5·36	5·1406
$2\frac{1}{2}$	3·89	4·49	2·18	2·1875	6	10	11·55	5·48	5·25
$2\frac{5}{8}$	4·05	4·67	2·30	2·2968					

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TABLE IV.

Safe Loads on Studs and Bolts.

Diameter of Stud or Bolt.	Safe Load that one Stud or Bolt will carry in Lbs. (B.B.I. or Mild Steel).	Diameter of Stud or Bolt.	Safe Load that one Stud or Bolt will carry in Lbs. (B.B.I. or Mild Steel).
Ins.	Lbs.	Ins.	Lbs.
$\frac{1}{2}$	250	$2\frac{1}{2}$	26,100
$\frac{5}{8}$	500	$2\frac{3}{4}$	31,200
$\frac{3}{4}$	900	3	38,100
$\frac{7}{8}$	1,500	$3\frac{1}{4}$	44,800
1	2,150	$3\frac{1}{2}$	53,000
$1\frac{1}{8}$	3,000	$3\frac{3}{4}$	60,500
$1\frac{1}{4}$	4,250	4	70,100
$1\frac{3}{8}$	5,500	$4\frac{1}{4}$	79,500
$1\frac{1}{2}$	7,000	$4\frac{1}{2}$	90,300
$1\frac{5}{8}$	8,500	$4\frac{3}{4}$	100,800
$1\frac{3}{4}$	11,000	5	113,000
$1\frac{7}{8}$	13,000	$5\frac{1}{4}$	124,600
2	16,000	$5\frac{1}{2}$	138,000
$2\frac{1}{4}$	20,400		

TABLE V.

Pipe Threads.

Whitworth's Standard Threads for Gas and Water Pipes.

Dimensions in Inches.			Number of Threads per Inch.	Dimensions in Inches.			Number of Threads per Inch.
Internal Diameter of Pipe.	External Diameter of Pipe.	Diameter at Bottom of Thread.		Internal Diameter of Pipe.	External Diameter of Pipe.	Diameter at Bottom of Thread.	
$\frac{1}{8}$	·382	·336	28	$1\frac{7}{8}$	2·245	2·128	11
$\frac{1}{4}$	·518	·451	19	2	2·347	2·23	11
$\frac{3}{8}$	·656	·589	19	$2\frac{1}{8}$	2·467	2·351	11
$\frac{1}{2}$	·826	·734	14	$2\frac{1}{4}$	2·587	2·47	11
$\frac{5}{8}$	·902	·811	14	$2\frac{3}{8}$	2·794	2·678	11
$\frac{3}{4}$	1·04	·943	14	$2\frac{1}{2}$	3	2·882	11
$\frac{7}{8}$	1·189	1·097	14	$2\frac{5}{8}$	3·124	3·009	11
1	1·309	1·192	11	$2\frac{3}{4}$	3·247	3·13	11
$1\frac{1}{8}$	1·492	1·375	11	$2\frac{7}{8}$	3·367	3·251	11
$1\frac{1}{4}$	1·65	1·533	11	3	3·485	3·368	11
$1\frac{3}{8}$	1·745	1·628	11	$3\frac{1}{4}$	3·698	3·581	11
$1\frac{1}{2}$	1·882	1·765	11	$3\frac{1}{2}$	3·912	3·795	11
$1\frac{5}{8}$	2·022	1·965	11	$3\frac{3}{4}$	4·125	4·008	11
$1\frac{3}{4}$	2·16	2·042	11	4	4·340	4·223	11

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FUELS.

The Heat developed by Fuels in burning, may be ascertained by calculation when their chemical composition is known. The chemical composition of various combustibles is given in the following Table.

TABLE VI.

Chemical Composition of Combustibles.

Description.	Carbon.	Hydrogen.	Oxygen.	Ash.
Coke, average of 25 analyses ...	0·935	—	—	0·065
Charcoal, from wood, average ...	0·925	—	—	0·075
Oil of turpentine	0·882	0·118	—	—
Pine-wood oil	0·867	0·106	0·027	—
Paraffin, or petroleum oil	0·855	0·145	—	—
Petroleum, average	0·840	0·110	0·050	—
Creosote, or tar-refuse	0·825	0·100	0·075	—
Spermaceti	0·817	0·123	0·060	—
Beeswax	0·815	0·139	0·046	—
Coal, average	0·804	0·052	0·079	0·041
Schist oil	0·800	0·117	0·083	—
Resin	0·791	0·102	0·107	—
Asphalte	0·790	0·095	0·087	0·028
Tallow	0·788	0·118	0·094	—
Sperm oil	0·787	0·110	0·103	—
Olive oil	0·770	0·134	0·096	—
Lignite, bituminous	0·749	0·075	0·130	0·046
Lignite, perfect	0·690	0·050	0·202	0·058
Lignite, imperfect	0·602	0·053	0·290	0·055
Peat, perfectly dry	0·600	0·060	0·300	0·040
Peat, dry, average	0·570	0·060	0·320	0·050
Peat, dry, medium quality	0·460	0·050	0·437	0·053
Alcohol	0·520	0·135	0·345	—
Wood-pimps, or bundles of twigs, dry	0·510	0·060	0·400	0·030
Wood, perfectly dry, average ...	0·500	0·060	0·418	0·022
Wood-bark, dry	0·477	0·063	0·432	0·028
Wood, pine, dry	0·445	0·055	0·485	0·015
Wood, oak, dry	0·430	0·053	0·496	0·021
Sawdust, dry	0·410	0·043	0·531	0·016
Straw, barley, dry	0·380	0·054	0·510	0·056
Straw, wheat, dry	0·360	0·050	0·540	0·050
Straw, oat, dry	0·348	0·050	0·557	0·045

Heating Power of Carbon.—One lb. of carbon burning to carbonic acid develops 14500 units of heat according to Peclet, and 12906 units of heat according to Dulong.

One lb. of hydrogen develops 62535 units of heat; the heat developed by hydrogen is $62535 \div 14500 = 4{\cdot}31$ times as great as that of carbon.

One lb. of hydrogen requires 8 lbs. of oxygen.

Atmospheric air contains 20 per cent. of oxygen.

The Heat developed by a Combustible, or its Calorific Power, may be calculated from its chemical constituents by this formula:—

$$\text{Units of heat} = (14500 \times \text{percentage of carbon}) + \left\{62535 \times \left(\% \text{ hydrogen} - \frac{\% \text{ oxygen}}{8}\right)\right\}.$$

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Fuels—*continued.*

Gaseous Fuel.—The composition of average coal-gas is as follows:—

Hydrogen	51·00
Marsh-gas	35·30
Carbon oxide	7·58
Olefines	3·55
Nitrogen	2·27
Oxygen	0·30
	100·00

The Maximum Heating Power of Average Coal-Gas has been found by experiment to be 700 heat-units per cubic foot of gas. The heating power of ordinary 16-candle-power coal-gas is 630 units of heat per cubic foot. As 30 cubic feet of coal-gas at 62° Fahr. = 1 lb., the maximum heating power of 1 lb. of coal-gas is $700 \times 30 = 21000$ heat-units, or more than 50 per cent, greater than that of coal of average quality.

TABLE VII.

Quantity of Gas in Cubic Feet obtained from Coal.

Description of Coal.	Specific Gravity of Gas.	Cubic feet of Gas obtained per Ton of Coal.	Weight in lbs. of Gas per Ton of Coal.
Boghead	0·752	15000	866
Capeldrae	0·577	14400	638
Lesmahago	0·618	13200	627
Arniston	0·626	12600	606
Newcastle	0·475	11648	423
Welsh	0·737	11424	645
Pelaw	0·444	11424	389
Pelton	0·437	11424	387
Bickerstaff, Liverpool	0·475	11424	415
Wigan	0·528	11400	461
Garesfield	0·398	10500	321
Powell	0·459	10165	357
Forest of Dean	0·360	10133	279
South Staffordshire	0·320	9600	235
Derby's Deep Main	0·424	9400	308
Derby's Soft Coal	0·528	7500	303
Leeds	0·530	6500	264

The volume of gas in cubic feet may be reduced to weight in lbs. by multiplying the volume by the specific gravity of the gas and by the constant number 0·076.

One Ton of Crude Petroleum will yield 21000 cubic feet of good gas.

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TABLE VIII.

COMPOSITION AND CALORIFIC POWER OF DOWSON'S WATER-GAS, AND OF ORDINARY 16 CANDLE-POWER COAL-GAS.

Composition of the Gas.	Ordinary Coal-Gas, 16 Candle-Power.			Dowson's Generator-Gas, or Water-Gas.		
	Volume per Cent. at 0 deg. Cent. and 760 mm.	Weight in Grammes of 100 Litres.	Calorific Power of 100 Litres.	Volume per Cent. at 0 deg. Cent. and 760 mm.	Weight in Grammes of 100 Litres.	Calorific Power of 100 Litres.
Hydrogen	51·81	4·63	159,559	18·73	1·67	57,689
Marsh gas	35·25	25·20	329,187	0·31	0·22	2,899
Olefiant gas ...	3·53	4·41	52,664	0·31	0·38	4,633
Carbonic oxide ...	8·95	11·20	27,854	25·07	31·36	77,992
,, acid ...	—	—	—	6·57	12·91	—
Oxygen	0·08	0·11	—	0·03	0·04	—
Nitrogen	0·38	0·47	—	48·98	61·27	—
	100·00	46·02	569,264	100·00	107·85	143,213

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TABLE IX.

TABLE FOR THE CONVERSION OF DEGREES ON THE CENTIGRADE THERMOMETER INTO THOSE OF FAHRENHEIT'S SCALE.

°Cent.	° Fahr.	°Cent.	° Fahr.	°Cent.	° Fahr.	°Cent.	° Fahr.	°Cent.	° Fahr.	°Cent.	° Fahr.
−100	−148·0	− 55	− 67·0	−10	14·0	35	95·0	80	176·0	125	257·0
− 99	−146·2	− 54	− 65·2	− 9	15·8	36	96·8	81	177·8	126	258·8
− 98	−144·4	− 53	− 63·4	− 8	17·6	37	98·6	82	179·6	127	260·6
− 97	−142·6	− 52	− 61·6	− 7	19·4	38	100·4	83	181·4	128	262·4
− 96	−140·8	− 51	− 59·8	− 6	21·2	39	102·2	84	183·2	129	264·2
− 95	−139·0	− 50	− 58·0	− 5	23·0	40	104·0	85	185·0	130	266·0
− 94	−137·2	− 49	− 56·2	− 4	24·8	41	105·8	86	186·6	131	267·8
− 93	−135·4	− 48	− 54·4	− 3	26·6	42	107·6	87	188·6	132	269·6
− 92	−133·6	− 47	− 52·6	− 2	28·4	43	109·4	88	190·4	133	271·4
− 91	−131·8	− 46	− 50·8	− 1	30·2	44	111·2	89	192·2	134	273·2
− 90	−130·0	− 45	− 49·0	0	32·0	45	113·0	90	194·0	135	275·0
− 89	−128·2	− 44	− 47·2	+ 1	33·8	46	114·8	91	195·8	136	276·8
− 88	−126·4	− 43	− 45·4	2	35·6	47	116·6	92	197·6	137	278·6
− 87	−124·6	− 42	− 43·6	3	37·4	48	118·4	93	199·4	138	280·4
− 86	−122·8	− 41	− 41·8	4	39·2	49	120·2	94	201·2	139	282·2
− 85	−121·0	− 40	− 40·0	5	41·0	50	122·0	95	203·0	140	284·0
− 84	−119·2	− 39	− 38·2	6	42·8	51	123·8	96	204·8	141	285·8
− 83	−117·4	− 38	− 36·4	7	44·6	52	125·6	97	206·6	142	287·6
− 82	−115·6	− 37	− 34·6	8	46·4	53	127·4	98	208·4	143	289·4
− 81	−113·8	− 36	− 32·8	9	48·2	54	129·2	99	210·2	144	291·2
− 80	−112·0	− 35	− 31·0	10	50·0	55	131·0	100	212·0	145	293·0
− 79	−110·2	− 34	− 29·2	11	51·8	56	132·8	101	213·8	146	294·8
− 78	−108·4	− 33	− 27·4	12	53·6	57	134·6	102	215·6	147	296·6
− 77	−106·6	− 32	− 25·6	13	55·4	58	136·4	103	217·4	148	298·4
− 76	−104·8	− 31	− 23·8	14	57·2	59	138·2	104	219·2	149	300·2
− 75	−103·0	− 30	− 22·0	15	59·0	60	140·0	105	221·0	150	302·0
− 74	−101·2	− 29	− 20·2	16	60·8	61	141·8	106	222·8	151	303·8
− 73	− 99·4	− 28	− 18·4	17	62·6	62	143·6	107	224·6	152	305·6
− 72	− 97·6	− 27	− 16·6	18	64·4	63	145·4	108	226·4	153	307·4
− 71	− 95·8	− 26	− 14·8	19	66·2	64	147·2	109	228·2	154	309·2
− 70	− 94·0	− 25	− 13·0	20	68·0	65	149·0	110	230·0	155	311·0
− 69	− 92·2	− 24	− 11·2	21	69·8	66	150·8	111	231·8	156	312·8
− 68	− 90·4	− 23	− 9·4	22	71·6	67	152·6	112	233·6	157	314·6
− 67	− 88·6	− 22	− 7·6	23	73·4	68	154·4	113	235·4	158	316·4
− 66	− 86·8	− 21	− 5·8	24	75·2	69	156·2	114	237·2	159	318·2
− 65	− 85·0	− 20	− 4·0	25	77·0	70	158·0	115	239·0	160	320·0
− 64	− 83·2	− 19	− 2·2	26	78·8	71	159·8	116	240·8	161	321·8
− 63	− 81·4	− 18	− 0·4	27	80·6	72	161·6	117	242·6	162	323·6
− 62	− 79·6	− 17	+ 1·4	28	82·4	73	163·4	118	244·4	163	325·4
− 61	− 77·8	− 16	3·2	29	84·2	74	165·2	119	246·2	164	327·2
− 60	− 76·0	− 15	5·0	30	86·0	75	167·0	120	248·0	165	329·0
− 59	− 74·2	− 14	6·8	31	87·8	76	168·8	121	249·8	166	330·8
− 58	− 72·4	− 13	8·6	32	89·6	77	170·6	122	251·6	167	332·6
− 57	− 70·6	− 12	10·4	33	91·4	78	172·4	123	253·4	168	334·4
− 56	− 68·8	− 11	12·2	34	93·2	79	174·2	124	255·2	169	336·2

TABLE FOR THE CONVERSION OF DEGREES (*continued*).

°Cent.	° Fahr.	°Cent.	° Fahr.	°Cent.	° Fahr.	°Cent.	° Fahr.	°Cent.	° Fahr.	°Cent.	° Fahr.
170	338·0	210	410·0	250	482·0	290	554·0	330	626·0	440	824
171	339·8	211	411·8	251	483·8	291	555·8	331	627·8	450	842
172	341·6	212	413·6	252	485·6	292	557·6	332	629·6	460	860
173	343·4	213	415·4	253	487·4	293	559·4	333	631·4	470	878
174	345·2	214	417·2	254	489·2	294	561·2	334	633·2	480	896
175	347·0	215	419·0	255	491·0	295	563·0	335	635·0	482	900
176	348·8	216	420·8	256	492·8	296	564·8	336	636·8	490	914
177	350·6	217	422·6	257	494·6	297	566·6	337	638·6	500	932
178	352·4	218	424·4	258	496·4	298	568·4	338	640·4	538	1000
179	354·2	219	426·2	259	498·2	299	570·2	339	642·2	593	1100
180	356·0	220	428·0	260	500·0	300	572·0	340	644·0	600	1112
181	357·8	221	429·8	261	501·8	301	573·8	341	645·8	649	1200
182	359·6	222	431·6	262	503·6	302	575·6	342	647·6	700	1292
183	361·4	223	433·4	263	505·4	303	577·4	343	649·4	704	1300
184	363·2	224	435·2	264	507·2	304	579·2	344	651·2	760	1400
185	365·0	225	437·0	265	509·0	305	581·0	345	653·0	800	1472
186	366·8	226	438·8	266	510·8	306	582·8	346	654·8	815	1500
187	368·6	227	440·6	267	512·6	307	584·6	347	656·6	871	1600
188	370·4	228	442·4	268	514·4	308	586·4	348	658·4	900	1652
189	372·2	229	444·2	269	516·2	309	588·2	349	660·2	926	1700
190	374·0	230	446·0	270	518·0	310	590·0	350	662·0	972	1800
191	375·8	231	447·8	271	519·8	311	591·8	351	663·8	1000	1832
192	377·6	232	449·6	272	521·6	312	593·6	352	665·6	1037	1900
193	379·4	233	451·4	273	523·4	313	595·4	353	667·4	1093	2000
194	381·2	234	453·2	274	525·2	314	597·2	354	669·2	1100	2012
195	383·0	235	455·0	275	527·0	315	599·0	355	671·0	1148	2100
196	384·8	236	456·8	276	528·8	316	600·8	356	672·8	1200	2192
197	386·6	237	458·6	277	530·6	317	602·6	357	674·6	1300	2372
198	388·4	238	460·4	278	532·4	318	604·4	358	676·4	1400	2552
199	390·2	239	462·2	279	534·2	319	606·2	359	678·2	1500	2732
200	392·0	240	464·0	280	536·0	320	608·0	360	680·0	1600	2912
201	393·8	241	465·8	281	537·8	321	609·8	370	698	1700	3092
202	395·6	242	467·6	282	539·6	322	611·6	371	700	1800	3272
203	397·4	243	469·4	283	541·4	323	613·4	380	716	1900	3452
204	399·2	244	471·2	284	543·2	324	615·2	390	734	2000	3632
205	401·0	245	473·0	285	545·0	325	617·0	400	752	2100	3812
206	402·8	246	474·8	286	546·8	326	618·8	410	770	2200	3992
207	404·6	247	476·6	287	548·6	327	620·6	420	788	2300	4172
208	406·4	248	478·4	288	550·4	328	622·4	426	800	2400	4352
209	408·2	249	480·2	289	552·2	329	624·2	430	806	2500	4532

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TABLE X.

TABLE FOR THE CONVERSION OF DEGREES ON THE FAHRENHEIT THERMOMETER INTO THOSE OF THE CENTIGRADE SCALE.

°Fahr.	° Cent.	°Fahr.	° Cent.	°Fahr.	° Cent.	°Fahr.	° Cent.	°Fahr.	° Cent.	°Fahr.	° Cent.
212	100·0	193	89·4	174	78·8	155	68·3	136	57·7	117	47·2
211	99·4	192	88·8	173	78·3	154	67·7	135	57·2	116	46·6
210	98·8	191	88·3	172	77·7	153	67·2	134	56·6	115	46·1
209	98·3	190	87·7	171	77·2	152	66·6	133	56·1	114	45·5
208	97·7	189	87·2	170	76·6	151	66·1	132	55·5	113	45·0
207	97·2	188	86·6	169	76·1	150	65·5	131	55·0	112	44·4
206	76·6	187	86·1	168	75·5	149	65·0	130	54·4	111	43·8
205	96·1	186	85·5	167	75·0	148	64·4	129	53·8	110	43·3
204	95·5	185	85·0	166	74·4	147	63·8	128	53·3	109	42·7
203	95·0	184	84·4	165	73·8	146	63·3	127	52·7	108	42·2
202	94·4	183	83·8	164	73·3	145	62·7	126	52·2	107	41·6
201	93·8	182	83·3	163	72·7	144	62·2	125	51·6	106	41·1
200	93·3	181	82·7	162	72·2	143	61·6	124	51·1	105	40·5
199	92·7	180	82·2	161	71·6	142	61·1	123	50·5	104	40·0
198	92·2	179	81·6	160	71·1	141	60·5	122	50·0	103	39·4
197	91·6	178	81·1	159	70·5	140	60·0	121	49·4	102	38·8
196	91·1	177	80·5	158	70·0	139	59·4	120	48·8	101	38·3
195	90·5	176	80·0	157	69·4	138	58·8	119	48·3	100	37·7
194	90·4	175	79·4	156	68·8	137	58·3	118	47·7	99	37·2

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TABLE XI.

METRIC PRESSURES IN KILOGRAMMES PER SQUARE CENTIMETRE WITH EQUIVALENTS IN POUNDS PER SQUARE INCH.

Kilos per Sq. Cm.	Lbs. per Sq. Inch.	Kilos per Sq. Cm.	Lbs. per Sq. Inch.	Kilos per Sq. Cm.	Lbs. per Sq. Inch.	Kilos per Sq. Cm.	Lbs. per Sq. Inch.
·1	1·422	3·1	44·091	6·1	86·761	9·1	129·431
·2	2·844	3·2	45·514	6·2	88·183	9·2	130·853
·3	4·266	3·3	46·936	6·3	89·606	9·3	132·275
·4	5·689	3·4	48·358	6·4	91·028	9·4	133·698
·5	7·111	3·5	49·781	6·5	92·450	9·5	135·120
·6	8·533	3·6	51·203	6·6	93·873	9·6	136·542
·7	9·956	3·7	52·625	6·7	95·295	9·7	137·965
·8	11·378	3·8	54·048	6·8	96·717	9·8	139·387
·9	12·800	3·9	55·470	6·9	98·140	9·9	140·809
1	14·223	4	56·892	7	99·562	10	142·232
1·1	15·645	4·1	58·315	7·1	100·984	10·5	149·343
1·2	17·067	4·2	59·737	7·2	102·407	11	156·455
1·3	18·490	4·3	61·159	7·3	103·829	11·5	163·566
1·4	19·912	4·4	62·582	7·4	105·251	12	170·678
1·5	21·334	4·5	64·004	7·5	106·674	12·5	177·790
1·6	22·757	4·6	65·426	7·6	108·096	13	184·901
1·7	24·179	4·7	66·849	7·7	109·518	13·5	192·013
1·8	25·601	4·8	68·271	7·8	110·940	14	199·124
1·9	27·024	4·9	69·693	7·9	112·363	14·5	206·236
2	28·446	5	71·116	8	113·785	15	213·348
2·1	29·868	5·1	72·538	8·1	115·207	15·5	220·459
2·2	31·291	5·2	73·960	8·2	116·630	16	227·571
2·3	32·713	5·3	75·382	8·3	118·052	16·5	234·682
2·4	34·135	5·4	76·805	8·4	119·474	17	241·794
2·5	35·558	5·5	78·227	8·5	120·897	17·5	248·906
2·6	36·980	5·6	79·649	8·6	122·319	18	256·017
2·7	38·402	5·7	81·072	8·7	123·741	18·5	263·129
2·8	39·824	5·8	82·494	8·8	125·164	19	270·240
2·9	41·247	5·9	83·916	8·9	126·586	19·5	277·352
3	42·669	6	85·339	9	128·008	20	284·464

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MOMENTS OF INERTIA, MODULI OF SECTIONS, BEAMS, AND BENDING MOMENTS.

TABLE XII.

TABLE OF MOMENTS OF INERTIA AND RESISTANCE OF VARIOUS SECTIONS.

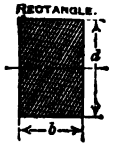
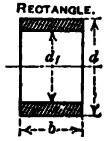
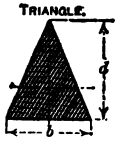
Form of Section	Moment of Inertia through Centre of Gravity	Modulus of Section $= Z_b$
RECTANGLE. 	$\frac{bd^3}{12}$	$\frac{bd^2}{6}$
RECTANGLE. 	$\frac{b}{12}\left(d^3-d_1^3\right)$	$\left\{\frac{b}{6d}\left(d^3-d_1^3\right)\right\}$
TRIANGLE. 	$\frac{bd^3}{36}$	$\frac{bd^2}{24}$

TABLE OF MOMENTS OF INERTIA, &C.—*cont.*

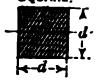
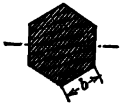
Form of Section	Moment of Inertia through Centre of Gravity	Modulus of Section $= Z_b$
SQUARE. 	$\frac{d^4}{12}$	$\frac{d^3}{6}$
SQUARE.	$\frac{d^4}{12}$	$\cdot 118d^3$
HEXAGON.	$\frac{5}{16}b^4\sqrt{3}$ $=\cdot 5413b^4$	$\frac{5}{8}b^3=\cdot 625b^3$
HEXAGON. 	$\frac{5}{16}b^4\sqrt{3}$ $=\cdot 5413b^4$	$\frac{5}{16}b^3\sqrt{3}=\cdot 5413b^3$
OCTAGON.	$\frac{1+2\sqrt{2}}{6}b^4$ $=\cdot 638b^4$	$\cdot 6906b^3$
Regular polygon with n sides. $b=$ side. $r=$ radius of circumscribed circle. $a=$ area $=\frac{nr^2}{2}\sin.\frac{2\pi}{n}$.	$\frac{a}{12}\left(3r^2-\frac{b^2}{2}\right)$	$\frac{ra}{4}$
CIRCLE.	$\frac{\pi d^4}{64}=\cdot 0491d^4$ $=\frac{\pi r^4}{4}=\cdot 7854r^4$	$\frac{\pi d^3}{32}=\cdot 0982d^3$ $=\frac{\pi r^3}{4}=\cdot 7854r^3$

Table of Moments of Inertia, &c.—*cont.*

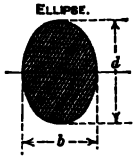
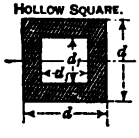
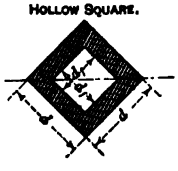
Form of Section	Moment of Inertia through Centre of Gravity	Modulus of Section $= Z_b$
Hollow Circle.	$\frac{\pi}{64}(d^4-d_1^4)$ $=\frac{\pi}{4}(r^4-r_1^4)$	$\frac{\pi}{32}\left(\frac{d^4-d_1^4}{d}\right)$ $\frac{\pi}{4}\left(\frac{r^4-r_1^4}{r}\right)$
Semicircle.	$\cdot 110d^4$	$\cdot 19r^3$
Hollow Semicircle.	$\cdot 110(r^4-r_1^4)$ $\frac{\cdot 283r^2r_1^2(r-r_1)}{r+r_1}$	$\frac{I}{y}$
Ellipse. 	$\frac{\pi bd^3}{64}=\cdot 0491bd^3$	$\frac{\pi bd^2}{32}=\cdot 0982bd^2$
Hollow Square. 	$\frac{d^4-d_1^4}{12}$	$\frac{1}{6}\left(\frac{d^4-d_1^4}{d}\right)$
Hollow Square. 	$\frac{d^4-d_1^4}{12}$	$\left(\frac{d^4-d_1^4}{12d}\right.\sqrt{2}$ $=\cdot 1178\left(\frac{d^4-d_1^4}{d}\right)$

Table of Moments of Inertia, &c.—*cont.*

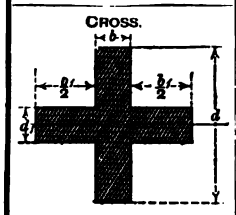
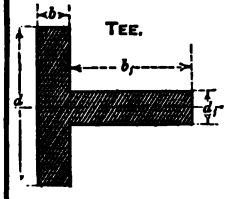
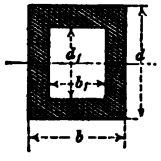
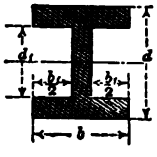
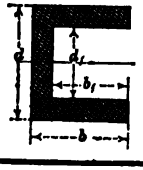
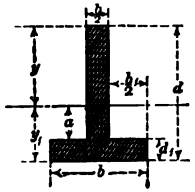
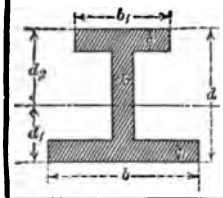
Form of Section	Moment of Inertia through Centre of Gravity	Modulus of Section $= Z_b$.
Cross. 	$\frac{bd^3+b_1d_1^3}{12}$	$\frac{bd^2+b_1d_1^2}{6d}$
Tee. 		
	*Note.*—The moment of inertia of a hollow section about any axis is equal to the moment of inertia of the whole section taken as solid about that axis and then deducting the moment of the hollow part from it about the same axis.	
	$\frac{bd^3-b_1d_1^3}{12}$	$\left(\frac{bd^2-b_1d_1^2}{6d}\right)$
		

TABLE XIII.

TABLE OF MOMENTS OF INERTIA, &C.—*cont.*

Form of Section	Moment of Inertia through Centre of Gravity	Modulus of Section = Z_b
	$\frac{1}{3}\left\{ by_1^3 - b_1a^3 + b_2y^3 \right\}$	$\frac{I}{y}$
	$\frac{1}{3}\left\{ bd_1^3 - (b-k)(d_1-c_1)^3 + b_1d_2^3 - (b_1-k)(d_2-c^3) \right\}$	

General Expressions for applying the above Table.

Let M_b = greatest bending moment.
Z_b = modulus of the section.
y = distance of neutral axis from part under greatest stress.
f = greatest tensile or compressive stress on unit of cross section.
I = moment of inertia of cross section.
E = modulus of elasticity of material.
r = radius of curvature.

Then

$$\frac{f}{y} = \frac{M}{I} = \frac{E}{r}$$

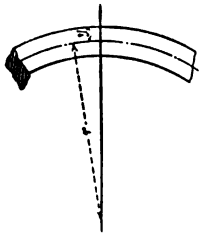
$$M = \frac{fI}{y} = \frac{EI}{r}$$

$$f = \frac{My}{I} = \frac{Ey}{r}$$

$$E = \frac{fr}{y} = \frac{Mr}{I}$$

$$r = \frac{EI}{M} = \frac{Ey}{f}$$

FIG. A.



Note.—All measurements to be taken in inches or pounds as required.

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TABLE OF MOMENTS OF INERTIA AND MODULI Z_b FOR CIRCULAR SECTIONS.

Diameter in Inches	Moment of Inertia	Modulus	Diameter in Inches	Moment of Inertia	Modulus	Diameter in Inches	Moment of Inertia	Modulus
1	0·0491	0·0982	35	73662	4209	69	1112660	32251
2	0·7854	0·7854	36	82448	4580	70	1178588	33674
3	3·976	2·651	37	91998	4973	71	1247393	35138
4	12·57	6·283	38	102354	5387	72	1319167	36644
5	30·68	12·27	39	113561	5824	73	1393995	38192
6	63·62	21·21	40	125664	6283	74	1471963	39783
7	117·9	33·67	41	138709	6766	75	1553156	41417
8	201·1	50·27	42	152745	7274	76	1637662	43096
9	322·1	71·57	43	167820	7806	77	1725571	44820
10	490·9	98·17	44	183984	8363	78	1816972	46589
11	718·7	130·7	45	201289	8946	79	1911967	48404
12	1018	169·6	46	219787	9556	80	2010619	50265
13	1402	215·7	47	239531	10193	81	2113051	52174
14	1886	269·4	48	260576	10857	82	2219347	54130
15	2485	331·3	49	282979	11550	83	2329605	56135
16	3217	402·1	50	306796	12272	84	2443920	58189
17	4100	482·3	51	332086	13023	85	2562392	60292
18	5153	572·6	52	358908	13804	86	2685120	62445
19	6397	673·4	53	387323	14616	87	2812205	64648
20	7854	785·4	54	417393	15459	88	2943748	66903
21	9547	909·2	55	449180	16334	89	3079853	69210
22	11499	1045	56	482750	17241	90	3220623	71569
23	13737	1194	57	518166	18181	91	3366165	73982
24	16286	1357	58	555497	19155	92	3516586	76448
25	19175	1534	59	594810	20163	93	3671992	78968
26	22432	1726	60	636172	21206	94	3832492	81542
27	26087	1932	61	679651	22284	95	3998198	84173
28	30172	2155	62	725332	23598	96	4169220	86859
29	34719	2394	63	773272	24548	97	4345671	89601
30	39761	2651	64	823550	25736	98	4527664	92401
31	45333	2925	65	876240	26961	99	4715315	95259
32	51472	3217	66	931420	28225	100	4908738	98175
33	58214	3528	67	989166	29527			
34	65597	3859	68	1049556	30869			

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TABLE XIV.

Table of Moments of Inertia and Constants for Moments of Resistance for Hollow Tubular Sections.

External Diameter in Inches	Thickness in Inches								
	1·2	1·5	1·8	2·0	2·2	2·5	2·8	3·0	3·5
10	327·1 65·42	373·0 74·59	408·5 81·70	427·3 85·45					
11	405·2 81·85	517·6 94·11	571·5 103·9	600·8 109·2	625·6 113·8				
12	601·0 100·2	695·8 116·0	773·5 128·9	816·8 136·1	854·1 142·4	900·0 150·0			
13	792·3 120·8	911·1 140·2	1019 156·8	1080 166·1	1134 174·4	1201 184·8			
14	996·9 142·4	1167 166·7	1311 187·4	1395 199·3	1469 209·9	1564 223·4			
15	1248 166·4	1467 195·6	1656 220·8	1766 235·5	1866 248·8	1994 265·9	2109 280·2		
16	1588 192·2	1815 226·9	2056 257·1	2199 274·9	2339 291·1	2498 312·3	2643 330·3		
17	1869 219·9	2214 260·5	2517 296·1	2698 317·4	2868 336·8	3082 362·6	3271 384·8	361 397·8	
18	2346 249·5	2668 296·4	3042 338·0	3267 363·0	3475 386·1	3751 416·8	3992 443·6	4185 465·5	
19		3180 334·8	3636 382·8	3912 411·8	4168 438·7	4511 474·9	4814 506·8	4995 525·8	
20		3754 375·4	4308 430·8	4637 463·7	4946 494·8	5369 536·9	5748 574·8	5968 596·8	6452 645·2
22·5			6319 561·7	6831 607·2	7311 650·0	7977 709·0	8575 762·3	8942 794·9	9747 866·4
25			8880 710·4	9628 770·2	10334 827·0	11320 905·7	12229 977·7	12778 1022	14022 1122
27·5				13602 952·9	14095 1025	15498 1127	16782 1231	17586 1279	19397 1411
30				17937 1155	18676 1246	20586 1372	22359 1491	23472 1565	26021 1735
32·5				22877 1377	24157 1487	26898 1648	29058 1788	30554 1880	34005 2093
35				28925 1619	30619 1750	33896 1937	36983 2114	38989 2225	43484 2485
37·5				35245 1880	38145 2035	42301 2256	46237 2466	48736 2599	54588 2912
40				42210 2161	46818 2341	51905 2600	56917 2846	60058 3003	67440 3372

Note.—The first figures given are the moments of inertia, and the second the modulus.

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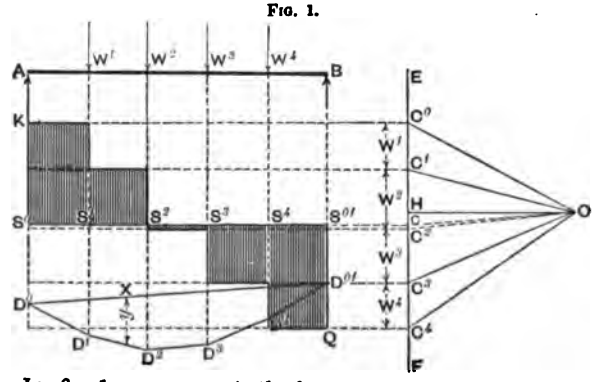
BEAMS.

Graphical Method of Determining the Bending Moments and Shearing Forces in a Beam.

Concentrated Loads acting at various Points.

Fig. 1.



In fig. 1 AB represents the beam supported at points A and B. Set off continuously along a line EF, the forces W^1, W^2, W^3, W^4 = C^0, C^4, the resultant of the forces. Take any point O, and join

OC^0, OC^1, . . . OC^4, &c. Draw the parallel lines AD^0, WD^1, . . . BD^0 through the lines of action of the forces. Take any point, D^0 in AD^0, and draw D^0, D^1 parallel to OC^0, D^1, D^2 parallel to OC^1, . . . D^1, D^{01} parallel to OC^4. Join D^0, D^{01}, completing the funicular polygon D^0, D^2, D^{01}. Draw a line OC parallel to D^0, D^{01}, cutting EF in C; then C^0C equals the supporting force at A, and CC^4 equals the supporting force at B. Also through O draw OH perpendicular to EF.

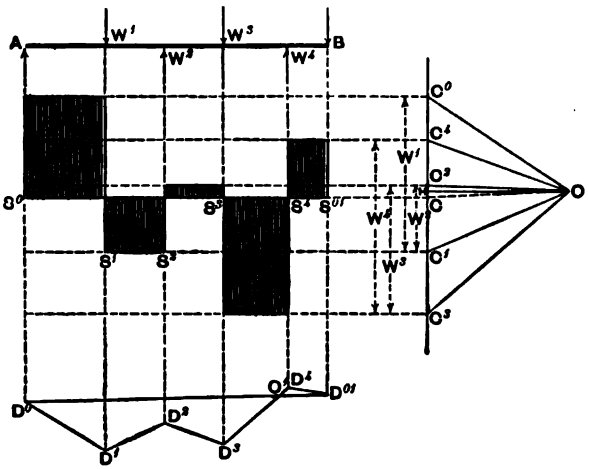
Through the points C^0, C^1, . . . C^4 drop perpendiculars on to AD^0, W^1D^1, . . . BD^{10}, and form the hatched figure $S^0KS^2QS^{01}$. Then the vertical ordinate of this diagram, measured at any point in the length of the beam, gives the shearing force at that point, measured on the same scale as used for setting off W^1, W^2, &c.

To obtain the bending moment at any point X make a scale as follows:—

If beam is drawn to a scale 1 foot $= x$ inches, and loads are drawn to a scale 1 pound $= z$ inches, then the ordinate 'y' of the funicular polygon is the bending moment at the point X on a scale such that 1 foot-pound is $= \frac{x \times y}{OH}$, in inches always; OH is measured on an inch scale.

Forces acting in different directions.

FIG. 2.



These diagrams are constructed in a similar manner to (fig. 1), the lengths of the forces being also set off in the direction of their line of action. C^0C is the supporting force at A, and CC^4 the holding down force at B as it lies to the left of C—that is, it is measured in the opposite direction to C^0C^4; the bending moment at any point O^1 where the sides of the funicular polygon cross is zero, and the bending moments to the left of O^1 are in the opposite direction to those on the right of O^1.

TABLE XV.

TABLE OF GRAPHICAL BENDING MOMENTS AND SHEARING FORCES OF BEAMS.

W = load. L = length of beam.

w = uniform intensity of load.

M = maximum bending moment.

m = bending moment at any section.

S_1 = maximum shearing force.

S = shearing force at any section.

The diagrams are constructed by setting off to scale M, and S_1, or S; then the ordinate measured at any point represents the moment or shearing force at that section.

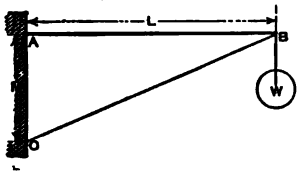
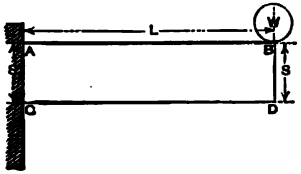
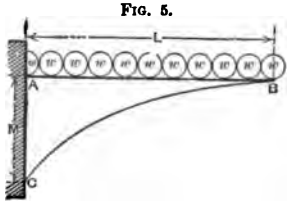
Bending Moment	Shearing Force
Fixed at one end, loaded at the other.	
FIG. 3. 	FIG. 4. 
Set down M = WL, and join BC.	Set down S = W parallel to AC.

TABLE OF GRAPHICAL BENDING MOMENTS, &c.—*cont.*

Bending Moments	Shearing Forces

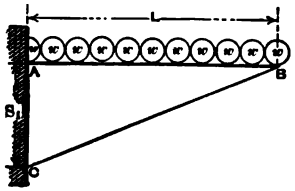
Fixed at one end and loaded uniformly.

FIG. 5.



Set off $M = \frac{wL^2}{2}$; draw the parabola BC, whose vertex is at B.

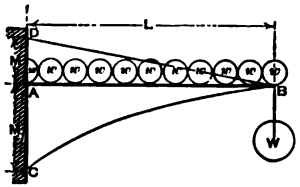
FIG. 6.



Set off $S_1 = wL$, and join BC.

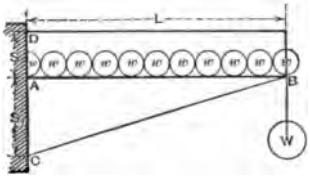
Fixed at one end, uniformly loaded, with additional weight at one end.

FIG. 7.



Set off $M = WL$, and $M_1 = \frac{wL^2}{2}$. Join DB, and construct the parabola BC whose vertex is at B.

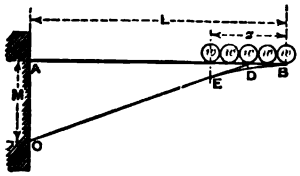
FIG. 8.



Set off $S = W$, and $S_1 = wL$. Join BC. Construct parallelogram ADEB.

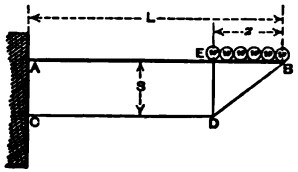
Fixed at one end, partially uniformly loaded.

FIG. 9.



Set off $M = wz\left(L - \frac{z}{2}\right)$. Join C to a point D at middle of load. Draw BE, a semiparabola, as for a beam uniformly loaded of a length z.

FIG. 10.



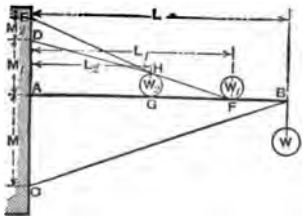
Set off the rectangle ACED, making $S = wz$, and join BD.

TABLE OF GRAPHICAL BENDING MOMENTS, &c.—*cont.*

Bending Moments	Shearing Forces

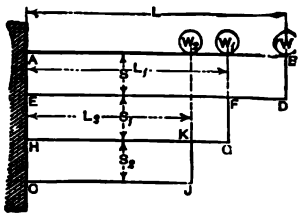
Fixed at one end with several concentrated loads.

FIG. 11.



Set off $M = WL$, $M_1 = W_1L_1$, $M_2 = W_2L_2$. Join B to C, F to D, and E to H. The bending moment is equal to the sum of the bending moments at the section produced by each load separately.

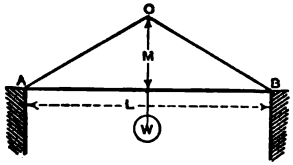
FIG. 12.



Set off $S = W$, $S_1 = W_1$, $S_2 = W_2$, and construct the rectangles ABDE, EFGH, and HKCJ.

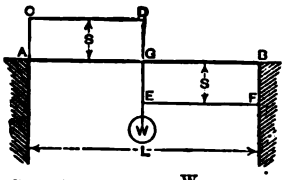
Supported at both ends, loaded at centre.

FIG. 13.



Set off $M = \frac{WL}{4}$. Join AC and BC.

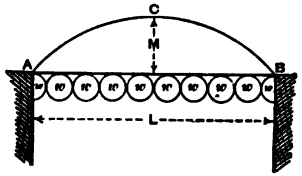
FIG. 14.



Set off S and $S = \frac{W}{2}$, and construct the rectangles ACDG and BGFE.

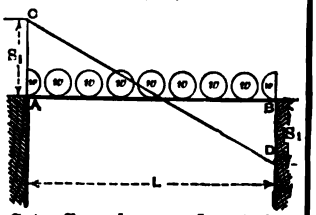
Supported at both ends, uniformly loaded.

FIG. 15.



Set off $M = \frac{wL^2}{8}$, and construct the parabola ACB, whose middle ordinate is at C.

FIG. 16.



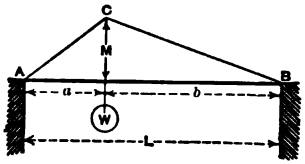
Set off S_1 above and S_1 below $= \frac{wL}{2}$, and join DC.

Table of Graphical Bending Moments, &c.—*cont.*

Bending Moments	Shearing Forces

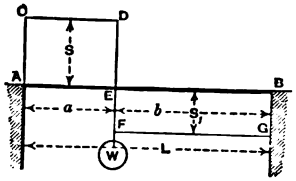
Supported at both ends, load out of centre.

Fig. 17.



Set off $M = \frac{Wab}{L}$, and join AC and BC.

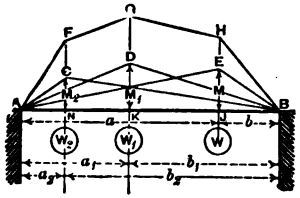
Fig. 18.



Set off $S = \frac{Wb}{L}$, and $S_1 = \frac{Wa}{L}$, and construct rectangles ACDE and EBGF.

Supported at both ends, unequally distributed loads.

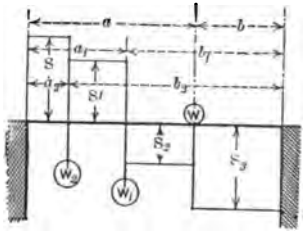
Fig. 19.



The bending moment at any point is equal to the sum of the bending moments produced at that point by each of the weights separately. Set off $M = \frac{Wab}{L}$, $M_1 = \frac{W_1a_1b_1}{L}$, $M_2 = \frac{W_2a_2b_2}{L}$.

Then set up JH, KG, and NF, making the length for whole ordinates equal to the sum of the three ordinates at those points due to the several bending moments.

Fig. 20.



Set up

$$S = \frac{Wb + W_1b_1 + W_2b_2}{L},$$

$$S_1 = \frac{Wb + W_1b_1 + W_2b_2}{L} - W_2,$$

$$S_2 = \frac{Wb + W_1b_1 + W_2b_2}{L} - W_2 - W_1,$$

$$S_3 = \frac{Wb + W_1b_1 + W_2b_2}{L} - W_2 - W_1 - W$$

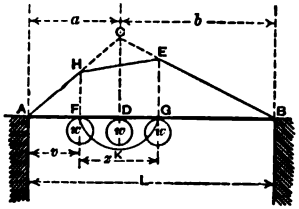
$$= \frac{Wa + W_1a_1 + W_2a_2}{L}.$$

Table of Graphical Bending Moments, &c.—*cont.*

Bending Moments	Shearing Forces

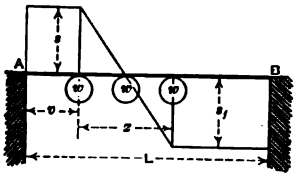
Supported at both ends, and partial uniform load not extending to either support.

Fig. 21.



Set off CD at centre of load $= \frac{wzab}{L}$, and join AC and BC; at ends of z erect perpendiculars HF and EG, cutting AC and BC in H and E. Join D and E, then on z draw the parabola FGK, whose middle ordinate equals $\frac{wz^2}{8}$.

Fig. 22.



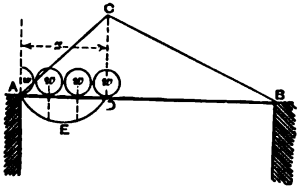
Set up $S = \frac{wz}{2L}(2L - 2v - z)$,

and $S_1 = \frac{wz}{2L}(2v + z)$,

and construct the diagram as in figure.

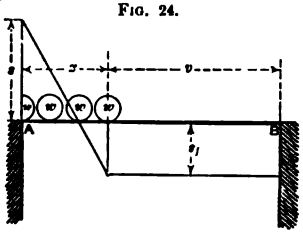
Supported at both ends, with partial uniform load extending to one support.

Fig. 23.



Set up $CD = \frac{wz(2L - z)}{2L}$; then draw underneath the parabola AED, whose ordinate at centre $= \frac{wz^2}{8}$.

Fig. 24.



Set up S at $A = \frac{wz(2L - z)}{2L}$,

and set down $S_1 = \frac{wz^2}{2L}$,

and construct the diagram.

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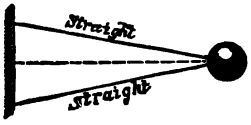
TABLE XVI.

Table of Beams of Equal Strength throughout their Length.

Note.—The sections are in all cases supposed to be rectangular.

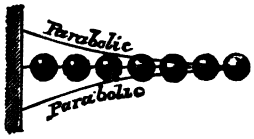
Fig. A.



Depth equal throughout.

Breadth proportional to distance from loaded end.

Fig. B.



Depth equal throughout.

Breadth proportional to square of distance from unsupported end.

Fig. C.

Breadth equal throughout.

Depth proportional to square root of distance from loaded end.

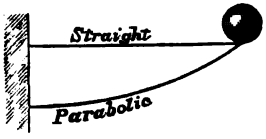


Fig. D.

Breadth equal throughout.

Depth proportional to distance from unsupported end.

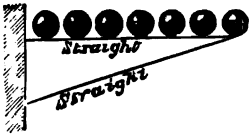


Fig. E.

Depth equal throughout.

Breadth proportional to distance from nearest point of support.

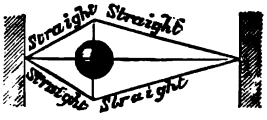


Fig. F.

Depth equal throughout.

Breadth proportional to product of distance from both points of support.

Fig. G.

Breadth equal throughout.

Depth proportional to the square root of the distance from the nearest point of support.

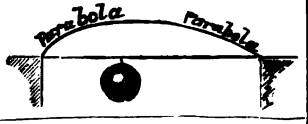


Fig. H.

Breadth equal throughout.

Depth proportional to the square root of the product of distance from both points of support.

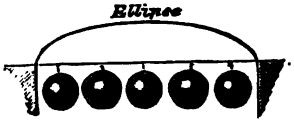


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TABLE XVII.

Liquid Measure.

		Litres	
	1 centilitre 10 cubic centimetres	0·01	0·61025 cubic inch. 0·0704 gill.
10 centilitres ...	1 décilitre ...	0·01	6·1025 cubic inches. 0·1761 pint.
10 décilitres ...	...1 LITRE (1 cubic decimetre)	1	61·02524 cubic ins. 0·2201 gallon.
10 litres ...	... 1 décalitre	...10	2·2009 gallons.
10 décalitres ...	... 1 hectolitre	...100	22·009 gallons.

Dry Measure.

10 litres ...	... 1 décalitre	...10	2·2009 gallons.
10 décalitres ...	... 1 hectolitre	...100	22·009 gallons. 2·7511 bushels.
10 hectolitres	...1 kilolitre (1 cubic metre)	1000	220·09 gallons. 27·511 bushels.

The use of measures equal to a *double-litre*, a *half-litre*, a *double-décilitre*, a *half-décilitre*, is sanctioned by law.

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TABLE XVIII.

French Measures of Length.

		Metres.	English Equivalents.
	1 millimetre =	0·001 =	0·03937 inch, or $\frac{1}{25}$ inch nearly.
10 millimetres	= 1 centimetre =	0·01 =	0·3937 inch.
10 centimetres	= 1 decimetre =	0·1 =	3·93704 inches, or 4 inches nearly.
10 decimetres 100 centimetres 1000 millimetres	= 1 METRE =	1 =	39·3704 inches 3·2809 feet.

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TABLE XIX.

French Measures of Surface.

		Square Metres.	English Equivalents.
	1 square millimetre	0·000001	0·00155 square inch.
100 square millimetres	1 square centimetre	0·0001	0·155 square inch.
100 square centimetres	1 square decimetre	0·01	15·5003 square ins.
100 square decimetres 10,000 square centimetres	1 square metre or centiare	1	10·7641 square feet. 1·1960 square yards

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TABLE XX.

French Measures of Weight.

		Grammes.	English Equivalents.
	1 milligramme	0·001	0·0154 grain.
10 milligrammes	1 centigramme	0·01	0·1543 grain.
10 centigrammes	1 decigramme	0·1	1·5432 grains.
10 decigrammes	1 GRAMME (unit of weight)	1	15·4323 grains.
10 grammes	1 décagramme	10	154·3235 grains. 0·3527 ounce.
10 décagrammes	1 hectogramme	100	1543·2349 grains. 3·5274 ounces.
10 hectogrammes	1 KILOGRAMME	1000	2·2046 pounds.
100 kilogrammes	1 metric quintal		220·4621 pounds.
10 quintals, or 1000 kilogrammes	1 millier, or tonne		2204·6212 pounds. 19·6841 cwts. 0·9842 ton.

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TABLE XXI.

MILLIMETRES TO INCHES.

TABLES GIVING THE ENGLISH EQUIVALENTS OF 1 MILLIMETRE TO 1,000.

Millimetres	Inches and Decimals of an Inch	Millimetres	Inches and Decimals of an Inch	Millimetres	Inches and Decimals of an Inch
		39	1·535447	78	3·070894
1	0·039370	40	1·574817	79	3·110264
2	0·078741	41	1·614188	80	3·149635
3	0·118111	42	1·653558	81	3·189005
4	0·157482	43	1·692929	82	3·228375
5	0·196852	44	1·732299	83	3·267746
6	0·236223	45	1·771669	84	3·307116
7	0·275593	46	1·811040	85	3·346487
8	0·314963	47	1·850410	86	3·385857
9	0·354334	48	1·889781	87	3·425228
10	0·393704	49	1·929151	88	3·464598
11	0·433075	50	1·968522	89	3·503968
12	0·472445	51	2·007892	90	3·543339
13	0·511816	52	2·047262	91	3·582709
14	0·551186	53	2·086633	92	3·622080
15	0·590556	54	2·126003	93	3·661450
16	0·629927	55	2·165374	94	3·700821
17	0·669297	56	2·204744	95	3·740191
18	0·708668	57	2·244115	96	3·779561
19	0·748038	58	2·283485	97	3·818932
20	0·787409	59	2·322855	98	3·858302
21	0·826779	60	2·362226	99	3·897673
22	0·866149	61	2·401596	100	3·937043
23	0·905520	62	2·440967	101	3·976414
24	0·944890	63	2·480337	102	4·015784
25	0·984261	64	2·519708	103	4·055155
26	1·023631	65	2·559078	104	4·094525
27	1·063002	66	2·598448	105	4·133895
28	1·102372	67	2·637819	106	4·173266
29	1·141742	68	2·677189	107	4·212636
30	1·181113	69	2·716560	108	4·252007
31	1·220483	70	2·755930	109	4·291377
32	1·259854	71	2·795301	110	4·330748
33	1·299224	72	2·834671	111	4·370118
34	1·338595	73	2·874041	112	4·409488
35	1·377965	74	2·913412	113	4·448859
36	1·417335	75	2·952782	114	4·488229
37	1·456706	76	2·992153	115	4·527600
38	1·496076	77	3·031523	116	4·566970

MILLIMETRES TO INCHES.

Millimetres	Inches and Decimals of an Inch	Millimetres	Inches and Decimals of an Inch	Millimetres	Inches and Decimals of an Inch
117	4·606341	165	6·496121	213	8·385902
118	4·645711	166	6·535492	214	8·425272
119	4·685081	167	6·574862	215	8·464643
120	4·724452	168	6·614233	216	8·504013
121	4·763822	169	6·653603	217	8·543384
122	4·803193	170	6·692973	218	8·582754
123	4·842563	171	6·732344	219	8·622125
124	4·881934	172	6·771714	220	8·661495
125	4·921304	173	6·811085	221	8·700866
126	4·960674	174	6·850455	222	8·740236
127	5·000045	175	6·889826	223	8·779606
128	5·039415	176	6·929196	224	8·818977
129	5·078786	177	6·968567	225	8·858347
130	5·118156	178	7·007937	226	8·897718
131	5·157527	179	7·047307	227	8·937088
132	5·196897	180	7·086678	228	8·976459
133	5·236267	181	7·126048	229	9·015829
134	5·275638	182	7·165419	230	9·055199
135	5·315008	183	7·204789	231	9·094570
136	5·354379	184	7·244160	232	9·133940
137	5·393749	185	7·283530	233	9·173311
138	5·433120	186	7·322900	234	9·212681
139	5·472490	187	7·362271	235	9·252052
140	5·511861	188	7·401641	236	9·291422
141	5·551231	189	7·441012	237	9·330792
142	5·590601	190	7·480382	238	9·370163
143	5·629972	191	7·519753	239	9·409533
144	5·669342	192	7·559123	240	9·448904
145	5·708713	193	7·598493	241	9·488274
146	5·748083	194	7·637864	242	9·527645
147	5·787454	195	7·677234	243	9·567015
148	5·826824	196	7·716605	244	9·606385
149	5·866194	197	7·755975	245	9·645756
150	5·905565	198	7·795346	246	9·685126
151	5·944935	199	7·834716	247	9·724497
152	5·984306	200	7·874086	248	9·763867
153	6·023676	201	7·913457	249	9·803238
154	6·063047	202	7·952827	250	9·842608
155	6·102417	203	7·992198	251	9·881978
156	6·141787	204	8·031568	252	9·921349
157	6·181158	205	8·070939	253	9·960719
158	6·220528	206	8·110309	254	10·000090
159	6·259899	207	8·149679	255	10·039460
160	6·299269	208	8·189050	256	10·078831
161	6·338640	209	8·228420	257	10·118201
162	6·378010	210	8·267791	258	10·157571
163	6·417380	211	8·307161	259	10·196942
164	6·456751	212	8·346532	260	10·236312

MILLIMETRES TO INCHES.

Millimetres	Inches and Decimals of an Inch	Millimetres	Inches and Decimals of an Inch	Millimetres	Inches and Decimals of an Inch
261	10·275683	309	12·165464	357	14·055244
262	10·315053	310	12·204834	358	14·094615
263	10·354424	311	12·244204	359	14·133985
264	10·393794	312	12·283575	360	14·173356
265	10·433165	313	12·322945	361	14·212726
266	10·472535	314	12·362316	362	14·252096
267	10·511905	315	12·401686	363	14·291467
268	10·551276	316	12·441057	364	14·330837
269	10·590646	317	12·480427	365	14·370208
270	10·630017	318	12·519797	366	14·409578
271	10·669387	319	12·559168	367	14·448949
272	10·708758	320	12·598538	368	14·488319
273	10·748128	321	12·637909	369	14·527689
274	10·787498	322	12·677279	370	14·567060
275	10·826869	323	12·716650	371	14·606430
276	10·866239	324	12·756200	372	14·645801
277	10·905610	325	12·795390	373	14·685171
278	10·944980	326	12·834761	374	14·724542
279	10·984351	327	12·874131	375	14·763912
280	11·023721	328	12·913502	376	14·803282
281	11·063091	329	12·952872	377	14·842653
282	11·102462	330	12·992243	378	14·882023
283	11·141832	331	13·031613	379	14·921394
284	11·181203	332	13·070983	380	14·960764
285	11·220573	333	13·110354	381	15·000135
286	11·259944	334	13·149724	382	15·039505
287	11·299314	335	13·189095	383	15·078875
288	11·338684	336	13·228465	384	15·118246
289	11·378055	337	13·267836	385	15·157616
290	11·417425	338	13·307206	386	15·196987
291	11·456796	339	13·346576	387	15·236357
292	11·496166	340	13·385947	388	15·275728
293	11·535537	341	13·425317	389	15·315098
294	11·574907	342	13·464688	390	15·354469
295	11·614277	343	13·504058	391	15·393839
296	11·653648	344	13·543429	392	15·433209
297	11·693018	345	13·582799	393	15·472580
298	11·732389	346	13·622170	394	15·511950
299	11·771759	347	13·661540	395	15·551321
300	11·811130	348	13·700910	396	15·590691
301	11·850500	349	13·740281	397	15·630062
302	11·889871	350	13·779651	398	15·669432
303	11·929241	351	13·819022	399	15·708802
304	11·968611	352	13·858392	400	15·748173
305	12·007982	353	13·897763	401	15·787543
306	12·047352	354	13·937133	402	15·826914
307	12·086723	355	13·976503	403	15·866284
308	12·126093	356	14·015874	404	15·905655

MILLIMETRES TO INCHES.

Millimetres	Inches and Decimals of an Inch	Millimetres	Inches and Decimals of an Inch	Millimetres	Inches and Decimals of an Inch
405	15·945025	453	17·834806	501	19·724586
406	15·984395	454	17·874176	502	19·763957
407	16·023766	455	17·913547	503	19·803327
408	16·063136	456	17·952917	504	19·842698
409	16·102507	457	17·992287	505	19·882068
410	16·141877	458	18·031658	506	19·921439
411	16·181248	459	18·071028	507	19·960809
412	16·220618	460	18·110399	508	20·000179
413	16·259988	461	18·149769	509	20·039550
414	16·299359	462	18·189140	510	20·078920
415	16·338729	463	18·228510	511	20·118291
416	16·378100	464	18·267880	512	20·157661
417	16·417470	465	18·307251	513	20·197032
418	16·456841	466	18·346621	514	20·236402
419	16·496211	467	18·385992	515	20·275773
420	16·535581	468	18·425362	516	20·315143
421	16·574952	469	18·464733	517	20·354513
422	16·614322	470	18·504103	518	20·393884
423	16·653693	471	18·543474	519	20·433254
424	16·693063	472	18·582844	520	20·472625
425	16·732444	473	18·622214	521	20·511995
426	16·771804	474	18·661585	522	20·551366
427	16·811175	475	18·700955	523	20·590736
428	16·850545	476	18·740326	524	20·630106
429	16·889915	477	18·779696	525	20·669477
430	16·929286	478	18·819067	526	20·708847
431	16·968656	479	18·858437	527	20·748218
432	17·008027	480	18·897807	528	20·787588
433	17·047397	481	18·937178	529	20·826959
434	17·086768	482	18·976548	530	20·866329
435	17·126138	483	19·015919	531	20·905699
436	17·165508	484	19·055289	532	20·945070
437	17·204879	485	19·094660	533	20·984440
438	17·244249	486	19·134030	534	21·023811
439	17·283620	487	19·173400	535	21·063181
440	17·322990	488	19·212771	536	21·102552
441	17·362361	489	19·252141	537	21·141922
442	17·401731	490	19·291512	538	21·181292
443	17·441101	491	19·330862	539	21·220663
444	17·480472	492	19·370253	540	21·260033
445	17·519842	493	19·409623	541	21·299404
446	17·559213	494	19·448993	542	21·338774
447	17·598583	495	19·488364	543	21·378145
448	17·637954	496	19·527734	544	21·417515
449	17·677324	497	19·567095	545	21·456885
450	17·716694	498	19·606465	546	21·496256
451	17·756065	499	19·645836	547	21·535626
452	17·795435	500	19·685216	548	21·574997

MILLIMETRES TO INCHES.

Millimetres	Inches and Decimals of an Inch	Millimetres	Inches and Decimals of an Inch	Millimetres	Inches and Decimals of an Inch
549	21·614367	597	23·504148	645	25·393929
550	21·653738	598	23·543518	646	25·433299
551	21·693108	599	23·582889	647	25·472670
552	21·732478	600	23·622259	648	25·512040
553	21·771849	601	23·661630	649	25·551410
554	21·811219	602	23·701000	650	25·590781
555	21·850590	603	23·740371	651	25·630151
556	21·889960	604	23·779741	652	25·669522
557	21·929331	605	23·819111	653	25·708892
558	21·968701	606	23·858482	654	25·748263
559	22·008072	607	23·897852	655	25·787633
560	22·047442	608	23·937223	656	25·827003
561	22·086812	609	23·976593	657	25·866374
562	22·126183	610	24·015964	658	25·905744
563	22·165553	611	24·055334	659	25·945115
564	22·204924	612	24·094704	660	25·984485
565	22·244294	613	24·134075	661	26·023856
566	22·283665	614	24·173445	662	26·063226
567	22·323035	615	24·212816	663	26·102596
568	22·362405	616	24·252186	664	26·141967
569	22·401776	617	24·291557	665	26·181337
570	22·441146	618	24·330927	666	26·220708
571	22·480517	619	24·370297	667	26·260078
572	22·519887	620	24·409668	668	26·299449
573	22·559928	621	24·449038	669	26·338819
574	22·598628	622	24·488409	670	26·378189
575	22·637998	623	24·527779	671	26·417560
576	22·677369	624	24·567150	672	26·456930
577	22·716739	625	24·606520	673	26·496301
578	22·756110	626	24·645890	674	26·535671
579	22·795480	627	24·685261	675	26·575042
580	22·834851	628	24·724631	676	26·614412
581	22·874221	629	24·764002	677	26·653782
582	22·913591	630	24·803372	678	26·693153
583	22·952962	631	24·842743	679	26·732523
584	22·992332	632	24·882113	680	26·771894
585	23·031703	633	24·921483	681	26·811264
586	23·071073	634	24·960854	682	26·850635
587	23·110444	635	25·000224	683	26·890005
588	23·149814	636	25·039595	684	26·929376
589	23·189184	637	25·078965	685	26·9 8746
590	23·228555	638	25·118336	686	27·008116
591	23·267925	639	25·157706	687	27·047487
592	23·307296	640	25·197077	688	27·086857
593	23·346666	641	25·236447	689	27·126228
594	23·386037	642	25·275817	690	27·165598
595	23·425407	643	25·315188	691	27·204969
596	23·464778	644	25·354558	692	27·244339

MILLIMETRES TO INCHES.

Millimetres	Inches and Decimals of an Inch	Millimetres	Inches and Decimals of an Inch	Millimetres	Inches and Decimals of an Inch
693	27·283709	741	29·173490	789	31·063271
694	27·323080	742	29·212861	790	31·102641
695	27·362450	743	29·252231	791	31·142012
696	27·401821	744	29·291601	792	31·181382
697	27·441191	745	29·330972	793	31·220752
698	27·480562	746	29·370342	794	31·260123
699	27·519932	747	29·409713	795	31·299493
700	27·559302	748	29·449083	796	31·338864
701	27·598673	749	29·488454	797	31·378234
702	27·638043	750	29·527824	798	31·417604
703	27·677414	751	29·567194	799	31·456975
704	27·716784	752	29·606565	800	31·496346
705	27·756155	753	29·645935	801	31·535716
706	27·795525	754	29·685306	802	31·575086
707	27·834895	755	29·724676	803	31·614457
708	27·874266	756	29·764047	804	31·653827
709	27·913636	757	29·803417	805	31·693198
710	27·953007	758	29·842787	806	31·732568
711	27·992377	759	29·882158	807	31·771938
712	28·031748	760	29·921528	808	31·811309
713	28·071118	761	29·960899	809	31·850679
714	28·110488	762	30·000269	810	31·890050
715	28·149859	763	30·039640	811	31·929420
716	28·189229	764	30·079010	812	31·968791
717	28·228600	765	30·118380	813	32·008161
718	28·267970	766	30·157751	814	32·047532
719	28·307341	767	30·197121	815	32·086902
720	28·346711	768	30·236492	816	32·126272
721	28·386081	769	30·275862	817	32·165643
722	28·425452	770	30·315233	818	32·205013
723	28·464822	771	30·354603	819	32·244384
724	28·504193	772	30·393973	820	32·283754
725	28·543563	773	30·433344	821	32·323125
726	28·582934	774	30·472714	822	32·362495
727	28·622304	775	30·512085	823	32·401866
728	28·661675	776	30·551455	824	32·441236
729	28·701045	777	30·590825	825	32·480606
730	28·740415	778	30·630196	826	32·519977
731	28·779786	779	30·669566	827	32·559347
732	28·819156	780	30·708937	828	32·598718
733	28·858527	781	30·748307	829	32·638088
734	28·897897	782	30·787678	830	32·677459
735	28·937268	783	30·827048	831	32·716829
736	28·976638	784	30·866419	832	32·756199
737	29·016008	785	30·905789	833	32·795570
738	29·055379	786	30·945159	834	32·834940
739	29·094749	787	30·984530	835	32·874311
740	29·134120	788	31·023900	836	32·913681

MILLIMETRES TO INCHES.

Millimetres	Inches and Decimals of an Inch	Millimetres	Inches and Decimals of an Inch	Millimetres	Inches and Decimals of an Inch
837	32·953052	885	34·842832	933	36·732613
838	32·992422	886	34·882203	934	36·771984
839	33·031792	887	34·921573	935	36·811354
840	33·071163	888	34·960944	936	36·850724
841	33·110533	889	35·000314	937	36·890095
842	33·149904	890	35·039684	938	36·929465
843	33·189274	891	35·079055	939	36·968836
844	33·228645	892	35·118425	940	37·008206
845	33·268015	893	35·157796	941	37·047576
846	33·307385	894	35·197166	942	37·086947
847	33·346756	895	35·236536	943	37·126317
848	33·386126	896	35·275907	944	37·165688
849	33·425497	897	35·315277	945	37·205058
850	33·464867	898	35·354648	946	37·244429
851	33·504238	899	35·394018	947	37·283799
852	33·543608	900	35·433389	948	37·323170
853	33·582979	901	35·472759	949	37·362540
854	33·622349	902	35·512130	950	37·410910
855	33·661719	903	35·551500	951	37·441281
856	33·701090	904	35·590971	952	37·480651
857	33·740460	905	35·630241	953	37·520022
858	33·779831	906	35·669611	954	37·559392
859	33·819201	907	35·708982	955	37·598765
860	33·858572	908	35·748352	956	37·638135
861	33·897942	909	35·787723	957	37·677503
862	33·937312	910	35·827093	958	37·716874
863	33·976683	911	35·866464	959	37·756244
864	34·016053	912	35·905834	960	37·795615
865	34·055424	913	35·945204	961	37·834985
866	34·094794	914	35·984575	962	37·874356
867	34·134165	915	36·023945	963	37·913726
868	34·173535	916	36·063316	964	37·953096
869	34·212905	917	36·102686	965	37·992467
870	34·252276	918	36·142057	966	38·031837
871	34·291646	919	36·181427	967	38·071208
872	34·331017	920	36·220797	968	38·110578
873	34·370387	921	36·260168	969	38·149949
874	34·409758	922	36·299538	970	38·189319
875	34·449128	923	36·338909	971	38·228689
876	34·448498	924	36·378279	972	38·268060
877	34·527869	925	36·417650	973	38·307430
878	34·567239	926	36·457020	974	38·346801
879	34·606610	927	36·496390	975	38·386171
880	34·645980	928	36·535761	976	38·425542
881	34·685351	929	36·575131	977	38·464912
882	34·724721	930	36·614502	978	38·504283
883	34·764091	931	36·653872	979	38·543653
884	34·803462	932	36·693243	980	38·583023

MILLIMETRES AND METRES TO INCHES.

Millimetres	Inches and Decimals of an Inch	Millimetres	Inches and Decimals of an Inch	Millimetres	Inches and Decimals of an Inch
981	38·622394	988	38·897987	995	39·173580
982	38·661764	989	38·937357	996	39·212950
983	38·701135	990	38·976728	997	39·252321
984	38·740505	991	39·016098	998	39·291691
985	38·779876	992	39·055469	999	39·331062
986	38·819246	993	39·094839	1000	39·370432
987	38·858616	994	39·134209		

TABLE XXII.

Table giving the English Equivalents of Metres in Inches and Decimals of an Inch.

Metres	Inches and Decimals of an Inch	Metres	Inches and Decimals of an Inch	Metres	Inches and Decimals of an Inch
1	39·370432	34	1338·594687	67	2637·818941
2	78·740864	35	1377·965119	68	2677·189373
3	118·111296	36	1417·335551	69	2716·559805
4	157·481728	37	1456·705983	70	2755·930237
5	196·852160	38	1496·076415	71	2795·300669
6	236·222592	39	1535·446846	72	2834·671101
7	275·593024	40	1574·817278	73	2874·041533
8	314·963456	41	1614·187710	74	2913·411965
9	354·333888	42	1653·558142	75	2952·782397
10	393·704320	43	1692·928574	76	2992·152829
11	433·074752	44	1732·299006	77	3031·523261
12	472·445184	45	1771·669438	78	3070·893693
13	511·815616	46	1811·039870	79	3110·264125
14	551·186047	47	1850·410302	80	3149·634557
15	590·556479	48	1889·780734	81	3189·004989
16	629·926911	49	1929·151166	82	3228·375421
17	669·297343	50	1968·521598	83	3267·745853
18	708·667775	51	2007·892030	84	3307·116285
19	748·038207	52	2047·262462	85	3346·486717
20	787·408639	53	2086·632894	86	3385·857149
21	826·779071	54	2126·003326	87	3425·227581
22	866·149503	55	2165·37358	88	3464·598013
23	905·519935	56	2204·744190	89	3503·968444
24	944·890367	57	2244·114622	90	3543·338876
25	984·260799	58	2283·485054	91	3582·709308
26	1023·631231	59	2322·855486	92	3622·079740
27	1063·001663	60	2362·225918	93	3661·450172
28	1102·372095	61	2401·596350	94	3700·820604
29	1141·742527	62	2440·966782	95	3740·191036
30	1181·112959	63	2480·337214	96	3779·561468
31	1220·483391	64	2519·707645	97	3818·931900
32	1259·853823	65	2559·078077	98	3858·302332
33	1299·224255	66	2598·448509	99	3897·672764

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TABLE XXIII.

FRACTIONS OF AN INCH TO MILLIMETRES.

TABLE GIVING THE EQUIVALENTS IN MILLIMETRES OF THE DIVISIONS OF THE INCH.

Divisions of the Inch	Millimetres	Divisions of the Inch	Millimetres
... ... 1/128	·198436	5/16 ... 1/64 1/128	8·532736
... ... 1/64 ...	·396871	5/16 1/32	8·731172
... ... 1/64 1/128	·595307	5/16 1/32 ... 1/128	8·929007
... 1/32	·793743	5/16 1/32 1/64 ...	9·128043
... 1/32 ... 1/128	·992179	5/16 1/32 1/64 1/128	9·326479
... 1/32 1/64 ...	1·190614	3/8	9·524915
... 1/32 1/64 1/128	1·389050	3/8 ... ... 1/128	9·723350
1/16	1·587486	3/8 ... 1/64 ...	9·921786
1/16 ... ... 1/128	1·785921	3/8 ... 1/64 1/128	10·120222
1/16 ... 1/64 ...	1·984357	3/8 1/32	10·318657
1/16 ... 1/64 1/128	2·182793	3/8 1/32 ... 1/128	10·517093
1/16 1/32	2·385129	3/8 1/32 1/64 ...	10·715529
1/16 1/32 ... 1/128	2·579664	3/8 1/32 1/64 1/128	10·913965
1/16 1/32 1/64 ...	2·778100	7/16	11·112400
1/16 1/32 1/64 1/128	2·976536	7/16 ... ... 1/128	11·310836
1/8	3·174972	7/16 ... 1/64 ...	11·509272
1/8 ... ... 1/128	3·373407	7/16 ... 1/64 1/128	11·707707
1/8 ... 1/64 ...	3·571843	7/16 1/32	11·906143
1/8 ... 1/64 1/128	3·770279	7/16 1/32 ... 1/128	12·104579
1/8 1/32	3·968714	7/16 1/32 1/64 ...	12·303015
1/8 1/32 ... 1/128	4·167150	7/16 1/32 1/64 1/128	12·501450
1/8 1/32 1/64 ...	4·365586	1/2	12·699886
1/8 1/32 1/64 1/128	4·564022	1/2 ... ... 1/128	12·898322
3/16	4·762457	1/2 ... 1/64 ...	13·096757
3/16 ... ... 1/128	4·960893	1/2 ... 1/64 1/128	13·295193
3/16 ... 1/64 ...	5·159329	1/2 1/32	13·493629
3/16 ... 1/64 1/128	5·357764	1/2 1/32 ... 1/128	13·692065
3/16 1/32	5·556200	1/2 1/32 1/64 ...	13·890500
3/16 1/32 ... 1/128	5·754636	1/2 1/32 1/64 1/128	14·088936
3/16 1/32 1/64 ...	5·953072	9/16	14·287372
3/16 1/32 1/64 1/128	6·151508	9/16 ... ... 1/128	14·485808
1/4	6·349943	9/16 ... 1/64 ...	14·684243
1/4 ... ... 1/128	6·548379	9/16 ... 1/64 1/128	14·882679
1/4 ... 1/64 ...	6·746814	9/16 1/32	15·081115
1/4 ... 1/64 1/128	6·945250	9/16 1/32 ... 1/128	15·279550
1/4 1/32	7·143686	9/16 1/32 1/64 ...	15·477986
1/4 1/32 ... 1/128	7·342122	9/16 1/32 1/64 1/128	15·676422
1/4 1/32 1/64 ...	7·540557	5/8	15·874858
1/4 1/32 1/64 1/128	7·738993	5/8 ... ... 1/128	16·073293
5/16	7·937429	5/8 ... 1/64 ...	16·271729
5/16 ... ... 1/128	8·135865	5/8 ... 1/64 1/128	16·470165
5/16 ... 1/64 ...	8·334300	5/8 1/32	16·668600

EQUIVALENTS OF ENGLISH AND METRICAL MEASURES.

Divisions of the Inch	Millimetres	Divisions of the Inch	Millimetres
5/8 1/32 ... 1/128	16·867036	13/16 ... 1/64 1/128	21·232622
5/8 1/32 1/64 ...	17·065472	13/16 1/32	21·431058
5/8 1/32 1/64 1/128	17·263908	13/16 1/32 ... 1/128	21·629493
11/16	17·462343	13/16 1/32 1/64 ...	21·827929
11/16 ... ... 1/128	17·660779	13/16 1/32 1/64 1/128	22·026365
11/16 ... 1/64 ...	17·859215	7/8	22·224801
11/16 ... 1/64 1/128	18·057650	7/8 ... ... 1/128	22·423236
11/16 1/32	18·256086	7/8 ... 1/64 ...	22·621672
11/16 1/32 ... 1/128	18·454522	7/8 ... 1/64 1/128	22·820108
11/16 1/32 1/64 ...	18·652958	7/8 1/32	23·018543
11/16 1/32 1/64 1/128	18·851393	7/8 1/32 ... 1/128	23·216979
3/4	19·049829	7/8 1/32 1/64 ...	23·415415
3/4 ... ... 1/128	19·248265	7/8 1/32 1/64 1/128	23·613851
3/4 ... 1/64 ...	19·446701	15/16	23·812286
3/4 ... 1/64 1/128	19·645136	15/16 ... ... 1/128	24·010722
3/4 1/32	19·843572	15/16 ... 1/64 ...	24·209158
3/4 1/32 ... 1/128	20·042008	15/16 ... 1/64 1/128	24·407594
3/4 1/32 1/64 ...	20·240443	15/16 1/32	24·606029
3/4 1/32 1/64 1/128	20·438879	15/16 1/32 ... 1/128	24·804465
13/16	20·637315	15/16 1/32 1/64 ...	25·002901
13/16 ... ... 1/128	20·835751	15/16 1/32 1/64 1/128	25·201336
13/16 ... 1/64 ...	21·034186	1	25·399772

TABLE XXIV.

TABLE GIVING THE EQUIVALENTS IN MILLIMETRES OF THE DIVISIONS OF THE FOOT.

In.	Millimetres	In.	Millimetres	In.	Millimetres	In.	Millimetres
1	25·39977	10	253·99772	19	482·59567	28	711·19362
2	50·79954	11	279·39749	20	507·99544	29	736·59339
3	76·19932	12	304·79727	21	533·39521	30	761·99316
4	101·59909	13	330·19704	22	558·79499	31	787·39294
5	126·99886	14	355·59681	23	584·19476	32	812·79271
6	152·39863	15	380·99658	24	609·59453	33	838·19248
7	177·79840	16	406·39635	25	634·99430	34	863·59225
8	203·19818	17	431·79613	26	660·39408	35	888·99202
9	228·59795	18	457·19590	27	685·78385	36	914·39180

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Table XXV.—Lineal Feet in Metres.

Feet.	Metres.	Feet.	Metres.	Feet.	Metres.	Feet.	Metres.
1	·3048	26	7·9248	51	15·5448	76	23·1648
2	·6096	27	8·2296	52	15·8496	77	23·4696
3	·9144	28	8·5344	53	16·1544	78	23·7744
4	1·2192	29	8·8392	54	16·4592	79	24·0792
5	1·5240	30	9·1440	55	16·7640	80	24·3840
6	1·8288	31	9·4488	56	17·0688	81	24·6888
7	2·1336	32	9·7536	57	17·3736	82	24·9936
8	2·4384	33	10·0584	58	17·6784	83	25·2984
9	2·7432	34	10·3632	59	17·9832	84	25·6032
10	3·0480	35	10·6680	60	18·2880	85	25·9080
11	3·3528	36	10·9728	61	18·5928	86	26·2128
12	3·6576	37	11·2776	62	18·8976	87	26·5176
13	3·9624	38	11·5824	63	19·2024	88	26·8224
14	4·2672	39	11·8872	64	19·5072	89	27·1272
15	4·5720	40	12·1920	65	19·8120	90	27·4320
16	4·8768	41	12·4968	66	20·1168	91	27·7368
17	5·1816	42	12·8016	67	20·4216	92	28·0416
18	5·4864	43	13·1064	68	20·7264	93	28·3464
19	5·7912	44	13·4112	69	21·0312	94	28·6512
20	6·0960	45	13·7160	70	21·3360	95	28·9560
21	6·4008	46	14·0208	71	21·6408	96	29·2608
22	6·7056	47	14·3256	72	21·9456	97	29·5656
23	7·0104	48	14·6304	73	22·2504	98	29·8704
24	7·3152	49	14·9352	74	22·5552	99	30·1752
25	7·6200	50	15·2400	75	22·8600	100	30·4800

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Tables XXVI., XXVII.—Metres in Lineal Feet and in Yards.

Metres.	Feet.	Yards.	Metres.	Feet.	Yards.
1	3·2809	1·0936	44	144·3596	48·1193
2	6·5618	2·1872	45	147·6405	49·2129
3	9·8427	3·2809	46	150·9214	50·3065
4	13·1236	4·3745	47	154·2023	51·4001
5	16·4045	5·4681	48	157·4832	52·4938
6	19·6854	6·5617	49	160·7641	53·5874
7	22·9663	7·6553	50	164·0450	54·6810
8	26·2472	8·7490	51	167·3259	55·7746
9	29·5281	9·8426	52	170·6068	56·8682
10	32·8090	10·9362	53	173·8877	57·9619
11	36·0899	12·0298	54	177·1686	59·0555
12	39·3708	13·1234	55	180·4495	60·1491
13	42·6517	14·2171	56	183·7304	61·2427
14	45·9326	15·3107	57	187·0113	62·3363
15	49·2135	16·4043	58	190·2922	63·4300
16	52·4944	17·4979	59	193·5731	64·5236
17	55·7753	18·5915	60	196·8540	65·6172
18	59·0562	19·6852	61	200·1349	66·7108
19	62·3371	20·7788	62	203·4158	67·8044
20	65·6180	21·8724	63	206·6967	68·8981
21	68·8989	22·9660	64	209·9776	69·9917
22	72·1798	24·0596	65	213·2585	71·0853
23	75·4607	25·1533	66	216·5394	72·1789
24	78·7416	26·2469	67	219·8203	73·2725
25	82·0225	27·3405	68	223·1012	74·3662
26	85·3034	28·4341	69	226·3821	75·4598
27	88·5843	29·5277	70	229·6630	76·5534
28	91·8652	30·6214	71	232·9439	77·6470
29	95·1461	31·7150	72	236·2248	78·7406
30	98·4270	32·8086	73	239·5057	79·8343
31	101·7079	33·9022	74	242·7866	80·9279
32	104·9888	34·9958	75	246·0675	82·0215
33	108·2697	36·0895	76	249·3484	83·1151
34	111·5506	37·1831	77	252·6293	84·2087
35	114·8315	38·2767	78	255·9102	85·3024
36	118·1124	39·3703	79	259·1911	86·3960
37	121·3933	40·4639	80	262·4720	87·4896
38	124·6742	41·5576	81	265·7529	88·5832
39	127·9551	42·5512	82	269·0338	89·6768
40	131·2360	43·7448	83	272·3147	90·7705
41	134·5169	44·8384	84	275·5956	91·8641
42	137·7978	45·9320	85	278·8765	92·9577
43	141·0787	47·0257	86	282·1574	94·0513

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EQUIVALENTS OF ENGLISH AND METRICAL WEIGHTS.

TABLE XXVIII.

TABLE GIVING THE EQUIVALENTS OF AVOIR. OZ. IN FRENCH KILOGRAMS.

Oz.	Kilograms	Oz.	Kilograms	Oz.	Kilograms	Oz.	Kilograms
1	·028349541	5	·141747704	9	·255145867	13	·368544030
2	·056699082	6	·170097245	10	·283495408	14	·396893571
3	·085048622	7	·198446785	11	·311844948	15	·425243112
4	·113398163	8	·226796326	12	·340194489	16	·453592652

TABLE XXIX.

TABLE GIVING THE EQUIVALENTS OF AVOIR. LBS. IN FRENCH KILOGRAMS.

Lbs.	Kilograms	Lbs.	Kilograms	Lbs.	Kilograms	lbs.	Kilograms
1	·45359265	8	3·62874122	15	6·80388978	22	9·97903835
2	·90718530	9	4·08233387	16	7·25748243	23	10·43263100
3	1·36077796	10	4·53592652	17	7·71107509	24	10·88622365
4	1·81437061	11	4·98951917	18	8·16466774	25	11·33981631
5	2·26796326	12	5·44311183	19	8·61826039	26	11·79340896
6	2·72155591	13	5·89670448	20	9·07185305	27	12·24700161
7	3·17514857	14	6·35029713	21	9·52544570	28	12·70059426

TABLE XXX.

TABLE GIVING THE EQUIVALENTS OF QUARTERS IN FRENCH KILOGRAMS.

Qr.	Kilograms	Qrs.	Kilograms	Qrs.	Kilograms	Qrs.	Kilograms
1	12·70059426	2	25·40118853	3	38·10178279	4	50·80237705

TABLE XXXI.

TABLE GIVING THE EQUIVALENTS OF CWTS. IN FRENCH KILOGRAMS.

Cwt	Kilograms	Cwt	Kilograms	Cwt	Kilograms	Cwt	Kilograms
1	50·80237705	6	304·81426231	11	558·82614757	16	812·83803283
2	101·60475410	7	355·61663936	12	609·62852462	17	863·64040988
3	152·40713116	8	406·41901642	13	660·43090168	18	914·44278694
4	203·20950821	9	457·22139347	14	711·23327873	19	965·24516399
5	254·01188526	10	508·02377052	15	762·03565578	20	1016·0475411

TABLE XXXII.

TABLE GIVING THE EQUIVALENTS OF TONS IN FRENCH KILOGRAMS.

Tons	Kilograms	Tons	Kilograms	Tons	Kilograms	Tons	Kilograms
1	1016·04754	20	20320·9508	300	304814·262	1300	1320861·80
2	2032·09508	30	30481·4262	400	406419·016	1400	1422466·56
3	3048·14262	40	40641·9016	500	508023·771	1500	1524071·31
4	4064·19016	50	50802·3771	600	609628·525	1600	1625676·07
5	5080·23771	60	60962·8525	700	711233·279	1700	1727280·82
6	6096·28525	70	71123·3279	800	812838·033	1800	1828885·57
7	7112·33279	80	81283·8033	900	914442·787	1900	1930490·33
8	8128·38033	90	91444·2787	1000	1016047·54	2000	2032095·08
9	9144·42787	100	101604·754	1100	1117652·30	3000	3048142·62
10	10160·4754	200	203209·508	1200	1219257·05	4000	4064190·16

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TABLE XXXIII.

Table giving the Equivalents of Kilograms in Avoirdupois Pounds and Tons.

Kilos.	Avoir. Lbs.	Ton	Kilos.	Avoir. Lbs.	Ton	Kilos.	Avoir. Lbs.	Ton	Kilos.	Avoir. Lbs.	Ton
1	2·20462	·00098421	26	57·32015	·02558935	51	112·43568	·05019450	76	167·55122	·07479965
2	4·40924	·00196841	27	59·52477	·02657356	52	114·64031	·05117871	77	169·75584	·07578386
3	6·61386	·00295262	28	61·72940	·02755777	53	116·84493	·05216291	78	171·96046	·07676806
4	8·81849	·00393682	29	63·93402	·02854197	54	119·04955	·05314712	79	174·16508	·07775227
5	11·02311	·00492103	30	66·13864	·02952618	55	121·25417	·05413133	80	176·36970	·07873647
6	13·22773	·00590524	31	68·34326	·03051038	56	123·45879	·05511553	81	178·57432	·07972068
7	15·43235	·00688944	32	70·54788	·03149459	57	125·66341	·05609974	82	180·77894	·08070489
8	17·63697	·00787365	33	72·75250	·03247880	58	127·86803	·05708394	83	182·98356	·08168909
9	19·84159	·00885785	34	74·95712	·03346300	59	130·07265	·05806815	84	185·18819	·08267330
10	22·04621	·00984206	35	77·16174	·03444721	60	132·27728	·05905235	85	187·39281	·08365750
11	24·25083	·01082627	36	79·36637	·03543141	61	134·48190	·06003656	86	189·59743	·08464171
12	26·45555	·01181047	37	81·57099	·03641562	62	136·68652	·06102077	87	191·80205	·08562591
13	28·66008	·01279468	38	83·77561	·03739982	63	138·89114	·06200497	88	194·00667	·08661012
14	30·86470	·01377888	39	85·98023	·03838403	64	141·09576	·06298918	89	196·21129	·08759433
15	33·06932	·01476309	40	88·18485	·03936824	65	143·30038	·06397338	90	198·41591	·08857853
16	35·27394	·01574729	41	90·38947	·04035244	66	145·50500	·06495759	91	200·62053	·08956274
17	37·47856	·01673150	42	92·59409	·04133665	67	147·70962	·06594180	92	202·82516	·09054694
18	39·68318	·01771571	43	94·79871	·04232085	68	149·91425	·06692600	93	205·02978	·09153115
19	41·88780	·01869991	44	97·00334	·04330506	69	152·11887	·06791021	94	207·23440	·09251536
20	44·09243	·01968412	45	99·20796	·04428927	70	154·32349	·06889441	95	209·43902	·09349956
21	46·29705	·02066832	46	101·41258	·04527347	71	156·52811	·06987862	96	211·64364	·09448377
22	48·50167	·02165253	47	103·61720	·04625768	72	158·73273	·07086283	97	213·84826	·09546797
23	50·70629	·02263674	48	105·82182	·04724188	73	160·93735	·07184703	98	216·05288	·09645218
24	52·91091	·02362094	49	108·02644	·04822609	74	163·14197	·07283124	99	218·25750	·09743639
25	55·11553	·02460515	50	110·23106	·04921030	75	165·34659	·07381544	100	220·46213	·09842059

Table giving the Equivalents of Kilograms in Avoirdupois Pounds and Tons (concluded).

Kilos.	Avoir. Lbs.	Ton	Kilos.	Avoir. Lbs.	Ton	Kilos.	Avoir. Lbs.	Ton	Kilos.	Avoir. Lbs.	Ton
101	222·66675	·09940480	126	277·78228	·12400995	151	332·89781	·14861509	176	388·01334	·17322024
102	224·87137	·10038900	127	279·98690	·12499415	152	335·10243	·14959930	177	390·21796	·17420445
103	227·07599	·10137321	128	282·29152	·12597836	153	337·30705	·15058351	178	392·42258	·17518865
104	229·28061	·10235742	129	284·49614	·12696256	154	339·51167	·15156771	179	394·62720	·17617286
105	231·48523	·10334162	130	286·60076	·12794677	155	341·71629	·15255192	180	396·83183	·17715706
106	233·68985	·10432583	131	288·80538	·12893097	156	343·92092	·15353612	181	399·03645	·17814127
107	235·89447	·10531003	132	291·01001	·12991518	157	346·12554	·15452033	182	401·24107	·17912548
108	238·09910	·10629424	133	293·21463	·13089939	158	348·33016	·15550453	183	403·44569	·18010968
109	240·30372	·10727844	134	295·41925	·13188359	159	350·53478	·15648874	184	405·65031	·18109389
110	242·50834	·10826265	135	297·62387	·13286780	160	352·73940	·15747295	185	407·85493	·18207809
111	244·71296	·10924686	136	299·82849	·13385200	161	354·94402	·15845715	186	410·05955	·18306230
112	246·91758	·11023106	137	302·03311	·13483621	162	357·14864	·15944136	187	412·26417	·18404651
113	249·12220	·11121527	138	304·23773	·13582042	163	359·35326	·16042556	188	414·46880	·18503071
114	251·32682	·11219947	139	306·44235	·13680462	164	361·55789	·16140977	189	416·67342	·18601492
115	253·53144	·11318368	140	308·64698	·13778883	165	363·76251	·16239398	190	418·87804	·18699912
116	255·73607	·11416789	141	310·85160	·13877303	166	365·96713	·16337818	191	421·08266	·18798333
117	257·94069	·11515209	142	313·05622	·13975724	167	368·17175	·16436239	192	423·28728	·18896754
118	260·14531	·11613630	143	315·26084	·14074145	168	370·37637	·16534659	193	425·49190	·18995174
119	262·34993	·11712050	144	317·46546	·14172565	169	372·58099	·16633080	194	427·69652	·19093595
120	264·55455	·11810471	145	319·67008	·14270986	170	374·78561	·16731501	195	429·90114	·19192015
121	266·75917	·11908892	146	321·87470	·14369406	171	376·99023	·16829921	196	432·10577	·19290436
122	268·96379	·12007312	147	324·07932	·14467827	172	379·19486	·16928342	197	434·31039	·19388857
123	271·16841	·12105733	148	326·28395	·14566248	173	381·39948	·17026762	198	436·51501	·19487277
124	273·37304	·12204153	149	328·48857	·14664668	174	383·60410	·17125183	199	438·71963	·19585698
125	275·57766	·12302574	150	330·69319	·14763089	175	385·80872	·17223604	200	440·92425	·19684118

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TABLE XXXIV.

Belt Velocity or Circumferential Speed of Pulleys.

Pulley Diam. in Inches.	Revolutions per Minute.												
	50	60	70	80	90	100	110	120	130	140	150	160	170
	Velocity in Feet per Minute.												
6	78·5	94·2	110	126	141	157	173	188	204	220	235	251	267
7	91·7	110	128	146	165	183	201	220	238	256	275	293	312
8	105	126	146	167	188	210	230	251	272	293	314	335	356
9	118	141	165	188	212	236	259	282	306	330	353	377	400
10	131	157	183	209	235	262	288	314	340	366	392	419	445
12	157	188	220	252	282	314	346	377	408	440	471	502	534
14	183	220	256	293	330	366	403	440	476	513	550	586	623
16	209	251	293	335	377	419	460	502	544	586	628	670	712
18	230	282	330	377	424	471	518	565	612	659	707	754	801
20	262	314	366	419	471	524	576	628	681	733	785	838	890
22	288	345	403	460	518	576	634	691	749	806	864	921	979
24	314	377	440	502	565	628	691	754	817	880	942	1005	1068
26	340	408	476	545	622	681	749	817	885	953	1021	1089	1157
28	380	440	513	586	659	733	806	880	953	1026	1100	1173	1246
30	393	471	550	628	706	785	864	942	1022	1100	1178	1256	1335
32	419	502	586	670	754	838	921	1005	1089	1173	1257	1340	1424
34	445	534	623	712	801	890	979	1068	1157	1246	1335	1424	1513
36	471	565	659	754	828	942	1037	1131	1225	1319	1414	1508	1602
40	523	628	733	837	942	1047	1152	1256	1361	1466	1571	1675	1780
48	628	754	879	1005	1131	1257	1382	1508	1633	1759	1885	2010	2136
54	707	848	989	1131	1272	1414	1555	1696	1838	1979	2120	2262	2403
60	785	942	1099	1256	1414	1571	1728	1885	2042	2199	2356	2513	2670
66	864	1036	1209	1382	1550	1728	1900	2073	2246	2419	2592	2764	2937
72	942	1131	1319	1508	1696	1885	2073	2262	2450	2639	2827	3016	**3204**
78	1021	1225	1429	1633	1838	2042	2245	2450	2655	2859	3063	**3267**	**3472**
84	1099	1319	1539	1754	1978	2199	2419	2639	2859	3079	**3298**	**3518**	**3738**

Pulley Diam. in Inches.	Revolutions per Minute.												
	180	190	200	210	220	230	240	250	260	270	280	290	300
	Velocity in Feet per Minute.												
6	282	298	314	330	346	361	377	392	408	424	440	455	471
7	330	348	367	385	403	421	440	458	477	495	513	531	550
8	377	398	419	440	461	481	503	523	545	565	586	607	628
9	424	447	471	495	518	542	565	588	613	630	660	683	707
10	471	497	524	549	576	602	628	654	681	707	733	759	785
12	560	597	628	659	691	722	754	785	817	848	880	911	942
14	659	696	733	769	806	843	880	916	953	989	1026	1063	1100
16	754	796	838	879	921	963	1005	1046	1089	1131	1173	1214	1257
18	848	895	942	989	1037	1084	1131	1178	1225	1272	1319	1366	1414
20	942	995	1047	1099	1152	1204	1256	1309	1361	1414	1466	1518	1571
22	1037	1094	1152	1209	1267	1325	1382	1440	1497	1555	1612	1670	1728
24	1131	1194	1257	1319	1382	1445	1508	1571	1633	1696	1759	1822	1885
26	1225	1293	1361	1429	1497	1565	1633	1701	1770	1838	1906	1974	2042
28	1319	1393	1466	1539	1613	1686	1759	1832	1906	1979	2052	2126	2199
30	1413	1492	1571	1649	1728	1806	1885	1963	2042	2120	2199	2277	2356
32	1508	1592	1675	1759	1843	1927	2010	2094	2178	2252	2345	2429	2513
34	1602	1691	1780	1869	1958	2047	2136	2225	2314	2403	2492	2581	2670
36	1696	1791	1885	1978	2073	2168	2262	1326	2450	2545	2639	2733	2827
40	1885	1989	2094	2199	2304	2513	2618	2723	2827	2932	3037	3141	**3246**
48	2262	2387	2513	2639	2765	2890	3016	3142	**3267**	**3393**	**3518**	**3644**	**3769**
54	2545	2686	2827	2969	3110	**3251**	**3393**	**3534**	**3676**	**3817**	**3959**	**4100**	**4240**
60	2827	2984	3141	3298	**3456**	**3613**	**3770**	**3927**	**4084**	**4241**	4398	4555	4712
66	3110	**3283**	**3455**	**3628**	**3801**	**3974**	**4147**	4319	4492	4665	4838	5010	5183
72	**3392**	**3581**	**3770**	**3958**	**4147**	4335	4524	4713	4900	5059	5278	5466	5654
78	**3676**	**3880**	**4084**	4288	4492	4696	4900	5059	5309	5513	5717	5921	6125
84	**3958**	**4178**	4398	4618	4838	5058	5277	5497	5717	5937	6157	6377	6597

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NOTE.—The speeds printed in heavy type are the best for Main Driving Belts.

TABLE XXXV.

Horse-power of Leather Belts per Inch of Width.

Velocity of Belt in Feet per Minute.	Best Oak-tanned Belts.			Best Link or Chain Belts.					
	Single Belts.	Light Double Belts.	Heavy Double Belts.	in. 3/8	in. 1/2	in. 5/8	in. 3/4	in. 7/8	in. 1
100	0·15	0·21	0·27	0·13	0·15	0·17	0·20	0·24	0·27
200	0·30	0·42	0·55	0·25	0·29	0·35	0·40	0·47	0·55
300	0·45	0·64	0·82	0·38	0·44	0·52	0·60	0·71	0·82
400	0·61	0·85	1·09	0·51	0·58	0·69	0·80	0·95	1·09
500	0·76	1·06	1·36	0·64	0·73	0·86	1·00	1·18	1·36
600	0·91	1·27	1·64	0·76	0·87	1·04	1·20	1·42	1·64
700	1·06	1·49	1·91	0·89	1·02	1·21	1·40	1·65	1·91
800	1·21	1·70	2·18	0·92	1·16	1·38	1·60	1·89	2·18
900	1·36	1·91	2·45	1·05	1·31	1·55	1·80	2·13	2·45
1000	1·51	2·12	2·73	1·27	1·45	1·73	2·00	2·36	2·73
1100	1·67	2·33	3·00	1·40	1·60	1·90	2·20	2·60	3·00
1200	1·82	2·55	3·27	1·53	1·75	2·07	2·40	2·84	3·27
1300	1·97	2·76	3·55	1·65	1·89	2·25	2·60	3·07	3·55
1400	2·12	2·97	3·82	1·78	2·04	2·42	2·80	3·31	3·82
1500	2·27	3·18	4·09	1·91	2·18	2·59	3·00	3·55	4·09
1600	2·42	3·39	4·36	2·04	2·33	2·76	3·20	3·78	4·36
1700	2·58	3·61	4·64	2·16	2·47	2·94	3·40	4·02	4·64
1800	2·73	3·82	4·91	2·29	2·62	3·11	3·60	4·25	4·91
1900	2·88	4·03	5·18	2·42	2·76	3·28	3·80	4·49	5·18
2000	3·03	4·24	5·45	2·55	2·91	3·45	4·00	4·73	5·45
2100	3·18	4·45	5·73	2·67	3·05	3·63	4·20	4·96	5·73
2200	3·33	4·67	6·00	2·80	3·20	3·80	4·40	5·20	6·00
2300	3·49	4·88	6·27	2·93	3·35	3·97	4·60	5·44	6·27
2400	3·64	5·09	6·55	3·05	3·49	4·15	4·80	5·67	6·55
2500	3·79	5·30	6·82	3·18	3·64	4·32	5·00	5·91	6·82
2600	3·94	5·52	7·09	3·24	3·78	4·49	5·20	6·15	7·09
2700	4·09	5·73	7·36	3·28	3·85	4·66	5·40	6·38	7·36
2800	4·24	5·94	7·64	3·31	3·86	4·73	5·60	6·62	7·64
2900	4·39	6·15	7·91	3·32	3·87	4·78	5·80	6·85	7·91
3000	4·50	6·36	8·18	3·31	3·86	4·75	5·97	7·09	8·18
3100	4·60	6·58	8·45	3·30	3·85	4·73	5·96	7·33	8·45
3200	4·69	6·79	8·70	3·28	3·82	4·71	5·94	7·37	8·73
3300	4·77	7·00	8·86	3·24	3·77	4·70	5·92	7·35	8·88
3400	4·84	7·21	8·96	3·19	3·71	4·64	5·87	7·32	8·86
3500	4·90	7·31	9·06	3·13	3·61	4·50	5·78	7·26	8·80
3600	4·95	7·40	9·16	3·05	3·50	4·37	5·67	7·16	8·73
3700	4·99	7·48	9·24	2·96	3·39	4·26	5·55	7·01	8·58
3800	5·03	7·54	9·29	2·84	3·28	4·15	5·41	6·87	8·41
3900	5·06	7·60	9·34	2·72	3·13	4·02	5·20	6·70	8·27
4000	5·08	7·64	9·37	2·58	2·95	3·84	5·01	6·48	8·04
4200	5·10	7·70	9·38	2·27	2·55	3·37	4·52	5·98	7·51
4500	5·07	7·69	9·27	1·64	1·77	2·45	3·68	5·05	6·55
5000	4·82	7·42	8·75	0·42	0·55	0·61	1·55	2·78	4·32

(*Reproduced by permission from the "Mechanical World Pocket Diary."*)

Note.—The speeds printed in heavy type are the best for Main Driving Belts.
For Table of Belt Velocities, see Table XXXIV., p. 271.

ROPE DRIVING.*

Undoubtedly every rope has its most suitable groove angle, but as the providing of a range of templates to agree with every advance in size would, perhaps, prove more inconvenient than advantageous, two leading angles have been fixed upon for all driving-ropes—viz. 40 deg. for those above 1¼ in. in diameter, and 30 deg. below that size; excepting, however, the small bands used for driving cotton machinery—say ½ in. in diameter and even less, when angles as sharp as 15 deg. are brought into requisition. In setting out a groove it should always be borne in mind that a rope is to all intents and purposes an elastic wedge, and reaches its highest driving capabilities when it is compressed to the shape of the groove itself. This cuniform ought, therefore, always to be anticipated, for if the sides of the groove are merely shown as tangents to the circle representing a given diameter of rope, there is always the danger of making the groove too shallow, so that when compression takes place the rope is liable to find its way to the bottom, or, what is almost as detrimental, work upon the semi-circle with which grooves generally terminate, when grip is considerably

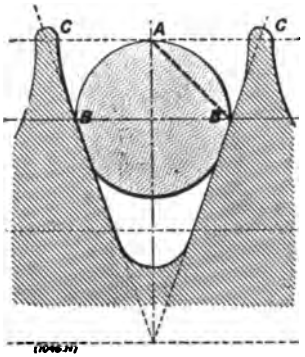

FIG. 1.—How to Set Out a Groove of 40 degrees.

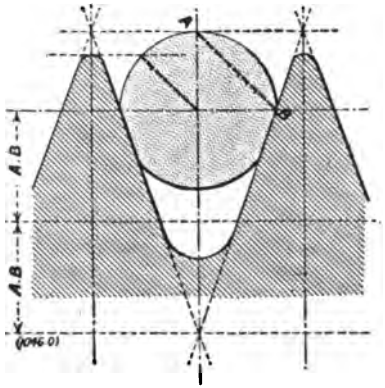

FIG. 2.—Flangeless Groove.

weakened and slipping supervenes, the true indication of which is heated pulley-rims.

How to Set Out a Groove of 40 Deg.—Without appealing to the protractor, a 40-deg. groove may be set out as per illustration (Fig. 1).

First let a circle of equal diameter represent the rope, draw the vertical and horizontal centre lines, the chord AB of the arc. AB then is the position measured downwards from the centre of the rope of the centre for rounding the bottom of the groove, and when doubled along the vertical line points the apex of the inverted angle, the arms of which cut through the circle at BB, and also through the upper horizontal line at CC. These two points mark the centres for the radii of the flange terminals, if such are required, and also indicate the thickness of the metal.

Flangless Grooves.—Finding that superposed flanges do not contribute in any way to better transmission, but rather prevent the rope falling direct into its working position if the driving is at all unsteady, besides crippling any extension in the direction of a larger diameter, many engineers are now adopting flangeless grooves, which are set out in precisely the same manner, only that the angles are extended and simply rounded off at the terminals, as shown in the section (Fig. 2).

The pitch is gained by measuring from centre to centre of mid-feathers along the horizontal line passing through the circle, and this multiplied by the number of ropes, adding the thickness of two apices to represent the extra metal required for outside flanges, will give the total width of the pulley.

Setting Out a 30-Deg. Groove (Fig. 3).—The only alteration in the above method when drawing a 30-deg. groove is the length of the measuring line, which, instead of being represented by the chord AB, is extended to a point where a line resting at 45 deg. from the centre of the rope (Fig. 2) intersects the circle at C (Fig. 3). The line (BC) measures the depth of the groove, and, repeated, points the inverted apex of the angle.

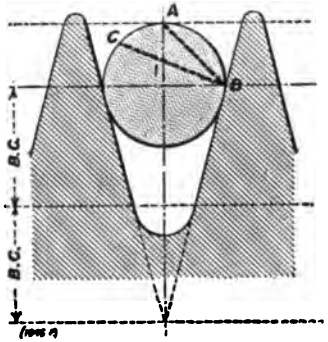

FIG. 3.—Setting Out a 30-degree Groove.

Grooves for Carrier Pulleys.—Grooves having no power to transmit, but which are merely used to bear up or conduct ropes out of the straight driving path, should be so constructed that they (the ropes) may rest easily at the bottom, while the mid-feathers open outwards. The pitch of these grooves should, of course, correspond with the transmitters to which they are tributary.

Plaster Casts of Grooves.—When templates of existing grooves are required, the quickest and surest way is to take plaster casts, the grooves being previously well greased. These give more readily measured definitions than may be secured by attempting to cut pieces of timber, tin, or other available material to the desired shape, which serve rather to perplex than to guide the judgment in the selection of suitable ropes, particularly when the pairs do not agree, although respectively intended to represent the grooves of driving and driven pulleys.

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* Abstract of a paper read before the South Wales Institute of Engineers by Mr. Edwin Kenyon, of Messrs. William Kenyon & Sons, Ltd., Dukinfield, near Manchester, inserted by kind permission of the author.

TABLE XXXVI.

To be used in estimating the size and number of ropes required for Main Driving Pulleys on shafts of FOUR CYLINDER DOUBLE-ACTING (OTTO CYCLE) GAS ENGINES. (For other types, see Tables XXXVII. and XXXVIII.)

Table of Horse-Powers which Good Three-Strand Cotton Driving-Ropes will transmit (allowing a good margin of safety), running upon Pulleys not less than thirty times their respective diameters.*

Rope Diameters	1"	1 1/8"	1 1/4"	1 3/8"	1 1/2"	1 5/8"	1 3/4"	1 7/8"	2"
Minimum Diameter of Smallest Pulley	2' 6"	2' 10"	3' 2"	3' 5"	3' 9"	4' 1"	4' 5"	4' 9"	5'
Velocity in Ft. per Min.									
1000	3·3	4·1	5·1	6·1	7·4	8·6	10	11·5	13
1100	3·6	4·5	5·6	6·7	8·1	9·5	11	12·6	14·3
1200	3·9	4·9	6·1	7·3	8·9	10·3	12	13·8	15·6
1300	4·2	5·3	6·6	8	9·7	11·2	13	14·9	16·9
1400	4·6	5·7	7·1	8·6	10·4	12	14	16	18·3
1500	4·9	6·1	7·6	9·2	11·1	12·9	15	17·2	19·6
1600	5·2	6·6	8·1	9·8	11·9	13·8	16	18·4	20·9
1700	5·5	7	8·6	10·4	12·6	14·6	17	19·5	22·2
1800	5·8	7·4	9·1	11	13·4	15·5	18	20·7	23·5
1900	6·2	7·8	9·6	11·6	14·1	16·3	19	21·8	24·8
2000	6·5	8·2	10·1	12·2	14·9	17·2	20	22·9	26·1
2100	6·8	8·6	10·6	12·8	15·6	18·1	21	24·1	27·4
2200	7·1	9	11·1	10·4	16·3	18·9	22	25·3	28·7
2300	7·5	9·5	11·6	14	17·1	19·8	23	26·4	30
2400	7·8	9·9	12·1	14·7	17·8	20·7	24	27·6	31·3
2500	8·1	10·3	12·6	15·2	18·5	21·5	25	28·7	32·6
2600	8·4	10·7	13·1	15·9	19·2	22·4	26	29·9	33·9
2700	8·7	11·1	13·6	16·5	20	23·3	27	31	35·2
2800	9·1	11·5	14·1	17·1	20·8	24·1	28	32·2	36·5
2900	9·4	11·9	14·6	17·7	21·5	25	29	33·3	37·8
3000	9·7	12·3	15·1	18·0	22·3	25·8	30	34·5	39·1
3100	10	12·7	15·6	18·9	23	26·7	31	35·6	40·4
3200	10·4	13·2	16·2	19·6	23·8	27·6	32	36·8	41·8
3300	10·7	13·6	16·7	20·2	24·6	28·4	33	37·9	43·1
3400	11	14	17·2	20·8	25·3	29·3	34	39·1	44·4
3500	11·3	14·4	17·7	21·4	26	30·1	35	40·2	45·7
3600	11.7	14·8	18·2	22	26·7	31	36	41·4	47
3700	12	15·2	18·7	22·6	27·5	31·9	37	42·5	48·3
3800	12·3	15·6	19·2	23·2	28·2	32·7	38	43·7	49·6

TABLE XXXVI.—*continued.*

Rope Diameters	1"	1 1/8"	1 1/4"	1 3/8"	1 1/2"	1 5/8"	1 3/4"	1 7/8"	2"
Minimum Diameter of Smallest Pulley	2' 6"	2' 10"	3' 2"	3' 5"	3' 9"	4' 1"	4' 5"	4' 9"	5'
Velocity in Ft. per Min.									
3900	12·6	16	19·7	23·8	29	33·6	39	44·8	50·9
4000	13	16·4	20·2	24·5	29·7	34·5	40	46	52·2
4100	13·3	16·9	20·7	25·1	30·4	35·3	41	47·1	53·5
4200	13·6	17·3	21·2	25·7	31·2	36·2	42	48·3	54·8
4300	13·9	17·7	21·7	26·3	31·9	37	43	49·4	56·1
4400	14·3	18·1	22·2	26·9	32·7	37·9	44	50·6	57·4
4500	14·6	18·5	22·7	27·5	33·4	38·8	45	51·7	58·7
4600	14·9	18·9	23·2	28·1	34·2	39·6	46	52·9	60
4700	15·2	19·3	23·7	28·7	34·9	40·5	47	54	61·3
4800	15·6	19·8	24·2	29·4	35·7	41·4	48	55·2	62·7
4900	15·9	20·2	24·7	30	36·4	42·2	49	56·3	64
5000	16·2	20·6	25·3	30·6	37·1	43·1	50	57·5	65·3
5100	16·5	21	25·8	31·2	37·9	43·9	51	58·6	66·6
5200	16·9	21·4	26·3	31·8	38·6	44·8	52	59·8	67·9
5300	17·2	21·8	26·8	32·4	39·4	45·7	53	60·9	69·2
5400	17·5	22·2	27·3	33	40·1	46·5	54	62·1	70·5
5500	17·8	22·6	27·8	33·6	40·9	47·4	55	63·2	71·8
5600	18·2	23·1	28·3	34·3	41·6	48·3	56	64·4	73·1
5700	18·5	23·5	28·8	34·9	42·3	49·1	57	65·5	74·4
5800	18·8	23·9	29·3	35·5	43·1	50	58	66·7	75·7
5900	19·1	24·3	29·8	36·1	43·8	50·8	59	67·8	77
6000	19·5	24·7	30·3	36·7	44·6	51·7	60	69	78·3
6100	19·8	25·1	30·8	37·3	45·3	52·6	61	70·1	79·6
6200	20·1	25·5	31·3	37·9	46·1	53·4	62	71·3	80·9
6300	20·4	25·9	31·8	38·5	46·8	54·3	63	72·4	82·2
6400	20·8	26·4	32·4	39·2	47·6	55·2	64	73·6	83·6
6500	21·1	26·8	32·9	32·8	48·3	56	65	74·7	84·9
6600	21·4	27·2	33·4	40·4	49	56·9	66	75·9	86·2
6700	21·7	27·6	33·9	41	49·8	57·7	67	77	87·5
6800	22·1	28	34·4	41·6	50·5	58·5	68	78·2	88·8
6900	22·4	28·4	34·9	42·2	51·3	59·4	69	79·3	90·1
7000	22·7	28·8	35·4	42·8	52	60·3	70	80·5	91·4

For circumferential speed of pulleys, see Table XXXIV., p. 271.

NOTE.—The most economical velocities for rope drives of this class are from 3000 to 4500 ft. per minute. In no case should the velocity exceed 5000 ft. per minute. The power given in the above Table should in no case be exceeded.

* Reproduced from a paper read before the South Wales Institute of Engineers, by Mr. Edwin Kenyon, of Messrs. William Kenyon & Sons, Ltd., Dukinfield, near Manchester, inserted by kind permission of the author.

TABLE XXXVII.*

To be used in estimating the size and number of ropes for Main Driving Pulleys on shafts of

2 cylinder double-acting (Otto cycle) tandem gas engines or
" " " " twin " " "
4 " single-acting " *vis-à-vis* " "

Table of Horse-Powers which good Three-Strand Cotton Driving Ropes will transmit (allowing a good margin of safety), running upon Pulleys not less than Thirty Times their Respective Diameters.

Diameter of Rope.	1"	1 1/8"	1 1/4"	1 3/8"	1 1/2"	1 5/8"	1 3/4"	1 7/8"	2"
Velocity in ft. per min.									
1000	2·5	3·1	3·9	4·6	5·6	6·5	7·5	8·7	9·8
1100	2·7	3·4	4·2	5·0	6·1	7·1	8·2	9·5	10·7
1200	3·0	3·7	4·6	5·5	6·7	7·8	9·0	10·4	11·6
1300	3·2	4·0	5·0	5·9	7·2	8·4	9·7	11·3	12·7
1400	3·5	4·3	5·4	6·4	7·8	9·1	10·5	12·1	13·7
1500	3·7	4·6	5·8	6·9	8·4	9·7	11·2	13·0	14·7
1600	4·0	4·9	6·2	7·3	8·9	10·4	12·0	13·9	15·6
1700	4·2	5·2	6·6	7·8	9·5	11·0	12·7	14·7	16·6
1800	4·5	5·5	7·0	8·2	10·0	11·7	13·5	15·6	17·6
1900	4·7	5·9	7·4	8·7	10·6	12·3	14·2	16·5	18·6
2000	5·0	6·2	7·8	9·2	11·2	13·0	15·0	17·4	19·6
2100	5·2	6·5	8·2	9·6	11·7	13·6	15·7	18·2	20·5
2200	5·5	6·8	8·5	10·1	12·3	14·3	16·5	19·1	21·5
2300	5·7	7·1	8·9	10·5	12·8	14·9	17·2	20·0	22·5
2400	6·0	7·4	9·3	11·0	13·4	15·6	18·0	20·8	23·5
2500	6·2	7·7	9·7	11·5	14·0	16·2	18·7	21·7	24·5
2600	6·5	8·0	10·1	11·9	14·5	16·9	19·5	22·6	25·4
2700	6·7	8·3	10·5	12·4	15·1	17·5	20·2	23·4	26·4
2800	7·0	8·6	10·9	12·8	15·6	18·2	21·0	24·3	27·4
2900	7·2	9·0	11·3	13·3	16·2	18·8	21·7	25·2	28·4
3000	7·5	9·3	11·7	13·8	16·8	19·5	22·5	26·1	29·4
3100	7·7	9·6	12·1	14·2	17·3	20·1	23·2	26·9	30·3
3200	8·0	9·9	12·4	14·7	17·9	20·8	24·0	27·8	31·3
3300	8·2	10·2	12·8	15·1	18·4	21·4	24·7	28·7	32·3
3400	8·5	10·5	13·2	15·5	19·0	22·1	25·5	29·5	33·3
3500	8·7	10·8	13·6	16·0	19·6	22·7	26·2	30·4	34·3
3600	9·0	11·1	14·0	16·5	20·1	23·4	27·0	31·3	35·2
3700	9·2	11·4	14·4	17·0	20·7	24·0	27·7	32·1	36·2
3800	9·5	11·7	14·8	17·4	21·2	24·7	28·5	33·0	37·2
3900	9·7	12·0	15·2	17·9	21·8	25·3	29·2	33·9	38·2
4000	10·0	12·4	15·6	18·4	22·4	26·0	30·0	34·8	39·2
4100	10·2	12·7	15·9	18·8	22·9	26·6	30·7	35·6	40·1
4200	10·5	13·0	16·3	19·3	23·5	27·2	31·5	36·5	41·1
4300	10·7	13·3	16·7	19·7	24·0	27·9	32·2	37·4	42·1
4400	11·0	13·6	17·1	20·2	24·6	28·6	33·0	38·2	43·1
4500	11·2	13·9	17·5	20·7	25·2	29·2	33·7	39·1	44·1
4600	11·5	14·2	17·9	21·1	25·7	29·9	34·5	40·0	45·0
4700	11·7	14·5	18·3	21·6	26·3	30·5	35·2	40·8	46·0
4800	12·0	14·8	18·7	22·0	26·8	31·2	36·0	41·7	47·0
4900	12·2	15·2	19·1	22·5	27·4	31·8	36·7	42·6	48·0
5000	12·5	15·5	19·5	23·0	28·0	32·5	37·5	43·5	49·0

For circumferential speed of pulleys, see Table XXXIV., p. 271.

NOTE.—The most economical velocities for rope drives of this class are from 3000 to 4500 feet per minute. In no case should the velocity exceed 5000 feet per minute.

In Table XXXVII. a very ample margin of safety has been allowed in order to secure a long life of the ropes; the powers may be increased by 15 per cent. with safety.

* This table has been specially prepared by Messrs. William Kenyon & Sons, Ltd., Dukinfield, near Manchester, for this class of drive.

TABLE XXXVIII.*

To be used in estimating the size and number of ropes for Main Driving Pulleys on shafts of—

Single-cylinder single-acting (Otto cycle) gas engines or
Vis-à-vis ,, ,, ,, ,,
Twin ,, ,, ,, ,,

Table of Horse-Powers which good Three-Strand Cotton Driving Ropes will transmit (allowing a good margin of safety), running upon Pulleys not less than Thirty Times their Respective Diameters.

Diameter of Rope.	1″	1⅛″	1¼″	1⅜″	1½″	1⅝″	1¾″	1⅞″	2″
Velocity in ft. per min.									
1000	1·6	2·0	2·5	3·0	3·7	4·3	5·0	5·7	6·5
1100	1·8	2·2	2·8	3·3	4·0	4·7	5·5	6·3	7·1
1200	1·9	2·4	3·0	3·6	4·4	5·1	6·0	6·9	7·8
1300	2·1	2·6	3·3	4·0	4·8	5·6	6·5	7·4	8·4
1400	2·3	2·8	3·5	4·3	5·2	6·0	7·0	8·0	9·1
1500	2·4	3·0	3·8	4·6	5·5	6·4	7·5	8·6	9·8
1600	2·6	3·3	4·0	4·9	5 9	6·9	8·0	9·2	10·4
1700	2·7	3·5	4·3	5·2	6·3	7·3	8·5	9·7	11·1
1800	2·9	3·7	4·5	5·5	6·7	7·7	9·0	10·3	11·7
1900	3·1	3·9	4·8	5·8	7·0	8·1	9·5	10·9	12·4
2000	3·2	4·1	5·1	6·1	7·4	8·6	10·0	11·4	13·0
2100	3·4	4·3	5·3	6·4	7·8	9·0	10·5	12·0	13·7
2200	3·5	4·5	5·6	6·7	8·1	9·4	11·0	12·6	14·3
2300	3·7	4·7	5·8	7·0	8·5	9·9	11·5	13·2	15·0
2400	3·9	4·9	6·1	7·3	8·9	10·3	12·0	13·8	15·6
2500	4·0	5·1	6·3	7·6	9·2	10·7	12·5	14·3	16·3
2600	4·2	5·3	6·6	7·8	9·6	11·2	13·0	14·9	16·9
2700	4·3	5·5	6·8	8·3	10·0	11·6	13·5	15·5	17·6
2800	4·5	5·7	7·1	8·5	10·4	12·0	14·0	16·1	18·2
2900	4·7	5·9	7·3	8·8	10·7	12·5	14·5	16·6	18·9
3000	4·8	6·1	7·6	9·1	11·1	12·9	15·0	17·2	19·5
3100	5·0	6·3	7·9	9·4	11·5	13·3	15·5	17·8	20·2
3200	5·2	6·6	8·1	9·8	11·9	13·8	16·0	18·4	20·9
3300	5·3	6·8	8·4	10·1	12·3	14·2	16·5	18 9	21·5
3400	5·5	7·0	8·6	10·4	12·6	14·6	17·0	19·5	22·2
3500	5·6	7·2	8·9	10·7	13·0	15·0	17·5	20·1	22·8
3600	5·8	7·4	9·1	11·0	13·4	15·5	18·0	20·7	23·5
3700	6·0	7·6	9·4	11·3	13·7	15·9	18·5	21·2	24·1
3800	6·1	7·8	9·6	11·6	14·1	16·3	19·0	21·8	24·8
3900	6·3	8·0	9·9	11·9	14·5	16·8	19·5	22·4	25·4
4000	6·5	8·2	10·2	12·2	14·8	17·2	20·0	23·0	26·1
4100	6·6	8·4	10·4	12·5	15·2	17·6	20·5	23·5	26·7
4200	6·8	8·6	10·7	12·8	15·6	18·1	21·0	24·1	27·4
4300	6·9	8·8	10·9	13·1	15·9	18·5	21·5	24·7	28·0
4400	7·1	9·0	11·2	13·4	16·4	18·9	22·0	25·3	28·7
4500	7·3	9·2	11·4	13·7	16·7	19·4	22·5	25·8	29·3
4600	7·4	9·4	11·7	14·0	17·1	19·8	23·0	26·4	30·0
4700	7·6	9·6	11·9	14·3	17·4	20·2	23·5	27·0	30·6
4800	7·8	9·9	12·2	14·7	17·8	20·7	24·0	27·6	31·3
4900	7·9	10·1	12·4	15·0	18·2	21·1	24·5	28·1	32·0
5000	8·1	10·3	12·7	15·3	18·5	21·5	25·0	28·7	32·7

For circumferential speed of pulleys, see Table XXXIV., p. 271.

NOTE.—The most economical velocities for rope drives of this class are from 3000 to 4500 feet per minute. In on case should the velocity exceed 5000 feet per minute.

In Table XXXVIII. a very ample margin of safety has been allowed in order to secure a long life of the ropes; the power may be increased by 15 per cent. with safety.

* This table has been specially prepared by Messrs. William Kenyon & Sons, Ltd., Dukinfield, near Manchester, for this class of drive.

TABLE XXXIX.

Table of the Circumferences and Areas of Circles, advancing by 8ths.

Diamr.	0		1/8		1/4		3/8		1/2		5/8		3/4		7/8		Diamr.
	Circm.	Area	Circm.	Area	Circm.	Area	Circm.	Area	Circm.	Area	Circm.	Area	Circm.	Area	Circm.	Area	
0	—	—	·3927	·0123	·7854	·0491	1·178	·1104	1·571	·1963	1·964	·3068	2·356	·4418	2·749	·6013	0
1	3·142	·7854	3·534	·9940	3·927	1·227	4·320	1·485	4·712	1·767	5·105	2·074	5·498	2·405	5·891	2·761	1
2	6·283	3·142	6·676	3·547	7·069	3·976	7·461	4·430	7·854	4·909	8·247	5·412	8·639	5·940	9·032	6·492	2
3	9·425	7·069	9·818	7·670	10·21	8·296	10·60	8·946	11·00	9·621	11·39	10·32	11·78	11·04	12·17	11·79	3
4	12·57	12·57	12·96	13·36	13·35	14·19	13·74	15·03	14·14	15·90	14·53	16·80	14·92	17·72	15·32	18·67	4
5	15·71	19·64	16·10	20·63	16·49	21·65	16·89	22·69	17·28	23·76	17·67	24·85	18·06	25·97	18·46	27·11	5
6	18·85	28·27	19·24	29·46	19·64	30·68	20·03	31·92	20·42	33·18	20·81	34·47	21·21	35·78	21·60	37·12	6
7	21·99	38·48	22·38	39·87	22·78	41·28	23·17	42·72	23·56	44·18	23·95	45·66	24·35	47·17	24·74	48·71	7
8	25·13	50·27	25·53	51·85	25·92	53·46	26·31	55·09	26·70	56·75	27·10	58·43	27·49	60·13	27·88	61·86	8
9	28·27	63·62	28·67	65·40	29·06	67·20	29·45	69·03	29·85	70·88	30·24	72·76	30·63	74·66	31·02	76·59	9
10	31·42	78·54	31·81	80·52	32·20	82·52	32·59	84·54	32·99	86·59	33·38	88·66	33·77	90·76	34·16	92·89	10
11	34·56	95·03	34·95	97·21	35·34	99·40	35·74	101·6	36·13	103·9	36·52	106·1	36·91	108·4	37·31	110·8	11
12	37·70	113·1	38·09	115·5	38·48	117·9	38·88	120·3	39·27	122·7	39·67	125·2	40·06	127·7	40·45	130·2	12
13	40·84	132·7	41·23	135·3	41·63	137·9	42·02	140·5	42·41	143·1	42·80	145·8	43·20	148·5	43·59	151·2	13
14	43·98	153·9	44·38	156·7	44·77	159·5	45·16	162·3	45·55	165·1	45·95	168·0	46·34	170·9	46·73	173·8	14
15	47·12	176·7	47·52	179·7	47·91	182·7	48·30	185·7	48·69	188·7	49·09	191·7	49·48	194·8	49·87	197·9	15
16	50·27	201·1	50·66	204·2	51·05	207·4	51·44	210·6	51·84	213·8	52·23	217·1	52·62	220·4	53·01	223·7	16
17	53·41	227·0	53·80	230·3	54·19	233·7	54·59	237·1	54·98	240·5	55·37	244·0	55·76	247·4	56·16	250·9	17
18	56·55	254·5	56·94	258·0	57·33	261·6	57·73	265·2	58·12	268·8	58·51	272·4	58·90	276·1	59·30	279·8	18
19	59·69	283·5	60·08	287·3	60·48	291·1	60·87	294·8	61·26	298·6	61·65	302·5	62·05	306·4	62·44	310·2	19
Diamr.	Circm.	Area	Circm.	Area	Circm.	Area	Circm.	Area	Circm.	Area	Circm.	Area	Circm.	Area	Circm.	Area	Diamr.
	0		1/8		1/4		3/8		1/2		5/8		3/4		7/8		

Table of the Circumferences and Areas of Circles, advancing by 8ths (continued).

Diamr.	0		1/8		1/4		3/8		1/2		5/8		3/4		7/8		Diamr.
	Circm.	Area	Circm.	Area	Circm.	Area	Circm.	Area	Circm.	Area	Circm.	Area	Circm.	Area	Circm.	Area	
20	62·83	314·2	63·22	318·1	63·62	322·1	64·01	326·1	64·40	330·1	64·80	334·1	65·19	338·2	65·58	342·3	20
21	65·97	346·4	66·37	350·5	66·76	354·7	67·15	358·8	67·54	363·1	67·94	367·3	68·33	371·5	68·72	375·8	21
22	69·12	380·1	69·51	384·5	69·90	388·8	70·29	393·2	70·69	397·6	71·08	402·0	71·47	406·5	71·86	411·0	22
23	72·26	415·5	72·65	420·0	73·04	424·6	73·43	429·1	73·83	433·7	74·22	438·4	74·61	443·0	75·01	447·7	23
24	75·40	452·4	75·79	457·1	76·18	461·9	76·58	466·6	76·97	471·4	77·36	476·3	77·75	481·1	78·15	486·0	24
25	78·54	490·9	78·93	495·8	79·33	500·7	79·72	505·7	80·11	510·7	80·50	515·7	80·90	520·8	81·29	525·8	25
26	81·68	530·9	82·07	536·0	82·47	541·2	82·86	546·4	83·25	551·5	83·64	556·8	84·04	562·0	84·43	567·3	26
27	84·82	572·6	85·22	577·9	85·61	583·2	86·00	588·6	86·39	594·0	86·79	599·4	87·18	604·8	87·57	610·3	27
28	87·96	615·8	88·36	621·3	88·75	626·8	89·14	632·4	89·54	637·9	89·93	643·5	90·32	649·2	90·71	654·8	28
29	91·11	660·5	91·50	666·2	91·89	672·0	92·28	677·7	92·68	683·5	93·07	689·3	93·46	695·1	93·86	701·0	29
30	94·25	706·9	94·64	712·8	95·03	718·7	95·43	724·6	95·82	730·6	96·21	736·6	96·60	742·6	97·00	748·7	30
31	97·39	754·8	97·78	760·9	98·17	767·0	98·57	773·1	98·96	779·3	99·35	785·5	99·75	791·7	100·1	798·0	31
32	100·5	804·2	100·9	810·5	101·3	816·9	101·7	823·2	102·1	829·6	102·5	836·0	102·9	842·4	103·3	848·8	32
33	103·7	855·3	104·1	861·8	104·5	868·3	104·9	874·8	105·2	881·4	105·6	888·0	106·0	894·6	106·4	901·3	33
34	106·8	907·9	107·2	914·6	107·6	921·3	108·0	928·1	108·4	934·8	108·8	941·6	109·2	948·4	109·6	955·3	34
35	110·0	962·1	110·3	969·0	110·7	975·9	111·1	982·8	111·5	989·8	111·9	996·8	112·3	1003·8	112·7	1010·8	35
36	113·1	1017·9	113·5	1025·0	113·9	1032·1	114·3	1039·2	114·7	1046·3	115·1	1053·5	115·5	1060·7	115·8	1068·0	36
37	116·2	1075·2	116·6	1082·5	117·0	1089·8	117·4	1097·1	117·8	1104·5	118·2	1111·8	118·6	1119·2	119·0	1126·7	37
38	119·4	1134·1	119·8	1141·6	120·2	1149·1	120·6	1156·6	121·0	1164·2	121·3	1171·7	121·7	1179·3	122·1	1186·9	38
39	122·5	1194·6	122·9	1202·3	123·3	1210·0	123·7	1217·7	124·1	1225·4	124·5	1233·2	124·9	1241·0	125·3	1248·8	39
Diamr.	Circm.	Area	Circm.	Area	Circm.	Area	Circm.	Area	Circm.	Area	Circm.	Area	Circm.	Area	Circm.	Area	Diamr.
	0		1/8		1/4		3/8		1/2		5/8		3/4		7/8		

TABLE XXXIX. (*continued*).

TABLE OF THE CIRCUMFERENCES AND AREAS OF CIRCLES, ADVANCING BY 8THS (continued).

Diamr.	0		1/8		1/4		3/8		1/2		5/8		3/4		7/8		Diamr.
	Circm.	Area	Circm.	Area	Circm.	Area	Circm.	Area	Circm.	Area	Circm.	Area	Circm.	Area	Circm.	Area	
40	125·7	1256·6	126·1	1264·5	126·4	1272·4	126·8	1280·3	127·2	1288·2	127·6	1296·2	128·0	1304·2	128·4	1312·2	40
41	128·8	1320·3	129·2	1328·3	129·6	1336·4	130·0	1344·5	130·4	1352·7	130·8	1360·8	131·2	1369·0	131·6	1377·2	41
42	131·9	1385·4	132·3	1393·7	132·7	1402·0	133·1	1410·3	133·5	1418·6	133·9	1427·0	134·3	1435·4	134·7	1443·8	42
43	135·1	1452·2	135·5	1460·7	135·9	1469·1	136·3	1477·6	136·7	1486·2	137·1	1494·7	137·4	1503·3	137·8	1511·9	43
44	138·2	1520·5	138·6	1529·2	139·0	1537·9	139·4	1546·6	139·8	1555·3	140·2	1564·0	140·6	1572·8	141·0	1581·6	44
45	141·4	1590·4	141·8	1599·3	142·2	1608·2	142·5	1617·0	142·9	1626·0	143·3	1634·9	143·7	1643·9	144·1	1652·9	45
46	144·5	1661·9	144·9	1671·0	145·3	1680·0	145·7	1689·1	146·1	1698·2	146·5	1707·4	146·9	1716·5	147·3	1725·7	46
47	147·7	1734·9	148·0	1744·2	148·4	1753·5	148·8	1762·7	149·2	1772·1	149·6	1781·4	150·0	1790·8	150·4	1800·1	47
48	150·8	1809·6	151·2	1819·0	151·6	1828·5	152·0	1837·9	152·4	1847·5	152·8	1857·0	153·2	1866·6	153·5	1876·1	48
49	153·9	1885·7	154·3	1895·4	154·7	1905·0	155·1	1914·7	155·5	1924·4	155·9	1934·2	156·3	1943·9	156·7	1953·7	49
50	157·1	1963·5	157·5	1973·3	157·9	1983·2	158·3	1993·1	158·7	2003·0	159·0	2012·9	159·4	2022·8	159·8	2032·8	50
51	160·2	2042·8	160·6	2052·8	161·0	2062·9	161·4	2073·0	161·8	2083·1	162·2	2093·2	162·6	2103·4	163·0	2113·5	51
52	163·4	2123·7	163·8	2133·9	164·1	2144·2	164·5	2154·5	164·9	2164·8	165·3	2175·1	165·7	2185·4	166·1	2195·8	52
53	166·5	2206·2	166·9	2216·6	167·3	2227·0	167·7	2237·5	168·1	2248·0	168·5	2258·5	168·9	2269·1	169·3	2279·6	53
54	169·6	2290·2	170·0	2300·8	170·4	2311·5	170·8	2322·1	171·2	2332·8	171·6	2343·5	172·0	2354·3	172·4	2365·0	54
55	172·8	2375·8	173·2	2386·6	173·6	2397·5	174·0	2408·3	174·4	2419·2	174·8	2430·1	175·1	2441·1	175·5	2452·0	55
56	175·9	2463·0	176·3	2474·0	176·7	2485·0	177·1	2496·1	177·5	2507·2	177·9	2518·3	178·3	2529·4	178·7	2540·6	56
57	179·1	2551·8	179·5	2563·0	179·9	2574·2	180·2	2585·4	180·6	2596·7	181·0	2608·0	181·4	2619·4	181·8	2630·7	57
58	182·2	2642·1	182·6	2653·5	183·0	2664·9	183·4	2676·4	183·8	2687·8	184·2	2699·3	184·6	2710·9	185·0	2722·4	58
59	185·4	2734·0	185·7	2745·6	186·1	2757·2	186·5	2768·8	186·9	2780·5	187·3	2792·2	187·7	2803·9	188·1	2815·7	59
Diamr.	Circm.	Area	Circm.	Area	Circm.	Area	Circm.	Area	Circm.	Area	Circm.	Area	Circm.	Area	Circm.	Area	Diamr.
	0		1/8		1/4		3/8		1/2		5/8		3/4		7/8		

TABLE OF THE CIRCUMFERENCES AND AREAS OF CIRCLES, ADVANCING BY 8THS (continued).

Diamr.	0		1/8		1/4		3/8		1/2		5/8		3/4		7/8		Diamr.
	Circm.	Area	Circm.	Area	Circm.	Area	Circm.	Area	Circm.	Area	Circm.	Area	Circm.	Area	Circm.	Area	
60	188·5	2827·4	188·9	2839·2	189·3	2851·0	169·7	2862·9	190·1	2874·8	190·5	2886·6	190·9	2898·6	191·2	2910·5	60
61	191·6	2922·5	192·0	2934·5	192·4	2946·5	192·8	2958·5	193·2	2970·6	193·6	2982·7	194·0	2994·8	194·4	3006·9	61
62	194·8	3019·1	195·2	3031·3	195·6	3043·5	196·0	3055·7	196·3	3068·0	196·7	3080·2	197·1	3092·6	197·5	3104·9	62
63	197·9	3117·2	198·3	3129·6	198·7	3142·0	199·1	3154·5	199·5	3166·9	199·9	3179·4	200·3	3191·9	200·7	3204·4	63
64	201·1	3217·0	201·5	3229·6	201·8	3242·2	202·2	3254·8	202·6	3267·5	203·0	3280·1	203·4	3292·8	203·8	3305·6	64
65	204·2	3318·3	204·6	3331·1	205·0	3343·9	205·4	3356·7	205·8	3369·6	206·2	3382·4	206·6	3395·3	207·0	3408·3	65
66	207·3	3421·2	207·7	3434·2	208·1	3447·2	208·5	3460·2	208·9	3473·2	209·3	3486·3	209·7	3499·4	210·1	3512·5	66
67	210·5	3525·7	210·9	3538·8	211·3	3552·0	211·7	3565·2	212·1	3578·5	212·5	3591·7	212·8	3605·0	213·2	3618·4	67
68	213·6	3631·7	214·0	3645·1	214·4	3658·4	214·8	3671·8	215·2	3685·3	215·6	3698·7	216·0	3712·2	216·4	3725·7	68
69	216·8	3739·3	217·2	3752·8	217·6	3766·4	217·9	3780·0	218·3	3793·7	218·7	3807·3	219·1	3821·0	219·5	3834·7	69
70	219·9	3848·5	220·3	3862·2	220·7	3876·0	221·1	3889·8	221·5	3903·6	221·9	3917·5	222·3	3931·4	222·7	3945·3	70
71	223·1	3959·2	223·4	3937·2	223·8	3987·1	224·2	4001·1	224·6	4015·2	225·0	4029·2	225·4	4043·3	225·8	4057·4	71
72	226·2	4071·5	226·6	4085·7	227·0	4099·8	227·4	4114·0	227·8	4128·2	228·2	4142·5	228·6	4156·8	228·9	4171·1	72
73	229·3	4185·4	229·7	4199·7	230·1	4214·1	230·5	4228·5	230·9	4242·9	231·3	4257·4	231·7	4271·8	232·1	4286·3	73
74	232·5	4300·9	232·9	4315·4	233·3	4330·0	233·7	4344·6	234·0	4359·2	234·4	4373·8	234·8	4388·5	235·2	4403·2	74
75	235·6	4417·9	236·0	4432·6	236·4	4447·4	236·8	4462·2	237·2	4477·0	237·6	4491·8	238·0	4506·7	238·4	4521·5	75
76	238·8	4536·5	239·2	4551·4	239·5	4566·4	240·0	4581·3	240·3	4596·3	240·7	4611·4	241·1	4626·4	241·5	4641·5	76
77	241·9	4656·6	242·3	4671·8	242·7	4686·9	243·1	4702·1	243·5	4717·3	243·9	4732·5	244·3	4747·8	244·7	4763·1	77
78	245·0	4778·4	245·4	4793·7	245·8	4809·0	246·2	4824·4	246·6	4839·8	247·0	4855·2	247·4	4870·7	247·8	4886·2	78
79	248·2	4901·7	248·6	4917·2	249·0	4932·7	249·4	4948·3	249·8	4963·9	250·1	4979·5	250·5	4995·2	250·9	5010·9	79
Diamr.	Circm.	Area	Circm.	Area	Circm.	Area	Circm.	Area	Circm.	Area	Circm.	Area	Circm.	Area	Circm.	Area	Diamr.
	0		1/8		1/4		3/8		1/2		5/8		3/4		7/8		

TABLE XXXIX. (*continued*).

Table of the Circumferences and Areas of Circles, advancing by 8ths (concluded).

Diamr.	0		⅛		¼		⅜		½		⅝		¾		⅞		Diamr.
	Circm.	Area	Circm.	Area	Circm.	Area	Circm.	Area	Circm.	Area	Circm.	Area	Circm.	Area	Circm.	Area	
80	251·3	5026·5	251·7	5042·3	252·1	5058·0	252·5	5073·8	252·9	5089·6	253·3	5105·4	253·7	5121·2	254·1	5137·1	80
81	254·5	5153·0	254·9	5168·9	255·3	5184·9	255·6	5200·8	256·0	5216·8	256·4	5232·8	256·8	5248·9	257·2	5264·9	81
82	257·6	5281·0	258·0	5297·1	258·4	5313·3	258·8	5329·4	259·2	5345·6	259·6	5361·8	260·0	5378·1	260·4	5394·3	82
83	260·8	5410·6	261·1	5426·9	261·5	5443·3	261·9	5459·6	262·3	5476·0	262·7	5492·4	263·1	5508·8	263·5	5525·3	83
84	263·9	5541·8	264·3	5558·3	264·7	5574·8	265·1	5591·4	265·5	5607·9	265·9	5624·5	266·3	5641·2	266·6	5657·8	84
85	267·0	5674·5	267·4	5691·2	267·8	5707·9	268·2	5724·7	268·6	5741·5	269·0	5758·3	269·4	5775·1	269·8	5791·9	85
86	270·2	5808·8	270·6	5825·7	271·0	5842·6	271·4	5859·6	271·7	5876·5	272·1	5893·5	272·5	5910·6	272·9	5927·6	86
87	273·3	5944·7	273·7	5961·8	274·1	5978·9	274·5	5996·0	274·9	6013·2	275·3	6030·4	275·7	6047·6	276·1	6064·9	87
88	276·5	6082·1	276·9	6099·4	277·2	6116·7	277·6	6134·1	278·0	6151·4	278·4	6168·8	278·8	6186·2	279·2	6203·7	88
89	279·6	6221·2	280·0	6238·6	280·4	6256·1	280·8	6273·7	281·2	6291·2	281·6	6308·8	282·0	6326·4	282·4	6344·1	89
90	282·7	6361·7	283·1	6379·4	283·5	6397·1	283·9	6414·8	284·3	6432·6	284·7	6450·4	285·1	6468·2	285·5	6486·0	90
91	285·9	6503·9	286·3	6521·8	286·7	6539·7	287·1	6557·6	287·5	6575·5	287·8	6593·5	288·2	6611·5	288·6	6629·6	91
92	289·0	6647·6	289·4	6665·7	289·8	6683·8	290·2	6701·9	290·6	6720·1	291·0	6738·2	291·4	6756·4	291·8	6774·7	92
93	292·2	6793·0	292·6	6811·2	293·0	6829·5	293·3	6847·8	293·7	6866·1	294·1	6884·5	294·5	6902·9	294·9	6921·3	93
94	295·3	6939·8	295·7	6958·2	296·1	6976·7	296·5	6995·3	296·9	7013·8	297·3	7032·4	297·7	7051·0	298·1	7069·6	94
95	298·5	7088·2	298·8	7106·9	299·2	7125·6	299·6	7144·3	300·0	7163·0	300·4	7181·8	300·8	7200·6	301·2	7219·4	95
96	301·6	7238·2	302·0	7257·1	302·4	7276·0	302·8	7294·9	303·2	7313·8	303·6	7332·8	303·9	7351·8	304·3	7370·8	96
97	304·7	7389·8	305·1	7408·9	305·5	7428·0	305·9	7447·1	306·3	7466·2	306·7	7485·3	307·1	7504·5	307·5	7523·7	97
98	307·9	7543·0	308·3	7562·2	308·7	7581·5	309·1	7600·8	309·4	7620·1	309·8	7639·5	310·2	7658·9	310·6	7678·3	98
99	311·0	7697·7	311·4	7717·2	311·8	7736·6	312·2	7756·1	312·6	7775·6	313·0	7795·2	313·4	7814·8	313·8	7834·4	99
Diamr.	Circm.	Area	Circm.	Area	Circm.	Area	Circm.	Area	Circm.	Area	Circm.	Area	Circm.	Area	Circm.	Area	Diamr.
	0		⅛		¼		⅜		½		⅝		¾		⅞		

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TABLE XL.

Table of the Circumferences of Circles, advancing by 10ths.

Diamr.	Circumferences										Diamr.
	·0	·1	·2	·3	·4	·5	·6	·7	8	·9	
0	·0000	·3142	·6823	·9425	1·2566	1·5708	1·8850	2·1991	2·5133	2·8274	0
1	3·1416	3·4557	3·7699	4·0840	4·3982	4·7124	5·0265	5·3407	5·6548	5·9690	1
2	6·2832	6·5973	6·9115	7·2256	7·5398	7·8540	8·1681	8·4823	8·7964	9·1106	2
3	9·4248	9·7389	10·0531	10·3672	10·6814	10·9956	11·3097	11·6239	11·9380	12·2522	3
4	12·5664	12·8805	13·1947	13·5088	13·8230	14·1372	14·4513	14·7655	15·0796	15·3938	4
5	15·7080	16·0221	16·3363	16·6504	16·9646	17·2788	17·5929	17·9071	18·2212	18·5354	5
6	18·8496	19·1637	19·4779	19·7920	20·1062	20·4204	20·7345	21·0487	21·3628	21·6770	6
7	21·9912	22·3053	22·6195	22·9336	23·2478	23·5620	23·8761	24·1903	24·5044	24·8186	7
8	25·1328	25·4469	25·7611	26·0752	26·3894	26·7036	27·0177	27·3319	27·6460	27·9602	8
9	28·2744	28·5885	28·9027	29·2168	29·5310	29·8452	30·1593	30·4735	30·7876	31·1018	9
10	31·4160	31·7301	32·0443	32·3580	32·6726	32·9868	33·3009	33·6150	33·9292	34·2434	10
11	34·5576	34·8717	35·1859	35·5010	35·8142	36·1284	36·4425	36·7567	37·0708	37·3840	11
12	37·6992	38·0133	38·3275	38·6416	38·9558	39·2700	39·5841	39·8983	40·2124	40·5266	12
13	40·8408	41·1549	41·4691	41·7832	42·0974	42·4116	42·7257	43·0399	43·3540	43·6682	13
14	43·9824	44·2965	44·6107	44·9248	45·2390	45·5532	45·8673	46·1815	46·4956	46·8098	14
15	47·1240	47·4381	47·7523	48·0664	48·3806	48·6948	49·0089	49·3321	49·6372	49·9514	15
16	50·2656	50·5797	50·8939	51·2080	51·5224	51·8364	52·1505	52·4647	52·7788	53·0930	16
17	53·4072	53·7213	54·0355	54·3496	54·6038	54·9780	55·2921	55·6063	55·9204	56·2346	17
18	56·5488	56·8629	57·1771	57·4912	57·8054	58·1196	58·4337	59·7479	59·0620	59·3762	18
19	59·6904	60·0045	60·3187	60·6328	60·9470	61·2612	61·5753	61·8895	62·2036	62·5178	19
Diamr.	·0	·1	·2	·3	·4	·5	·6	·7	·8	·9	Diamr.
	Circumferences										

TABLE XL. (*continued*).

TABLE OF THE CIRCUMFERENCES OF CIRCLES, ADVANCING BY 10THS (continued).

Diamr.	Circumferences ·0	·1	·2	·3	·4	·5	·6	·7	·8	·9	Diamr.
20	62·8320	63·1461	63·4603	63·7744	64·0886	64·4028	64·7161	65·0311	65·3452	65·6594	20
21	65·9736	66·2870	66·6012	66·7916	67·2930	67·5444	67·8585	68·1727	68·4868	68·8010	21
22	69·1152	69·4293	69·7435	70·0576	70·3718	70·6860	71·0001	71·3143	71·6284	71·9426	22
23	72·2568	72·5709	72·8851	73·1992	73·5134	73·8276	74·1417	74·4559	74·7680	75·0882	23
24	75·3984	75·7125	76·0267	76·3408	76·6523	76·9692	77·2833	77·5975	77·9116	78·2258	24
25	78·5400	78·8541	79·1683	79·4824	79·7966	80·8108	80·4249	80·7391	81·0532	81·3674	25
26	81·6816	82·3099	82·6240	82·6240	82·9382	83·2524	83·5665	83·8807	84·1948	84·5090	26
27	84·8232	85·1373	85·4515	85·7656	86·0798	86·3940	86·7081	87·0223	87·3364	87·6506	27
28	87·9648	88·2789	88·5931	88·9072	89·2214	89·5315	89·8497	90·1639	90·4780	90·7922	28
29	91·1064	91·4205	91·7347	92·0488	92·3630	92·6772	92·9913	93·3055	93·6196	93·9338	29
30	94·2480	94·5621	94·8763	95·1904	95·5046	95·8188	96·1329	96·4471	96·7612	97·0754	30
31	97·3896	97·7037	98·0179	98·3320	98·6452	98·9604	99·2745	99·5887	99·9028	100·217	31
32	100·531	100·845	101·160	101·474	101·748	102·102	102·416	102·730	103·044	103·359	32
33	103·673	103·987	104·301	104·615	104·929	105·244	105·558	105·872	106·186	106·500	33
34	106·814	107·129	107·427	107·757	108·071	108·385	108·699	109·035	109·308	109·642	34
35	109·956	110·270	110·584	110·898	112·213	111·527	111·841	112·155	112·469	112·783	35
36	113·098	113·412	113·726	114·040	114·354	114·668	114·983	115·297	115·611	115·925	36
37	116·239	116·553	116·868	117·182	117·496	117·810	118·124	118·438	118·752	119·067	37
38	119·381	119·695	120·009	120·323	120·637	120·952	121·266	121·580	121·894	122·208	38
39	122·522	122·837	123·151	123·465	123·779	124·093	124·407	124·722	125·036	125·350	39
Diamr.	·0	·1	·2	·3	·4	·5	·6	·7	·8	·9	Diamr.
	Circumferences										

TABLE OF THE CIRCUMFERENCES OF CIRCLES, ADVANCING BY 10THS (continued).

Diamr.	Circumferences ·0	·1	·2	·3	·4	·5	·6	·7	·8	·9	Diamr.
40	125·664	125·978	126·292	126·606	126·921	127·235	127·549	127·863	128·177	128·491	40
41	128·806	129·120	129·432	129·748	130·062	130·376	130·691	131·005	131·319	131·632	41
42	131·947	132·261	132·576	132·890	133·204	133·518	133·832	134·146	134·460	134·775	42
43	135·089	135·403	135·717	136·033	136·345	136·660	136·974	137·288	137·602	137·916	43
44	138·230	138·545	138·859	139·173	139·487	139·801	140·115	140·430	140·744	141·058	44
45	141·372	141·686	142·000	142·314	142·629	142·943	143·257	143·571	143·885	144·199	45
46	144·514	144·828	145·142	145·456	145·770	146·084	146·399	146·713	147·027	147·341	46
47	147·655	147·969	148·284	148·598	148·912	149·226	149·536	149·854	150·168	150·483	47
48	150·797	151·111	151·425	151·739	152·053	152·368	152·682	152·996	153·310	153·624	48
49	153·938	154·253	154·567	154·881	155·195	155·509	155·823	156·138	156·452	156·756	49
50	157·080	157·394	157·708	158·022	158·337	158·651	158·965	159·279	159·593	159·907	50
51	160·222	160·536	160·850	161·164	161·478	161·792	162·107	162·421	162·734	163·049	51
52	163·363	163·677	163·994	164·306	164·620	164·934	165·248	165·562	165·876	166·191	52
53	166·505	166·819	167·133	167·447	167·761	168·076	168·390	168·705	169·018	169·332	53
54	169·646	169·961	170·275	170·589	170·903	171·217	171·531	171·846	172·160	172·474	54
55	172·788	173·102	173·416	173·730	174·045	174·359	174·673	174·977	175·309	175·615	55
56	175·930	176·244	176·558	176·872	177·186	177·500	177·815	178·129	178·443	178·757	56
57	179·071	179·385	179·700	180·014	180·328	180·642	180·956	181·280	181·584	181·899	57
58	182·213	182·527	182·841	183·155	183·469	183·784	184·098	184·412	184·726	185·040	58
59	185·354	185·669	185·983	186·270	186·611	186·925	187·239	187·554	187·868	188·182	59
Diamr.	·0	·1	·2	·3	·4	·5	·6	·7	·8	·9	Diamr.
	Circumferences										

TABLE XL. (*continued*).

Table of the Circumferences of Circles, advancing by 10ths (continued).

Diamr.	Circumferences ·0	·1	·2	·3	·4	·5	·6	·7	·8	·9	Diamr.
60	188·496	188·810	189·124	189·438	189·753	190·067	190·381	190·695	191·009	191·323	60
61	191·638	191·952	192·266	192·580	192·894	193·208	193·523	193·837	194·151	194·465	61
62	194·779	195·093	195·408	195·722	196·036	196·350	196·664	196·978	197·292	197·607	62
63	197·921	198·235	198·549	198·863	199·177	199·492	199·806	200·120	200·434	200·748	63
64	201·062	201·377	201·691	202·005	202·319	202·633	202·947	203·262	203·576	203·890	64
65	204·204	204·518	204·832	205·146	205·461	205·775	206·089	206·403	206·717	207·031	65
66	207·346	207·660	207·974	208·288	208·602	208·916	209·231	209·545	209·859	210·173	66
67	210·487	210·801	211·116	211·430	211·744	212·058	212·372	212·686	213·000	213·315	67
68	213·629	213·943	214·257	214·571	214·885	215·200	215·514	215·828	216·142	216·456	68
69	216·770	217·085	217·399	217·713	218·027	218·341	218·655	218·970	219·284	219·598	69
70	219·912	220·226	220·540	220·854	221·169	221·483	221·797	222·111	222·425	222·739	70
71	223·054	223·368	223·682	223·996	224·310	224·624	224·939	225·253	225·567	225·881	71
72	226·195	226·509	226·824	227·138	227·452	227·766	228·080	228·394	228·708	229·023	72
73	229·337	229·651	229·965	230·279	230·593	230·908	231·222	231·536	231·850	232·164	73
74	232·478	232·793	233·107	233·421	233·735	234·049	234·363	234·678	234·992	235·306	74
75	235·620	235·934	236·248	236·562	236·877	237·191	237·505	237·819	238·133	238·447	75
76	238·762	239·076	239·390	239·704	240·018	240·332	240·647	240·961	241·275	241·599	76
77	241·903	242·217	242·532	242·846	243·160	243·474	243·788	244·102	244·416	244·731	77
78	245·045	245·359	245·673	245·987	246·301	246·616	246·930	247·244	247·548	247·872	78
79	248·186	248·501	248·815	249·129	249·443	249·757	250·071	250·386	250·700	251·014	79
Diamr.	·0	·1	·2	·3	·4	·5	·6	·7	·8	·9	Diamr.
	Circumferences										

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Table of the Circumferences of Circles, advancing by 10ths (concluded).

Diamr.	Circumferences ·0	·1	·2	·3	·4	·5	·6	·7	·8	·9	Diamr.
80	251·328	251·624	251·956	252·270	252·585	252·899	253·213	253·527	253·841	254·155	80
81	254·470	254·784	255·098	255·412	255·726	256·040	256·355	256·669	256·983	257·297	81
82	257·611	257·925	258·240	258·554	258·865	259·182	259·496	259·810	260·124	260·439	82
83	260·753	261·067	261·381	261·695	262·009	262·324	262·638	262·952	263·264	263·580	83
84	263·894	264·209	264·523	264·837	235·151	265·465	265·779	266·094	266·408	266·722	84
85	267·036	267·350	267·664	267·978	268·293	268·607	268·921	269·235	269·549	269·863	85
86	270·178	270·492	270·806	271·120	271·434	271·748	272·067	272·377	272·691	273·005	86
87	273·319	273·633	273·988	274·262	274·576	274·890	275·204	275·518	275·832	276·147	87
88	276·461	276·775	277·089	277·403	277·717	278·032	278·346	278·660	278·975	279·288	88
89	279·602	279·917	280·231	280·545	280·859	281·173	281·487	281·883	282·116	282·430	89
90	282·744	283·058	283·372	283·686	284·001	284·315	284·629	284·943	285·257	285·571	90
91	285·886	286·200	286·514	286·829	287·142	287·456	287·771	288·085	288·399	288·713	91
92	289·027	289·341	289·656	289·970	290·284	290·598	290·912	291·226	291·540	291·855	92
93	292·169	292·483	292·797	293·111	293·425	293·740	294·054	294·368	294·682	294·996	93
94	295·310	295·625	295·939	296·244	296·567	296·881	297·195	297·510	297·824	298·138	94
95	298·452	298·766	299·072	299·394	299·709	300·023	300·337	300·651	300·965	301·279	95
96	301·594	301·908	302·222	302·536	302·850	303·164	303·479	303·793	304·107	304·421	96
97	304·735	305·049	305·364	305·678	305·992	306·306	306·620	306·936	307·248	307·563	97
98	307·877	308·191	308·505	308·819	309·133	309·448	309·762	310·076	310·396	310·704	98
99	311·018	311·333	311·647	311·961	312·275	312·589	312·903	313·218	313·512	313·846	99
Diamr.	·0	·1	·2	·3	·4	·5	·6	·7	·8	·9	Diamr.
	Circumferences										

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TABLE XLI.

Table of the Areas of Circles, advancing by 10ths.

Diamr.	Areas ·0	·1	·2	·3	·4	·5	·6	·7	·8	·9	Diamr.
0	·0000	·0078	·0314	·0706	·1256	·1963	·2827	·3848	·5026	·6361	0
1	·7854	·9503	1·1309	1·3273	1·5393	1·7671	2·0106	2·2698	2·5446	2·8352	1
2	3·1416	3·4636	3·8013	4·1547	4·5239	4·9087	5·3093	5·7255	6·1575	6·6052	2
3	7·0686	7·5476	8·0424	8·5530	9·0792	9·6211	10·1787	10·7521	11·3411	11·9459	3
4	12·5664	13·2025	13·8544	14·5220	15·2053	15·9043	16·6190	17·3494	18·0956	18·8574	4
5	19·6350	20·4282	21·2372	22·0618	22·9022	23·7583	24·6301	25·5176	26·4208	27·3397	5
6	28·2744	29·2247	30·1907	31·1725	32·1699	33·1831	34·2120	35·2566	36·3168	37·3928	6
7	38·4846	39·5920	40·7151	41·8539	43·0085	44·1787	45·3647	46·5663	47·7837	49·0168	7
8	50·2656	51·5300	52·8102	54·1062	55·4178	56·7451	58·0881	59·4469	60·8213	62·2115	8
9	63·6174	65·0389	66·4762	67·9292	69·3979	70·8823	72·3824	73·8982	75·4298	76·9770	9
10	78·5400	80·1186	81·7130	83·2320	84·9488	86·5903	88·2475	89·9204	91·6090	93·3133	10
11	95·0334	96·7691	98·5205	100·2877	102·0705	103·8691	105·6834	107·5134	109·3590	111·2204	11
12	113·0976	114·9904	116·8989	118·8231	120·7631	122·7187	124·6901	126·6771	128·6799	130·6984	12
13	132·7326	134·7824	136·8480	138·9294	141·0264	143·1391	145·2675	147·4117	149·5715	151·7471	13
14	153·9384	156·1453	158·3680	160·6064	162·8605	165·1303	167·4158	169·7170	172·0340	174·3666	14
15	176·7150	179·0790	181·4588	183·8542	186·2654	188·6923	191·1349	193·5932	196·0672	198·5569	15
16	201·0624	203·5835	206·1209	208·6723	211·1411	213·8251	216·4248	219·0402	221·6712	224·3189	16
17	226·9086	229·6588	232·3527	235·0623	237·7877	240·5287	243·2855	246·0579	248·8461	251·6500	17
18	254·4696	257·3048	260·1558	263·0226	265·9050	268·8031	271·7169	274·6465	277·5917	280·5527	18
19	283·5294	286·5217	289·5298	292·5536	295·5931	298·6483	301·7192	304·8060	307·9082	311·0252	19
Diamr.	·0	·1	·2	·3	·4	·5	·6	·7	·8	·9	Diamr.
	Areas										

Table of the Areas of Circles, advancing by 10ths (continued).

Diamr.	Areas ·0	·1	·2	·3	·4	·5	·6	·7	·8	·9	Diamr.
20	314·1600	317·3094	320·4746	323·6554	326·8520	330·0643	333·2923	336·3360	339·7954	343·0705	20
21	346·3614	349·6679	352·9901	356·3281	359·6817	363·0511	366·4362	369·8370	373·2534	376·6856	21
22	380·1336	383·5972	387·0765	390·5751	394·0823	397·6087	401·1509	404·7087	408·2823	411·8716	22
23	415·4766	419·0972	422·7336	426·3858	430·0536	433·7371	437·4363	441·1511	444·8819	448·6283	23
24	452·3904	456·1681	459·9616	463·7708	467·5957	471·4363	475·2926	479·1646	483·0524	486·9558	24
25	490·8750	494·8098	498·7604	502·7266	506·7086	510·7063	514·7196	518·7488	522·7936	526·8541	25
26	530·9304	535·0223	539·1299	543·2533	547·3923	551·5471	555·7176	559·9038	564·1056	568·3232	26
27	572·5566	576·8056	581·0703	585·3507	589·6469	593·9587	598·2863	602·6295	606·9885	611·3632	27
28	615·7536	620·1596	624·5814	629·0190	633·4722	637·9411	642·4257	646·9261	651·4421	655·9739	28
29	660·5214	665·0845	669·6634	674·2580	678·8683	683·4943	688·1360	692·7934	697·4666	702·1554	29
30	706·8600	711·5802	716·3162	721·0678	725·8352	730·6183	735·4171	740·2316	745·0618	749·9077	30
31	754·7694	759·6467	764·5397	769·4485	774·3729	779·3131	784·2689	789·2406	794·2278	799·2308	31
32	804·2496	809·2840	814·3341	819·3999	824·4815	829·5787	834·6917	839·8203	844·9647	850·1248	32
33	855·3006	860·4920	865·6992	870·9222	876·1608	881·4151	886·6851	891·9709	897·2723	902·5895	33
34	907·9224	913·2709	918·6352	924·0115	929·4109	934·8223	940·2494	945·6922	951·1508	956·6250	34
35	962·1150	967·6206	973·1420	978·6790	984·2318	989·8003	995·3845	1000·984	1006·600	1012·231	35
36	1017·878	1023·541	1029·220	1034·913	1040·624	1046·349	1052·090	1057·8474	1063·620	1069·408	36
37	1075·213	1081·032	1086·868	1092·719	1098·586	1104·469	1110·367	1116·281	1122·211	1128·156	37
38	1134·118	1140·095	1146·087	1152·095	1158·119	1164·159	1170·215	1176·286	1182·373	1188·465	38
39	1194·539	1200·727	1206·877	1213·042	1219·224	1225·420	1231·633	1237·861	1244·121	1250·365	39
Diamr.	·0	·1	·2	·3	·4	·5	·6	·7	·8	·9	Diamr.
	Areas										

TABLE XLI. (*continued*).

Table of the Areas of Circles, advancing by 10ths (continued).

Diamr.	Areas ·0	·1	·2	·3	·4	·5	·6	·7	·8	·9	Diamr.
40	1256·640	1262·931	1269·239	1275·560	1281·898	1288·252	1294·622	1301·007	1307·408	1313·825	40
41	1320·257	1326·706	1333·169	1339·649	1346·144	1352·655	1359·182	1365·724	1372·282	1378·856	41
42	1385·446	1392·051	1398·672	1405·308	1411·961	1418·629	1425·313	1432·012	1438·727	1445·458	42
43	1452·205	1458·967	1465·745	1472·539	1479·348	1486·173	1493·014	1499·871	1506·743	1513·629	43
44	1520·534	1527·454	1534·389	1541·340	1548·306	1555·288	1562·286	1569·300	1576·329	1583·374	44
45	1590·435	1597·511	1604·604	1611·711	1618·835	1625·974	1633·129	1640·302	1647·486	1654·689	45
46	1661·906	1669·140	1676·389	1683·654	1690·935	1698·231	1705·543	1712·871	1720·214	1727·574	46
47	1734·949	1742·339	1749·746	1757·168	1764·605	1772·059	1779·528	1787·013	1794·513	1802·030	47
48	1809·562	1817·109	1824·673	1832·252	1839·847	1847·457	1855·083	1862·725	1870·383	1878·056	48
49	1885·745	1893·450	1901·171	1908·907	1916·659	1924·426	1932·210	1940·009	1947·823	1955·654	49
50	1963·500	1971·362	1979·239	1987·133	1995·042	2002·966	2010·907	2018·863	2026·835	2034·877	50
51	2042·825	2050·844	2058·878	2066·929	2074·995	2083·077	2091·175	2099·288	2107·417	2115·561	51
52	2123·722	2131·898	2140·089	2148·297	2156·520	2164·759	2173·013	2181·284	2189·570	2197·871	52
53	2206·189	2214·522	2222·870	2231·235	2239·615	2248·011	2256·423	2264·870	2273·293	2281·752	53
54	2290·226	2298·717	2307·222	2315·744	2324·281	2332·834	2341·403	2349·987	2358·588	2367·203	54
55	2375·835	2384·482	2393·145	2401·824	2410·518	2419·228	2427·954	2436·696	2445·453	2454·226	55
56	2463·014	2471·819	2480·639	2489·475	2498·326	2507·193	2516·076	2524·974	2533·889	2542·819	56
57	2551·765	2560·726	2569·703	2578·696	2587·705	2596·729	2605·769	2614·824	2623·896	2632·983	57
58	2642·086	2651·205	2660·338	2669·488	2678·654	2687·835	2697·032	2706·245	2715·473	2724·718	58
59	2733·977	2743·253	2752·544	2761·851	2771·174	2780·512	2789·866	2799·236	2808·622	2818·023	59
Diamr.	·0	·1	·2	·3	·4	·5	·6	·7	·8	·9	Diamr.
	Areas										

Table of the Areas of Circles, advancing by 10ths (continued).

Diamr.	Areas ·0	·1	·2	·3	·4	·5	·6	·7	·8	·9	Diamr.
60	2827·440	2836·873	2846·321	2855·785	2865·265	2874·760	2884·262	2893·798	2903·341	2912·899	60
61	2922·473	2932·063	2941·669	2957·290	2960·927	2970·579	2980·247	2989·931	2999·630	3009·346	61
62	3019·078	3028·824	3038·587	3048·365	3058·159	3067·969	3077·794	3087·634	3097·492	3107·364	62
63	3117·253	3127·156	3137·076	3147·011	3156·966	3166·929	3176·912	3186·910	3196·924	3206·953	63
64	3216·998	3227·059	3237·136	3247·228	3257·337	3267·460	3277·600	3287·755	3297·926	3308·113	64
65	3318·315	3328·534	3338·767	3349·016	3359·281	3369·562	3379·859	3390·171	3400·499	3410·843	65
66	3421·202	3431·578	3441·963	3452·375	3462·797	3473·235	3483·689	3494·164	3504·643	3515·143	66
67	3525·661	3536·193	3546·741	3557·304	3567·884	3578·479	3589·090	3599·716	3610·358	3621·016	67
68	3631·690	3642·379	3653·084	3663·804	3674·541	3685·293	3696·006	3706·845	3717·644	3728·459	68
69	3739·289	3750·136	3760·998	3771·876	3782·769	3793·678	3804·603	3815·544	3826·500	3837·472	69
70	3848·460	3859·495	3870·483	3881·517	3892·568	3903·634	3914·716	3925·814	3936·927	3948·057	70
71	3959·201	3970·362	3981·538	3992·730	4003·937	4015·161	4026·400	4037·655	4048·925	4060·212	71
72	4071·514	4082·833	4094·165	4105·513	4116·879	4128·259	4139·652	4151·067	4162·494	4173·938	72
73	4185·397	4196·871	4208·361	4219·868	4231·390	4242·927	4254·480	4266·049	4277·634	4289·234	73
74	4300·850	4312·482	4324·130	4335·793	4347·472	4359·166	4370·877	4382·603	4394·345	4406·102	74
75	4417·875	4429·664	4441·468	4453·289	4465·125	4476·976	4488·844	4500·727	4512·626	4524·540	75
76	4536·470	4548·416	4560·379	4572·355	4584·358	4596·357	4608·382	4620·422	4632·478	4644·549	76
77	4656·637	4668·740	4680·858	4692·993	4705·143	4717·309	4729·490	4741·688	4753·961	4766·129	77
78	4778·374	4790·634	4802·909	4815·201	4827·508	4839·831	4852·170	4864·524	4876·897	4889·280	78
79	4901·681	4914·099	4926·531	4938·982	4951·444	4963·924	4976·484	4988·931	5001·459	5014·001	79
Diamr.	·0	·1	·2	·3	·4	·5	·6	·7	·8	·9	Diamr.
	Areas										

TABLE XLI. (*continued*).

Table of the Areas of Circles, advancing by 10ths (concluded).

Diamr.	Areas ·0	·1	·2	·3	·4	·5	·6	·7	·8	·9	Diamr.
80	5026·560	5039·134	5051·724	5064·330	5076·955	5089·588	5102·241	5114·910	5127·594	5140·294	80
81	5153·009	5165·741	5178·488	5191·251	5204·029	5216·823	5229·633	5242·459	5255·300	5268·157	81
82	5281·030	5293·918	5306·822	5319·744	5332·678	5345·629	5358·596	5371·598	5384·576	5397·591	82
83	5410·621	5423·666	5436·727	5449·804	5462·897	5476·005	5489·129	5502·269	5515·424	5528·596	83
84	5541·702	5554·985	5568·203	5581·437	5594·687	5607·952	5621·233	5634·568	5647·843	5661·171	84
85	5674·515	5687·875	5701·250	5714·641	5728·048	5741·470	5754·909	5768·362	5781·832	5795·317	85
86	5808·818	5822·335	5835·868	5849·416	5862·980	5876·559	5890·154	5903·765	5917·392	5931·034	86
87	5944·693	5958·364	5972·056	5985·769	5999·482	6013·219	6026·971	6040·739	6054·515	6068·322	87
88	6082·138	6095·968	6109·815	6123·677	6137·555	6151·449	6165·359	6179·284	6193·225	6207·181	88
89	6221·153	6235·141	6249·145	6263·164	6277·200	6291·204	6305·317	6319·399	6333·497	6347·681	89
90	6361·740	6375·885	6390·046	6404·222	6418·414	6432·622	6446·844	6461·085	6475·340	6489·611	90
91	6503·897	6518·200	6532·517	6546·891	6561·208	6575·565	6589·946	6604·322	6618·754	6633·182	91
92	6647·626	6662·085	6676·560	6691·016	6705·557	6720·079	6734·617	6749·170	6763·739	6778·324	92
93	6792·925	6807·541	6822·173	6836·821	6851·484	6866·163	6880·858	6895·569	6910·295	6925·037	93
94	6939·794	6954·568	6969·357	6984·161	6998·982	7013·818	7028·670	7043·503	7058·418	7073·320	94
95	7088·235	7103·165	7118·112	7133·073	7148·051	7163·044	7178·053	7193·078	7208·118	7223·175	95
96	7238·246	7253·334	7268·437	7283·556	7298·691	7313·841	7329·007	7344·189	7359·386	7374·600	96
97	7389·829	7405·073	7420·334	7435·610	7450·901	7466·209	7481·532	7496·871	7512·225	7527·596	97
98	7542·982	7558·383	7573·801	7589·234	7604·683	7620·147	7635·627	7651·193	7666·635	7682·162	98
99	7697·705	7713·264	7728·839	7744·429	7760·035	7775·656	7791·294	7806·947	7822·615	7838·300	99
Diamr.	·0	·1	·2	·3	·4	·5	·6	·7	·8	·9	Diamr.
	Areas										

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TABLE XLII.

Table of the Circumferences of Circles, advancing by 12ths.

Diamr.	Circumferences 0	$\frac{1}{12}$	$\frac{2}{12}$	$\frac{3}{12}$	$\frac{4}{12}$	$\frac{5}{12}$	$\frac{6}{12}$	$\frac{7}{12}$	$\frac{8}{12}$	$\frac{9}{12}$	$\frac{10}{12}$	$\frac{11}{12}$	Diamr.
0	·00000	·26180	·52360	·78540	1·0472	1·3090	1·5708	1·8326	2·0944	2·3562	2·6180	2·8798	0
1	3·1416	3·4034	3·6652	3·9270	4·1888	4·4506	4·7124	4·9742	5·2360	5·4978	5·7596	6·0214	1
2	6·2832	6·5450	6·8068	7·0686	7·3304	7·5922	7·8540	8·1158	8·3776	8·6394	8·9012	9·1630	2
3	9·4248	9·6866	9·9484	10·2102	10·4720	10·7338	10·9956	11·2574	11·5192	11·7810	12·0428	12·3046	3
4	12·5664	12·8282	13·0900	13·3518	13·6136	13·8754	14·1372	14·3990	14·6608	14·9226	15·1844	15·4462	4
5	15·7080	15·9698	16·2316	16·4934	16·7552	17·0170	17·2788	17·5406	17·8024	18·0642	18·3260	18·5878	5
6	18·8496	19·1114	19·3732	19·6350	19·8968	20·1586	20·4204	20·6822	20·9440	21·2058	21·4676	21·7294	6
7	21·9912	22·2530	22·5148	22·7766	23·0384	23·3002	23·5620	23·8238	24·0856	24·3474	24·6092	24·8710	7
8	25·1328	25·3946	25·6564	25·9182	26·1800	26·4418	26·7036	26·9654	27·2272	27·4890	27·7508	28·0126	8
9	28·2744	28·5362	28·7980	29·0598	29·3216	29·5834	29·8452	30·1070	30·3688	30·6306	30·8924	31·1542	9
10	31·4160	31·6778	31·9396	32·2014	32·4632	32·7250	32·9868	32·2486	33·5104	33·7722	34·0340	34·2958	10
11	34·5576	34·8194	35·0812	35·3430	35·6048	35·8666	36·1284	36·3902	36·6520	36·9138	37·1756	34·4374	11
12	37·6992	37·9610	38·2228	38·4846	38·7464	39·0082	39·2700	39·5318	39·7936	40·0554	40·3172	40·5790	12
13	40·8408	41·1026	41·3644	41·6262	41·8880	42·1498	42·4116	42·6734	42·9352	43·1970	43·4588	43·7206	13
14	43·9824	44·2442	44·5060	44·7678	45·0296	45·2914	45·5532	45·8150	46·0678	46·3386	46·6004	46·8622	14
15	47·1240	47·3858	47·6476	47·9094	48·1712	48·4330	48·6948	48·9566	49·2184	49·4802	49·7420	50·0038	15
16	50·2656	50·5274	50·7892	51·0510	51·3128	51·5746	51·8364	52·0982	52·3600	52·6218	52·8836	53·1454	16
17	53·4072	53·6690	53·9308	54·1926	54·4544	54·7162	54·9780	55·2398	55·5016	55·7634	56·0252	56·2870	17
18	56·5488	56·8106	57·0724	57·3342	57·5960	57·8578	58·1196	58·3814	58·6432	58·9050	59·1668	59·4286	18
19	59·6904	59·9522	60·2140	60·4758	60·7376	60·9994	61·2612	61·5230	61·7848	62·0466	62·3084	62·5702	19
Diamr.	0	$\frac{1}{12}$	$\frac{2}{12}$	$\frac{3}{12}$	$\frac{4}{12}$	$\frac{5}{12}$	$\frac{6}{12}$	$\frac{7}{12}$	$\frac{8}{12}$	$\frac{9}{12}$	$\frac{10}{12}$	$\frac{11}{12}$	Diamr.
	Circumferences												

TABLE XLII. (*continued*).

Table of the Circumferences of Circles, advancing by 12ths (concluded).

Diamr.	Circumferences												Diamr.
	0	1/12	2/12	3/12	4/12	5/12	6/12	7/12	8/12	9/12	10/12	11/12	
20	62·8320	63·0938	63·3556	63·6174	63·8792	64·1410	64·4028	64·6646	64·9264	65·1882	65·4500	65·7118	20
21	65·9736	66·2354	66·4972	66·7590	67·0208	67·2826	67·5444	67·8062	68·0680	68·3298	68·5916	68·8534	21
22	69·1152	69·3770	69·6388	69·9006	70·1624	70·4242	70·6860	70·9478	71·2096	71·4714	71·7332	71·9950	22
23	72·2568	72·5186	72·7804	73·0422	73·3040	73·5658	73·8276	74·0894	74·3512	74·6130	74·8748	75·1366	23
24	75·3984	75·6602	75·9220	76·1838	76·4456	76·7074	76·9692	77·2310	77·4928	77·7546	78·0164	78·2782	24
25	78·5400	78·8018	79·0636	79·3254	79·5872	79·8490	80·1108	80·3726	80·6344	80·8962	81·1580	81·4198	25
26	81·6816	81·9434	82·2052	82·4670	82·7288	82·9906	83·2524	83·5142	83·7760	84·0378	84·2996	84·5614	26
27	84·8232	85·0850	85·3468	85·6086	85·8704	86·1322	86·3940	86·6558	86·9176	87·1794	87·4412	87·7030	27
28	87·9648	88·2266	88·4884	88·7502	89·0120	89·2738	89·5356	89·7974	90·0592	90·3210	90·5828	90·8446	28
29	91·1064	91·3682	91·6300	91·8918	92·1536	92·4154	92·6772	92·9390	93·2008	93·4626	93·7244	93·9862	29
30	94·2480	94·5098	94·7716	95·0334	95·2952	95·5570	95·8186	96·0806	96·3424	96·6042	96·8660	92·1278	30
31	97·3896	97·6514	97·9132	98·1750	98·4368	98·6986	98·9604	99·2222	99·4840	99·7458	100·008	100·269	31
32	100·531	100·793	101·055	101·317	101·578	101·840	102·102	102·364	102·626	102·887	103·149	103·411	32
33	103·673	103·935	104·196	104·458	104·720	104·982	105·244	105·505	105·767	106·029	106·291	106·553	33
34	106·814	107·076	107·338	107·600	107·862	108·123	108·385	108·647	108·909	109·171	109·432	109·694	34
35	109·956	110·218	110·480	110·741	111·003	111·265	111·527	111·789	112·050	112·312	112·574	112·836	35
36	113·098	113·359	113·621	113·883	114·145	114·407	114·668	114·930	115·192	115·454	115·776	115·977	36
37	116·239	116·501	116·763	117·025	117·286	117·548	117·810	118·072	118·334	118·595	118·857	119·119	37
38	119·381	119·643	119·904	120·166	120·428	120·690	120·952	121·213	121·475	121·737	121·999	122·261	38
39	122·522	122·784	123·046	123·308	123·570	123·831	124·093	124·355	124·617	124·879	125·140	125·402	39
Diamr.	0	1/12	2/12	3/12	4/12	5/12	6/12	7/12	8/12	9/12	10/12	11/12	Diamr.
	Circumferences												

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TABLE XLIII.

Table of the Areas of Circles, advancing by 12ths.

Diamr.	Areas												Diamr.
	0	1/12	2/12	3/12	4/12	5/12	6/12	7/12	8/12	9/12	10/12	11/12	
0	·0000	·0055	·0218	·0491	·0873	·1364	·1963	·2673	·3491	·4418	·5454	·6600	0
1	·7854	·9218	1·0690	1·2272	1·3963	1·5763	1·7671	1·9689	2·1817	2·4053	2·6398	2·8852	1
2	3·1416	3·4088	3·6870	3·9761	4·2761	4·5869	4·9087	5·2414	5·5851	5·9396	6·3050	6·6813	2
3	7·0686	7·4667	7·8758	8·2958	8·7266	9·1684	9·6211	10·0847	10·5592	11·0447	11·5410	12·0482	3
4	12 5664	13·0954	13·6354	14·1863	14·7480	15·3207	15·9043	16·4988	17·1042	17·7205	18·3478	18·9859	4
5	19·6350	20·2949	20·9658	21·6475	22·3402	23·0438	23·7583	24·4837	25·2200	25·9672	26·7254	27·4944	5
6	28·2743	29·0652	29·8669	30·6796	31·5032	32·3377	33·1831	34·0394	34·9066	35·7847	36·6737	37·5737	6
7	38·4846	39·4063	40·3389	41·2825	42·2370	43·2024	44·1786	45·1658	46·1640	47·1730	48·1929	49·2237	7
8	50·2655	51·3181	52·3817	53·4562	54·5415	55·6378	56·7450	57·8631	58·9921	60·1320	61·2829	62·4446	8
9	63·6173	64·8008	65·9953	67·2006	68·4169	69·6441	70·8822	72·1312	73·3911	74·6619	75·9436	77·2363	9
10	78·5398	79·8543	81·1796	82·5159	83·8631	85·2212	86·5901	87·9700	89·3609	90·7626	92·1752	93·5987	10
11	95·0332	96·4785	97·9348	99·4020	100·880	102·369	103·869	105·380	106·901	108·434	109·978	111·532	11
12	113·097	114·674	116·261	117·859	119·468	121·088	122·719	124·360	126·013	127·676	129·351	131·036	12
13	132·732	134·439	136·158	137·887	139·626	141·377	143·139	144·911	146·695	148·489	150·295	152·111	13
14	153·938	155·776	157·625	159·485	161·356	163·237	165·130	167·034	168·948	170·873	172·809	174·757	14
15	176·715	178·684	180·663	182·654	184·656	186·663	188·692	190·726	192·772	194·828	196·895	198·973	15
16	201·062	203·162	205·273	207·394	209·527	211·670	213·825	215·990	218·166	220·353	222·551	224·760	16
17	226·980	229·211	231·453	233·705	235·969	238·243	240·528	242·824	245·132	247·450	249·778	252·118	17
18	254·469	256·831	259·203	261·587	263·981	266·386	268·803	271·230	273·668	276·117	278·576	281·047	18
19	283·529	286·021	288·525	291·039	293·564	296·111	298·648	301·206	303·775	306·354	308·945	311·547	19
Diamr.	0	1/12	2/12	3/12	4/12	5/12	6/12	7/12	8/12	9/12	10/12	11/12	Diamr.
	Areas												

TABLE XLIII. (*continued*).

Table of the Areas of Circles, advancing by 12ths (concluded).

Diamr.	Areas												Diamr.
	0	$\frac{1}{12}$	$\frac{2}{12}$	$\frac{3}{12}$	$\frac{4}{12}$	$\frac{5}{12}$	$\frac{6}{12}$	$\frac{7}{12}$	$\frac{8}{12}$	$\frac{9}{12}$	$\frac{10}{12}$	$\frac{11}{12}$	
20	314·159	316·783	319·417	322·062	324·719	327·386	330·064	332·753	335·452	338·163	340·885	343·617	20
21	346·361	349·115	351·880	354·656	357·443	360·241	363·050	365·870	368·701	371·542	374·395	377·258	21
22	380·133	383·018	385·914	388·821	391·739	394·668	397·608	400·559	403·520	406·493	409·476	412·470	22
23	415·476	418·492	421·519	424·557	427·606	430·665	433·736	436·818	439·910	443·014	446·128	449·253	23
24	452·389	455·536	458·694	461·863	465·043	468·234	471·435	474·648	477·871	481·106	484·351	487·607	24
25	490·874	494·152	497·441	500·740	504·051	507·373	510·705	514·049	517·403	520·768	524·144	527·531	25
26	530·929	534·338	537·758	541·188	544·630	548·083	551·546	555·020	558·505	562·002	565·509	569·026	26
27	572·555	576·095	579·646	583·207	586·780	590·363	593·957	597·563	601·179	604·806	608·444	612·092	27
28	615·752	619·423	623·104	626·797	630·500	634·215	637·940	641·676	645·423	649·181	652·950	656·729	28
29	660·520	664·321	668·134	671·957	675·792	679·637	683·493	687·360	691·238	695·127	699·026	702·937	29
30	706·858	710·791	714·734	718·688	722·654	726·630	730·617	734·615	738·623	742·643	746·674	750·715	30
31	754·768	758·831	762·905	766·990	771·087	775·193	779·311	783·440	787·580	791·730	795·892	800·064	31
32	804·248	808·442	812·647	816·863	821·090	825·328	829·577	833·837	838·107	842·389	846·681	850·984	32
33	855·299	859·624	863·960	868·307	872·665	877·033	881·413	885·804	890·205	894·618	899·041	903·475	33
34	907·920	912·376	916·843	921·321	925·810	930·310	934·820	939·342	943·874	948·417	952·972	957·537	34
35	962·113	966·700	971·298	975·906	980·526	985·157	989·798	994·450	999·114	1003·79	1008·47	1013·17	35
36	1017·88	1022·59	1027·32	1032·06	1036·81	1041·57	1046·35	1051·13	1055·92	1060·73	1060·55	1070·37	36
37	1075·21	1080·06	1084·92	1089·79	1094·67	1099·56	1104·47	1109·38	1114·31	1119·24	1124·19	1129·15	37
38	1134·11	1139·09	1144·09	1149·09	1154·10	1159·12	1164·16	1169·20	1174·26	1179·32	1184·40	1189·49	38
39	1194·59	1199·70	1204·82	1209·96	1215·10	1220·25	1225·42	1230·59	1235·78	1240·98	1246·19	1251·41	39
Diamr.	0	$\frac{1}{12}$	$\frac{2}{12}$	$\frac{3}{12}$	$\frac{4}{12}$	$\frac{5}{12}$	$\frac{6}{12}$	$\frac{7}{12}$	$\frac{8}{12}$	$\frac{9}{12}$	$\frac{10}{12}$	$\frac{11}{12}$	Diamr.
	Areas												

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TABLE XLIV.

Table of the Areas of the Segments of a Circle, the Diameter being Unity.

To find the area of the segment of any circle from the following table.

Rule.—Divide the height of the segment by the diameter, take out the corresponding tabular area, which multiply by the square of the diameter for the result.

$\frac{H}{D}$	Area	$\frac{H}{D}$	Area	$\frac{H}{D}$	Area	$\frac{H}{D}$	Area
·001	·000042	·038	·009763	·075	·026761	·112	·048262
·002	·000119	·039	·010148	·076	·027289	·113	·048894
·003	·000219	·040	·010537	·077	·027821	·114	·049528
·004	·000337	·041	·010931	·078	·028356	·115	·050165
·005	·000470	·042	·011330	·079	·028894	·116	·050804
·006	·000618	·043	·011734	·080	·029435	·117	·051446
·007	·000779	·044	·012142	·081	·029979	·118	·052090
·008	·000951	·045	·012554	·082	·030526	·119	·052736
·009	·001135	·046	·012971	·083	·031076	·120	·053385
·010	·001329	·047	·013392	·084	·031629	·121	·054036
·011	·001533	·048	·013818	·085	·032186	·122	·054689
·012	·001746	·049	·014247	·086	·032745	·123	·055345
·013	·001968	·050	·014681	·087	·033307	·124	·056003
·014	·002199	·051	·015119	·088	·033872	·125	·056663
·015	·002438	·052	·015561	·089	·034441	·126	·057326
·016	·002685	·053	·016007	·090	·035011	·127	·057991
·017	·002940	·054	·016457	·091	·035585	·128	·058658
·018	·003202	·055	·016911	·092	·036162	·129	·059327
·019	·003471	·056	·017369	·093	·036741	·130	·059999
·020	·003748	·057	·017831	·094	·037323	·131	·060672
·021	·004031	·058	·018296	·095	·037909	·132	·061348
·022	·004322	·059	·018766	·096	·038496	·133	·062026
·023	·004618	·060	·019239	·097	·039087	·134	·062707
·024	·004921	·061	·019716	·098	·039680	·135	·063389
·025	·005230	·062	·020196	·099	·040276	·136	·064074
·026	·005546	·063	·020680	·100	·040875	·137	·064760
·027	·005867	·064	·021168	·101	·041476	·138	·065449
·028	·006194	·065	·021659	·102	·042080	·139	·066140
·029	·006527	·066	·022154	·103	·042687	·140	·066833
·030	·006865	·067	·022652	·104	·043296	·141	·067528
·031	·007209	·068	·023154	·105	·043908	·142	·068225
·032	·007558	·069	·023659	·106	·044522	·143	·068924
·033	·007913	·070	·024168	·107	·045139	·144	·069625
·034	·008273	·071	·024680	·108	·045759	·145	·070328
·035	·008638	·072	·025195	·109	·046381	·146	·071033
·036	·009008	·073	·025714	·110	·047005	·147	·071741
·037	·009383	·074	·026236	·111	·047632	·148	·072450

TABLE XLIV. (*continued*).

Table of the Areas of the Segments of a Circle, the Diameter being Unity (continued).

H/D	Area	H/D	Area	H/D	Area	H/D	Area
·149	·073161	·193	·106261	·237	·142387	·281	·180918
·150	·073874	·194	·107051	·238	·143238	·282	·181817
·151	·074589	·195	·107842	·239	·144091	·283	·182718
·152	·075306	·196	·108636	·240	·144944	·284	·183619
·153	·076026	·197	·109430	·241	·145799	·285	·184521
·154	·076747	·198	·110226	·242	·146655	·286	·185425
·155	·077469	·199	·111024	·243	·147512	·287	·186329
·156	·078194	·200	·111823	·244	·148371	·288	·187234
·157	·078921	·201	·112624	·245	·149230	·289	·188140
·158	·079649	·202	·113426	·246	·150091	·290	·189047
·159	·080380	·203	·114230	·247	·150953	·291	·189955
·160	·081112	·204	·115035	·248	·151816	·292	·190864
·161	·081846	·205	·115842	·249	·152680	·293	·191775
·162	·082582	·206	·116650	·250	·153546	·294	·192684
·163	·083320	·207	·117460	·251	·154412	·295	·193596
·164	·084059	·208	·118271	·252	·155280	·296	·194509
·165	·084801	·209	·119083	·253	·156149	·297	·195422
·166	·085544	·210	·119897	·254	·157019	·298	·196337
·167	·086289	·211	·120712	·255	·157890	·299	·197252
·168	·087036	·212	·121529	·256	·158762	·300	·198168
·169	·087785	·213	·122347	·257	·159636	·301	·199085
·170	·088535	·214	·123167	·258	·160510	·302	·200003
·171	·089287	·215	·123988	·259	·161386	·303	·200922
·172	·090041	·216	·124810	·260	·162263	·304	·201841
·173	·090797	·217	·125634	·261	·163140	·305	·202761
·174	·091554	·218	·126459	·262	·164019	·306	·203683
·175	·092313	·219	·127285	·263	·164899	·307	·204605
·176	·093074	·220	·128113	·264	·165780	·308	·205527
·177	·093836	·221	·128942	·265	·166663	·309	·206451
·178	·094601	·222	·129773	·266	·167546	·310	·207376
·179	·095366	·223	·130605	·267	·168430	·311	·208301
·180	·096134	·224	·131438	·268	·169315	·312	·209227
·181	·096903	·225	·132272	·269	·170202	·313	·210154
·182	·097674	·226	·133108	·270	·171089	·314	·211082
·183	·098447	·227	·133945	·271	·171978	·315	·212011
·184	·099221	·228	·134784	·272	·172867	·316	·212940
·185	·099997	·229	·135624	·273	·173758	·317	·213871
·186	·100774	·230	·136465	·274	·174649	·318	·214802
·187	·101553	·231	·137307	·275	·175542	·319	·215733
·188	·102334	·232	·138150	·276	·176435	·320	·216666
·189	·103116	·233	·138995	·277	·177330	·321	·217599
·190	·103900	·234	·139841	·278	·178225	·322	·218533
·191	·104685	·235	·140688	·279	·179122	·323	·219468
·192	·105472	·236	·141537	·280	·180019	·324	·220404

Table of the Areas of the Segments of a Circle, the Diameter being Unity (concluded).

H/D	Area	H/D	Area	H/D	Area	H/D	Area
·325	·221340	·369	·263213	·413	·306140	·457	·349752
·326	·222277	·370	·264178	·414	·307125	·458	·350748
·327	·223215	·371	·265144	·415	·308110	·459	·351745
·328	·224154	·372	·266111	·416	·309095	·460	·352742
·329	·225093	·373	·267078	·417	·310081	·461	·353739
·330	·226033	·374	·268045	·418	·311068	·462	·354736
·331	·226974	·375	·269013	·419	·312054	·463	·355732
·332	·227915	·376	·269982	·420	·313041	·464	·356730
·333	·228858	·377	·270951	·421	·314029	·465	·357727
·334	·229801	·378	·271920	·422	·315016	·466	·358725
·335	·230745	·379	·272890	·423	·316004	·467	·359723
·336	·231689	·380	·273861	·424	·316992	·468	·360721
·337	·232634	·381	·274832	·425	·317981	·469	·361719
·338	·233580	·382	·275803	·426	·318970	·470	·362717
·339	·234526	·383	·276775	·427	·319959	·471	·363715
·340	·235473	·384	·277748	·428	·320948	·472	·364713
·341	·236421	·385	·278721	·429	·321938	·473	·365712
·342	·237369	·386	·279694	·430	·322928	·474	·366710
·343	·238318	·387	·280668	·431	·323918	·475	·367709
·344	·239268	·388	·281642	·432	·324909	·476	·368708
·345	·240218	·389	·282617	·433	·325900	·477	·369707
·346	·241169	·390	·283592	·434	·326892	·478	·370706
·347	·242121	·391	·284568	·435	·327882	·479	·371705
·348	·243074	·392	·285544	·436	·328874	·480	·372704
·349	·244026	·393	·286521	·437	·329866	·481	·373703
·350	·244980	·394	·287498	·438	·330858	·482	·374702
·351	·245934	·395	·288476	·439	·331850	·483	·375702
·352	·246889	·396	·289453	·440	·332843	·484	·376702
·353	·247845	·397	·290432	·441	·333836	·485	·377701
·354	·248801	·398	·291411	·442	·334829	·486	·378701
·355	·249757	·399	·292390	·443	·335822	·487	·379700
·356	·250715	·400	·293369	·444	·336816	·488	·380700
·357	·251673	·401	·294349	·445	·337810	·489	·381699
·358	·252631	·402	·295330	·446	·338804	·490	·382699
·359	·253590	·403	·296311	·447	·339798	·491	·383699
·360	·254550	·404	·297292	·448	·340793	·492	·384699
·361	·255510	·405	·298273	·449	·341787	·493	·385699
·362	·256471	·406	·299255	·450	·342782	·494	·386699
·363	·257433	·407	·300238	·451	·343777	·495	·387699
·364	·258395	·408	·301220	·452	·344772	·496	·388699
·365	·259357	·409	·302203	·453	·345768	·497	·389699
·366	·260320	·410	·303187	·454	·346764	·498	·390699
·367	·261284	·411	·304171	·455	·347759	·499	·391699
·368	·262248	·412	·305155	·456	·348755	·500	·392699

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TABLE XLV.

Table of Squares, Cubes, Square Roots, Cube Roots, and Reciprocals of all Integer Numbers from 1 to 2200.

No.	Square	Cube	Square Root	Cube Root	Reciprocal
1	1	1	1·0000000	1·0000000	1·000000000
2	4	8	1·4142136	1·2599210	·500000000
3	9	27	1·7320508	1·4422496	·333333333
4	16	64	2·0000000	1·5874011	·250000000
5	25	125	2·2360680	1·7099759	·200000000
6	36	216	2·4494897	1·8171206	·166666667
7	49	343	2·6457513	1·9129312	·142857143
8	64	512	2·8284271	2·0000000	·125000000
9	81	729	3·0000000	2·0800837	·111111111
10	1 00	1 000	3·1622777	2·1544347	·100000000
11	1 21	1 331	3·3166248	2·2239801	·090909091
12	1 44	1 728	3·4641016	2·2894286	·083333333
13	1 69	2 197	3·6055513	2·3513347	·076923077
14	1 96	2 744	3·7416574	2·4101422	·071428571
15	2 25	3 375	3·8729833	2·4662121	·066666667
16	2 56	4 096	4·0000000	2·5198421	·062500000
17	2 89	4 913	4·1231056	2·5712816	·058823529
18	3 24	5 832	4·2426407	2·6207414	·055555556
19	3 61	6 859	4·3588989	2·6684016	·052631579
20	4 00	8 000	4·4721360	2·7144177	·050000000
21	4 41	9 261	4·5825757	2·7589243	·047619048
22	4 84	10 648	4·6904158	2·8020393	·045454545
23	5 29	12 167	4·7958315	2·8438670	·043478261
24	5 76	13 824	4·8989795	2·8844991	·041666667
25	6 25	15 625	5·0000000	2·9240177	·040000000
26	6 76	17 576	5·0990195	2·9624960	·038461538
27	7 29	19 683	5·1961524	3·0000000	·037037037
28	7 84	21 952	5·2915026	3·0365889	·035714286
29	8 41	24 389	5·3851648	3·0723168	·034482759
30	9 00	27 000	5·4772256	3·1072325	·033333333
31	9 61	29 791	5·5677644	3·1413806	·032258065
32	10 24	32 768	5·6568542	3·1748021	·031250000
33	10 89	35 937	5·7445626	3·2075343	·030303030
34	11 56	39 304	5·8309519	3·2396118	·029411765
35	12 25	42 875	5·9160798	3·2710663	·028571429
36	12 96	46 656	6·0000000	3·3019272	·027777778
37	13 69	50 653	6·0827625	3·3322218	·027027027
38	14 44	54 872	6·1644140	3·3619754	·026315789
39	15 21	59 319	6·2449980	3·3912114	·025641026
40	16 00	64 000	6·3245553	3·4199519	·025000000
41	16 81	68 921	6·4031242	3·4482172	·024390244
42	17 64	74 088	6·4807407	3·4760266	·023809524
43	18 49	79 507	6·5574385	3·5033981	·023255814
44	19 36	85 184	6·6332496	3·5303483	·022727273
45	20 25	91 125	6·7082039	3·5568933	·022222222
46	21 16	97 336	6·7823300	3·5830479	·021739130
47	22 09	103 823	6·8556546	3·6088261	·021276600
48	23 04	110 592	6·9282032	3·6342411	·020833333
49	24 01	117 649	7·0000000	3·6593057	·020408163
50	25 00	125 000	7·0710678	3·6840314	·020000000
51	26 01	132 651	7·1414284	3·7084298	·019607843
52	27 04	140 608	7·2111026	3·7325111	·019230769
53	28 09	148 877	7·2801099	3·7562858	·018867925
54	29 16	157 464	7·3484692	3·7797631	·018518519
55	30 25	166 375	7·4161985	3·8029525	·018181818
56	31 36	175 616	7·4833148	3·8258624	·017857143
57	32 49	185 193	7·5498344	3·8485011	·017543860
58	33 64	195 112	7·6157731	3·8708766	·017241379
59	34 81	205 379	7·6811457	3·8929965	·016949153
60	36 00	216 000	7·7459667	3·9148676	·016666667
61	37 21	226 981	7·8102497	3·9364972	·016393443
62	38 44	238 328	7·8740079	3·9578915	·016129032
63	39 69	250 047	7·9372539	3·9790571	·015873016
64	40 96	262 144	8·0000000	4·0000000	·015625000
65	42 25	274 625	8·0622577	4·0207256	·015384615
66	43 56	287 496	8·1240384	4·0412401	·015151515
67	44 89	300 763	8·1853528	4·0615480	·014925373
68	46 24	314 432	8·2462113	4·0816551	·014705882
69	47 61	328 509	8·3066239	4·1015661	·014492754
70	49 00	343 000	8·3666003	4·1212853	·014285714
71	50 41	357 911	8·4261498	4·1408178	·014084507
72	51 84	373 248	8·4852814	4·1601676	·013888889
73	53 29	389 017	8·5440037	4·1793392	·013698630
74	54 76	405 224	8·6023253	4·1983364	·013513514
75	56 25	421 875	8·6602540	4·2171633	·013333333
76	57 76	438 976	8·7177979	4·2358236	·013157895
77	59 29	456 533	8·7749644	4·2543210	·012987013
78	60 84	474 552	8·8317609	4·2726586	·012820513
79	62 41	493 039	8·8881944	4·2908404	·012658228
80	64 00	512 000	8·9442719	4·3088695	·012500000
81	65 61	531 441	9·0000000	4·3267487	·012345679
82	67 24	551 368	9·0553851	4·3444815	·012195122
83	68 89	571 787	9·1104336	4·3620707	·012048193
84	70 56	592 704	9·1651514	4·3795191	011904762
85	72 25	614 125	9·2195445	4·3968296	·011764706
86	73 96	636 056	9·2736185	4·4140049	·011627907
87	75 69	658 503	9·3273791	4·4310476	·011494253
88	77 44	681 472	9·3808315	4·4479602	·011363636
89	79 21	704 969	9·4339811	4·4647451	·011235955
90	81 00	729 000	9·4868330	4·4814047	·011111111
91	82 81	753 571	9·5393920	4·4979414	·010989011
92	84 64	778 688	9·5916630	4·5143574	·010869565
93	86 49	804 357	9·6436508	4·5306549	·010752688
94	88 36	830 584	9·6953597	4·5468359	·010638298

TABLE XLV. (*continued*).

No.	Square	Cube	Square Root	Cube Root	Reciprocal
95	90 25	857 375	9·7467943	4·5629026	·010526316
96	92 16	884 736	9·7979590	4·5788570	·010416667
97	94 09	912 673	9·8488578	4·5947009	·010309278
98	96 04	941 192	9·8994949	4·6104363	·010204082
99	98 01	970 299	9·9498744	4·6260650	·010101010
100	1 00 00	1 000 000	10·0000000	4·6415888	·010000000
101	1 02 01	1 030 301	10·0498756	4·6570095	·009900990
102	1 04 04	1 061 208	10·0995049	4·6723287	·009803922
103	1 06 09	1 092 727	10·1488916	4·6875482	·009708738
104	1 08 16	1 124 864	10·1980390	4·7026694	·009615385
105	1 10 25	1 157 625	10·2469508	4·7176940	·009523810
106	1 12 36	1 191 016	10·2956301	4·7326235	·009433962
107	1 14 49	1 225 043	10·3440804	4·7474594	·009345794
108	1 16 64	1 259 712	10·3923048	4·7622032	·009259259
109	1 18 81	1 295 029	10·4403065	4·7768562	·009174312
110	1 21 00	1 331 000	10·4880885	4·7914199	·009090909
111	1 23 21	1 367 631	10·5356538	4·8058955	·009009009
112	1 25 44	1 404 928	10·5830052	4·8202845	·008928571
113	1 27 69	1 442 897	10·6301458	4·8345881	·008849558
114	1 29 96	1 481 544	10·6770783	4·8488076	·008771930
115	1 32 25	1 520 875	10·7238053	4·8629442	·008695652
116	1 34 56	1 560 896	10·7703296	4·8769990	·008620690
117	1 36 89	1 601 613	10·8166538	4·8909732	·008547009
118	1 39 24	1 643 032	10·8627805	4·9048681	·008474576
119	1 41 61	1 685 159	10·9087121	4·9186847	·008403361
120	1 44 00	1 728 000	10·9544512	4·9324242	·008333333
121	1 46 41	1 771 561	11·0000000	4·9460874	·008264463
122	1 48 84	1 815 848	11·0453610	4·9596757	·008196721
123	1 51 29	1 860 867	11·0905365	4·9731898	·008130081
124	1 53 76	1 906 624	11·1355287	4·9866310	·008064516
125	1 56 25	1 953 125	11·1803399	5·0000000	·008000000
126	1 58 76	2 000 376	11·2249722	5·0132979	·007936508
127	1 61 29	2 048 383	11·2694277	5·0265257	·007874016
128	1 63 84	2 097 152	11·3137085	5·0396842	·007812500
129	1 66 41	2 146 689	11·3578167	5·0527743	·007751938
130	1 69 00	2 197 000	11·4017543	5·0657970	·007692308
131	1 71 61	2 248 091	11·4455231	5·0787531	·007633588
132	1 74 24	2 299 968	11·4891253	5·0916434	·007575758
133	1 76 89	2 352 637	11·5325626	5·1044687	·007518797
134	1 79 56	2 406 104	11·5758369	5·1172299	·007462687
135	1 82 25	2 460 375	11·6189500	5·1299278	·007407407
136	1 84 96	2 515 456	11·6619038	5·1425632	·007352941
137	1 87 69	2 571 353	11·7046999	5·1551367	·007299270
138	1 90 44	2 628 072	11·7473401	5·1676493	·007246377
139	1 93 21	2 685 619	11·7898261	5·1801015	·007194245
140	1 96 00	2 744 000	11·8321596	5·1924941	·007142857
141	1 98 81	2 803 221	11·8743422	5·2048279	·007092199
142	2 01 64	2 863 288	11·9163753	5·2171034	·007042254
143	2 04 49	2 924 207	11·9582607	5·2293215	·006993007
144	2 07 36	2 985 984	12·0000000	5·2414828	·006944444
145	2 10 25	3 048 625	12·0415946	5·2535879	·006896552
146	2 13 16	3 112 136	12·0830460	5·2656374	·006849315
147	2 16 09	3 176 523	12·1243557	5·2776321	·006802721
148	2 19 04	3 241 792	12·1655251	5·2895725	·006756757
149	2 22 01	3 307 949	12·2065556	5·3014592	·006711409
150	2 25 00	3 375 000	12·2474487	5·3132928	·006666667
151	2 28 01	3 442 951	12·2882057	5·3250740	·006622517
152	2 31 04	3 511 808	12·3288280	5·3368033	·006578947
153	2 34 09	3 581 577	12·3693169	5·3484812	·006535948
154	2 37 16	3 652 264	12·4096736	5·3601084	·006493506
155	2 40 25	3 723 875	12·4498996	5·3716854	·006451613
156	2 43 36	3 796 416	12·4899960	5·3832126	·006410256
157	2 46 49	3 869 893	12·5299641	5·3946907	·006369427
158	2 49 64	3 944 312	12·5698051	5·4061202	·006329114
159	2 52 81	4 019 679	12·6095202	5·4175015	·006289308
160	2 56 00	4 096 000	12·6491106	5·4288352	·006250000
161	2 59 21	4 173 281	12·6885775	5·4401218	·006211180
162	2 62 44	4 251 528	12·7279221	5·4513618	·006172840
163	2 65 69	4 330 747	12·7671453	5·4625556	·006134969
164	2 68 96	4 410 944	12·8062485	5·4737037	·006097561
165	2 72 25	4 492 125	12·8452326	5·4848066	·006060606
166	2 75 56	4 574 296	12·8840987	5·4958647	·006024096
167	2 78 89	4 657 463	12·9228480	5·5068784	·005988024
168	2 82 24	4 741 632	12·9614814	5·5178484	·005952381
169	2 85 61	4 826 809	13·0000000	5·5287748	·005917160
170	2 89 00	4 913 000	13·0384048	5·5396583	·005882353
171	2 92 41	5 000 211	13·0766968	5·5504991	·005847953
172	2 95 84	5 088 448	13·1148770	5·5612978	·005813953
173	2 99 29	5 177 717	13·1529464	5·5720546	·005780347
174	3 02 76	5 268 024	13·1909060	5·5827702	·005747126
175	3 06 25	5 359 375	13·2287566	5·5934447	·005714286
176	3 09 76	5 451 776	13·2664992	5·6040787	·005681818
177	3 13 29	5 545 233	13·3041347	5·6146724	·005649718
178	3 16 84	5 639 752	13·3416641	5·6252263	·005617978
179	3 20 41	5 735 339	13·3790882	5·6357408	·005586592
180	3 24 00	5 832 000	13·4164079	5·6462162	·005555556
181	3 27 61	5 929 741	13·4536240	5·6566528	·005524862
182	3 31 24	6 028 568	13·4907376	5·6670511	·005494505
183	3 34 89	6 128 487	13·5277493	5·6774114	·005464481
184	3 38 56	6 229 504	13·5646600	5·6877340	·005434783
185	3 42 25	6 331 625	13·6014705	5·6980192	·005405405
186	3 45 96	6 434 856	13·6381817	5·7082675	·005376344
187	3 49 69	6 539 203	13·6747943	5·7184791	·005347594
188	3 53 44	6 644 672	13·7113092	5·7286543	·005319149
189	3 57 21	6 751 269	13·7477271	5·7387936	·005291005
190	3 61 00	6 859 000	13·7840488	5·7488971	·005263158
191	3 64 81	6 967 871	13·8202750	5·7589652	·005235602
192	3 68 64	7 077 888	13·8564065	5·7689982	·005208333

TABLE XLV. (*continued*).

No.	Square	Cube	Square Root	Cube Root	Reciprocal
193	8 72 49	7 189 057	13·8924440	5·7789966	·005181347
194	8 76 36	7 301 384	13·9283883	5·7889604	·005154639
195	8 80 25	7 414 875	13·9642400	5·7988900	·005128205
196	8 84 16	7 529 536	14·0000000	5·8087857	·005102041
197	8 88 09	7 645 373	14·0356688	5·8186479	·005076142
198	8 92 04	7 762 392	14·0712473	5·8284767	·005050505
199	8 96 01	7 880 599	14·1067360	5·8382725	·005025126
200	4 00 00	8 000 000	14·1421356	5·8480355	·005000000
201	4 04 01	8 120 601	14·1774469	5·8577660	·004975124
202	4 08 04	8 242 408	14·2126704	5·8674643	·004950495
203	4 12 09	8 365 427	14·2478068	5·8771307	·004926108
204	4 16 16	8 489 664	14·2828569	5·8867653	·004901961
205	4 20 25	8 615 125	14·3178211	5·8963685	·004878049
206	4 24 36	8 741 816	14·3527001	5·9059406	·004854369
207	4 28 49	8 869 743	14·3874946	5·9154817	·004830918
208	4 32 64	8 998 912	14·4222051	5·9249921	·004807692
209	4 36 81	9 129 329	14·4568323	5·9344721	·004784689
210	4 41 00	9 261 000	14·4913767	5·9439220	·004761905
211	4 45 21	9 393 931	14·5258390	5·9533418	·004739336
212	4 49 44	9 528 128	14·5602198	5·9627320	·004716981
213	4 53 69	9 663 597	14·5945195	5·9720926	·004694836
214	4 57 96	9 800 344	14·6287388	5·9814240	·004672897
215	4 62 25	9 938 375	14·6628783	5·9907264	·004651163
216	4 66 56	10 077 696	14·6969385	6·0000000	·004629630
217	4 70 89	10 218 313	14·7309199	6·0092450	·004608295
218	4 75 24	10 360 232	14·7648231	6·0184617	·004587156
219	4 79 61	10 503 459	14·7986486	6·0276502	·004566210
220	4 84 00	10 648 000	14·8323970	6·0368107	·004545455
221	4 88 41	10 793 861	14·8660687	6·0459435	·004524887
222	4 92 84	10 941 048	14·8996644	6·0550489	·004504505
223	4 97 29	11 089 567	14·9331845	6·0641270	·004484305
224	5 01 76	11 239 424	14·9666295	6·0731779	·004464286
225	5 06 25	11 390 625	15·0000000	6·0822020	·004444444
226	5 10 76	11 543 176	15·0332964	6·0911994	·004424779
227	5 15 29	11 697 083	15·0665192	6·1001702	·004405286
228	5 19 84	11 852 352	15·0996689	6·1091147	·004385965
229	5 24 41	12 008 989	15·1327460	6·1180332	·004366812
230	5 29 00	12 167 000	15·1657509	6·1269257	·004347826
231	5 33 61	12 326 391	15·1986842	6·1357924	·004329004
232	5 38 24	12 487 168	15·2315462	6·1446337	·004310345
233	5 42 89	12 649 337	15·2643375	6·1534495	·004291845
234	5 47 56	12 812 904	15·2970585	6·1622401	·004273504
235	5 52 25	12 977 875	15·3297097	6·1710058	·004255319
236	5 56 96	13 144 256	15·3622915	6·1797466	·004237288
237	5 61 69	13 312 053	15·3948043	6·1884628	·004219409
238	5 66 44	13 481 272	15·4272486	6·1971544	·004201681
239	5 71 21	13 651 919	15·4596248	6·2058218	·004184100
240	5 76 00	13 824 000	15·4919334	6·2144650	·004166667
241	5 80 81	13 997 521	15·5241747	6·2230843	·004149378
242	5 85 64	14 172 488	15·5563492	6·2316797	·004132231
243	5 90 49	14 348 907	15·5884573	6·2402515	·004115226
244	5 95 36	14 526 784	15·6204994	6·2487998	·004098361
245	6 00 25	14 706 125	15·6524758	6·2573248	·004081633
246	6 05 16	14 886 936	15·6843871	6·2658266	·004065041
247	6 10 09	15 069 223	15·7162336	6·2743054	·004048583
248	6 15 04	15 252 992	15·7480157	6·2827613	·004032258
249	6 20 01	15 438 249	15·7797338	6·2911946	·004016064
250	6 25 00	15 625 000	15·8113883	6·2996053	·004000000
251	6 30 01	15 813 251	15·8429795	6·3079935	·003984064
252	6 35 04	16 003 008	15·8745079	6·3163596	·003968254
253	6 40 09	16 194 277	15·9059737	6·3247035	·003952569
254	6 45 16	16 387 064	15·9373775	6·3330256	·003937008
255	6 50 25	16 581 375	15·9687194	6·3413257	·003921569
256	6 55 36	16 777 216	16·0000000	6·3496042	·003906250
257	6 60 49	16 974 593	16·0312195	6·3578611	·003891051
258	6 65 64	17 173 512	16·0623784	6·3660968	·003875969
259	6 70 81	17 373 979	16·0934769	6·3743111	·003861004
260	6 76 00	17 576 000	16·1245155	6·3825043	·003846154
261	6 81 21	17 779 581	16·1554944	6·3906765	·003831418
262	6 86 44	17 984 728	16·1864141	6·3988279	·003816794
263	6 91 69	18 191 447	16·2172747	6·4069585	·003802281
264	6 96 96	18 399 744	16·2480768	6·4150687	·003787879
265	7 02 25	18 609 625	16·2788206	6·4231583	·003773585
266	7 07 56	18 821 096	16·3095064	6·4312276	·003759398
267	7 12 89	19 034 163	16·3401346	6·4392767	·003745318
268	7 18 24	19 248 832	16·3707055	6·4473057	·003731343
269	7 23 61	19 465 109	16·4012195	6·4553148	·003717472
270	7 29 00	19 683 000	16·4316767	6·4633041	·003703704
271	7 34 41	19 902 511	16·4620776	6·4712736	·003690037
272	7 39 84	20 123 648	16·4924225	6·4792236	·003676471
273	7 45 29	20 346 417	16·5227116	6·4871541	·003663004
274	7 50 76	20 570 824	16·5529454	6·4950653	·003649635
275	7 56 25	20 796 875	16·5831240	6·5029572	·003636364
276	7 61 76	21 024 576	16·6132477	6·5108300	·003623188
277	7 67 29	21 253 933	16·6433170	6·5186839	·003610108
278	7 72 84	21 484 952	16·6733320	6·5265189	·003597122
279	7 78 41	21 717 639	16·7032931	6·5343351	·003584229
280	7 84 00	21 952 000	16·7332005	6·5421326	·003571429
281	7 89 61	22 188 041	16·7630546	6·5499116	·003558719
282	7 95 24	22 425 768	16·7928556	6·5576722	·003546099
283	8 00 89	22 665 187	16·8226038	6·5654144	·003533569
284	8 06 56	22 906 304	16·8522995	6·5731385	·003521127
285	8 12 25	23 149 125	16·8819430	6·5808443	·003508772
286	8 17 96	23 393 656	16·9115345	6·5885323	·003496503
287	8 23 69	23 639 903	16·9410743	6·5962023	·003484321
288	8 29 44	23 887 872	16·9705627	6·6038545	·003472222
289	8 35 21	24 137 569	17·0000000	6·6114890	·003460208
290	8 41 00	24 389 000	17·0293864	6·6191060	·003448276

TABLE XLV. (*continued*).

No.	Square	Cube	Square Root	Cube Root	Reciprocal
291	8 46 81	24 642 171	17·0587221	6·6267054	·003436426
292	8 52 64	24 897 088	17·0880075	6·6342874	·003424658
293	8 58 49	25 153 757	17·1172428	6·6418522	·003412969
294	8 64 36	25 412 184	17·1464282	6·6493998	·003401861
295	8 70 25	25 672 375	17·1755640	6·6569302	·003389831
296	8 76 16	25 934 336	17·2046505	6·6644437	·003378378
297	8 82 09	26 198 073	17·2336879	6·6719403	·003367003
298	8 88 04	26 463 592	17·2626765	6·6794200	·003355705
299	8 94 01	26 730 899	17·2916165	6·6868831	·003344482
300	9 00 00	27 000 000	17·3205081	6·6943295	·003333333
301	9 06 01	27 270 901	17·3493516	6·7017593	·003322259
302	9 12 04	27 543 608	17·3781472	6·7091729	·003311258
303	9 18 09	27 818 127	17·4068952	6·7165700	·003300330
304	9 24 16	28 094 464	17·4355958	6·7239508	·003289474
305	9 30 25	28 372 625	17·4642492	6·7313155	·003278689
306	9 36 36	28 652 616	17·4928557	6·7386641	·003267974
307	9 42 49	28 934 443	17·5214155	6·7459967	·003257329
308	9 48 64	29 218 112	17·5499288	6·7533134	·003246753
309	9 54 81	29 503 629	17·5783958	6·7606143	·003236246
310	9 61 00	29 791 000	17·6068169	6·7678995	·003225806
311	9 67 21	30 080 231	17·6351921	6·7751690	·003215434
312	9 73 44	30 371 328	17·6635217	6·7824229	·003205128
313	9 79 69	30 664 297	17·6918060	6·7896613	·003194888
314	9 85 96	30 959 144	17·7200451	6·7968844	·003184713
315	9 92 25	31 255 875	17·7482393	6·8040921	·003174603
316	9 98 56	31 554 496	17·7763888	6·8112847	·003164557
317	10 04 89	31 855 013	17·8044938	6·8184620	·003154574
318	10 11 24	32 157 432	17·8325545	6·8256242	·003144654
319	10 17 61	32 461 759	17·8605711	6·8327714	·003134796
320	10 24 00	32 768 000	17·8885438	6·8399037	·003125000
321	10 30 41	33 076 161	17·9164729	6·8470213	·003115265
322	10 36 84	33 386 248	17·9443584	6·8541240	·003105590
323	10 43 29	33 698 267	17·9722008	6·8612120	·003095975
324	10 49 76	34 012 224	18·0000000	6·8682855	·003086420
325	10 56 25	34 328 125	18·0277564	6·8753443	·003076923
326	10 62 76	34 645 976	18·0554701	6·8823888	·003067485
327	10 69 29	34 965 783	18·0831413	6·8894188	·003058104
328	10 75 84	35 287 552	18·1107703	6·8964345	·003048780
329	10 82 41	35 611 289	18·1383571	6·9034359	·003039514
330	10 89 00	35 937 000	18·1659021	6·9104232	·003030303
331	10 95 61	36 264 691	18·1934054	6·9173964	·003021148
332	11 02 24	36 594 368	18·2208672	6·9243556	·003012048
333	11 08 89	36 926 037	18·2482876	6·9313008	·003003003
334	11 15 56	37 259 704	18·2756669	6·9382321	·002994012
335	11 22 25	37 595 375	18·3030052	6·9451496	·002985075
336	11 28 96	37 933 056	18·3303028	6·9520533	·002976190
337	11 35 69	38 272 753	18·3575598	6·9589434	·002967359
338	11 42 44	38 614 472	18·3847763	6·9658198	·002958580
339	11 49 21	38 958 219	18·4119526	6·9726826	·002949853
340	11 56 00	39 304 000	18·4390889	6·9795321	·002941176
341	11 62 81	39 651 821	18·4661853	6·9863681	·002932551
342	11 69 64	40 001 688	18·4932420	6·9931906	·002923977
343	11 76 49	40 353 607	18·5202592	7·0000000	·002915452
344	11 83 36	40 707 584	18·5472370	7·0067962	·002906977
345	11 90 25	41 063 625	18·5741756	7·0135791	·002898551
346	11 97 16	41 421 736	18·6010752	7·0203490	·002890173
347	12 04 09	41 781 923	18·6279360	7·0271058	·002881844
348	12 11 04	42 144 192	18·6547581	7·0338497	·002873563
349	12 18 01	42 508 549	18·6815147	7·0405806	·002865330
350	12 25 00	42 875 000	18·7082869	7·0472987	·002857143
351	12 32 01	43 243 551	18·7349940	7·0540041	·002849003
352	12 39 04	43 614 208	18·7616630	7·0606967	·002840909
353	12 46 09	43 986 977	18·7882942	7·0673767	·002832861
354	12 53 16	44 361 864	18·8148877	7·0740440	·002824859
355	12 60 25	44 738 875	18·8414437	7·0806988	·002816901
356	12 67 36	45 118 016	18·8679623	7·0873411	·002808989
357	12 74 49	45 499 293	18·8944436	7·0939709	·002801120
358	12 81 64	45 882 712	18·9208879	7·1005885	·002793296
359	12 88 81	46 268 279	18·9472953	7·1071937	·002785515
360	12 96 00	46 656 000	18·9736660	7·1137866	·002777778
361	13 03 21	47 045 881	19·0000000	7·1203674	·002770083
362	13 10 44	47 437 928	19·0262976	7·1269360	·002762431
363	13 17 69	47 832 147	19·0525589	7·1334925	·002754821
364	13 24 96	48 228 544	19·0787840	7·1400370	·002747253
365	13 32 25	48 627 125	19·1049732	7·1465695	·002739726
366	13 39 56	49 027 896	19·1311265	7·1530901	·002732240
367	13 46 89	49 430 863	19·1572441	7·1595988	·002724796
368	13 54 24	49 836 032	19·1833261	7·1660957	·002717391
369	13 61 61	50 243 409	19·2093727	7·1725809	·002710027
370	13 69 00	50 653 000	19·2353841	7·1790544	·002702703
371	13 76 41	51 064 811	19·2613603	7·1855162	·002695418
372	13 83 84	51 478 848	19·2873015	7·1919663	·002688172
373	13 91 29	51 895 117	19·3132079	7·1984050	·002680965
374	13 98 76	52 313 624	19·3390796	7·2048322	·002673797
375	14 06 25	52 734 375	19·3649167	7·2112479	·002666667
376	14 13 76	53 157 376	19·3907194	7·2176522	·002659574
377	14 21 29	53 582 633	19·4164878	7·2240450	·002652520
378	14 28 84	54 010 152	19·4422221	7·2304268	·002645503
379	14 36 41	54 439 939	19·4679223	7·2367972	·002638522
380	14 44 00	54 872 000	19·4935887	7·2431565	·002631579
381	14 51 61	55 306 341	19·5192213	7·2495045	·002624672
382	14 59 24	55 742 968	19·5448203	7·2558415	·002617801
383	14 66 89	56 181 887	19·5703858	7·2621675	·002610966
384	14 74 56	56 623 104	19·5959179	7·2684824	·002604167
385	14 82 25	57 066 625	19·6214169	7·2747864	·002597403
386	14 89 96	57 512 456	19·6468827	7·2810794	·002590674
387	14 97 69	57 960 603	19·6723156	7·2873617	·002583979
388	15 05 44	58 411 072	19·6977156	7·2936330	·002577320

TABLE XLV. (*continued*).

No.	Square	Cube	Square Root	Cube Root	Reciprocal
389	15 13 21	58 863 869	19·7230829	7·2998936	·002570694
390	15 21 00	59 319 000	19·7484177	7·3061436	·002564103
391	15 28 81	59 776 471	19·7737199	7·3123828	·002557545
392	15 36 64	60 236 288	19·7989899	7·3186114	·002551020
393	15 44 49	60 698 457	19·8242276	7·3248295	·002544529
394	15 52 36	61 162 984	19·8494332	7·3310369	·002538071
395	15 60 25	61 629 875	19·8746069	7·3372339	·002531646
396	15 68 16	62 099 136	19·8997487	7·3434205	·002525253
397	15 76 09	62 570 773	19·9248588	7·3495966	·002518892
398	15 84 04	63 044 792	19·9499373	7·3557624	·002512563
399	15 92 01	63 521 199	19·9749844	7·3619178	·002506266
400	16 00 00	64 000 000	20·0000000	7·3680630	·002500000
401	16 08 01	64 481 201	20·0249844	7·3741979	·002493766
402	16 16 04	64 964 808	20·0499377	7·3803227	·002487562
403	16 24 09	65 450 827	20·0748599	7·3864373	·002481390
404	16 32 16	65 939 264	20·0997512	7·3925418	·002475248
405	16 40 25	66 430 125	20·1246118	7·3986363	·002469136
406	16 48 36	66 923 416	20·1494417	7·4047206	·002463054
407	16 56 49	67 419 143	20·1742410	7·4107950	·002457002
408	16 64 64	67 917 312	20·1990099	7·4168595	·002450980
409	16 72 81	68 417 929	20·2237484	7·4229142	·002444988
410	16 81 00	68 921 000	20·2484567	7·4289589	·002439024
411	16 89 21	69 426 531	20·2731349	7·4349938	·002433090
412	16 97 44	69 934 528	20·2977831	7·4410189	·002427184
413	17 05 69	70 444 997	20·3224014	7·4470342	·002421308
414	17 13 96	70 957 944	20·3469899	7·4530399	·002415459
415	17 22 25	71 473 375	20·3715488	7·4590359	·002409639
416	17 30 56	71 991 296	20·3960781	7·4650223	·002403846
417	17 38 89	72 511 713	20·4205779	7·4709991	·002398082
418	17 47 24	73 034 632	20·4450483	7·4769664	·002392344
419	17 55 61	73 560 059	20·4694895	7·4829242	·002386635
420	17 64 00	74 088 000	20·4939015	7·4888724	·002380952
421	17 72 41	74 618 461	20·5182845	7·4948113	·002375297
422	17 80 84	75 151 448	20·5426386	7·5007406	·002369668
423	17 89 29	75 686 967	20·5669638	7·5066607	·002364066
424	17 97 76	76 225 024	20·5912603	7·5125715	·002358491
425	18 06 25	76 765 625	20·6155281	7·5184730	·002352941
426	18 14 76	77 308 776	20·6397674	7·5243652	·002347418
427	18 23 29	77 854 483	20·6639783	7·5302482	·002341920
428	18 31 84	78 402 752	20·6881609	7·5361221	·002336449
429	18 40 41	78 953 589	20·7123152	7·5419867	·002331002
430	18 49 00	79 507 000	20·7364414	7·5478423	·002325581
431	18 57 61	80 062 991	20·7605395	7·5536888	·002320186
432	18 66 24	80 621 568	20·7846097	7·5595263	·002314815
433	18 74 89	81 182 737	20·8086520	7·5653548	·002309469
434	18 83 56	81 746 504	20·8326667	7·5711743	·002304147
435	18 92 25	82 312 875	20·8566536	7·5769849	·002298851
436	19 00 96	82 881 856	20·8806130	7·5827865	·002293578
437	19 09 69	83 453 453	20·9045450	7·5885793	·002288330
438	19 18 44	84 027 672	20·9284495	7·5943633	·002283105
439	19 27 21	84 604 519	20·9523268	7·6001385	·002277904
440	19 36 00	85 184 000	20·9761770	7·6059049	·002272727
441	19 44 81	85 766 121	21·0000000	7·6116626	·002267574
442	19 53 64	86 350 888	21·0237960	7·6174116	·002262443
443	19 62 49	86 938 307	21·0475652	7·6231519	·002257336
444	19 71 36	87 528 384	21·0713075	7·6288837	·002252252
445	19 80 25	88 121 125	21·0950231	7·6346067	·002247191
446	19 89 16	88 716 536	21·1187121	7·6403213	·002242152
447	19 98 09	89 314 623	21·1423745	7·6460272	·002237136
448	20 07 74	89 915 392	21·1660105	7·6517247	·002232143
449	20 16 01	90 518 849	21·1896201	7·6574138	·002227171
450	20 25 00	91 125 000	21·2132034	7·6630943	·002222222
451	20 34 01	91 733 851	21·2367606	7·6687665	·002217295
452	20 43 04	92 345 408	21·2602916	7·6744303	·002212389
453	20 52 09	92 959 677	21·2837967	7·6800857	·002207506
454	20 61 16	93 576 664	21·3072758	7·6857328	·002202643
455	20 70 25	94 196 375	21·3307290	7·6913717	·002197802
456	20 79 36	94 818 816	21·3541565	7·6970023	·002192982
457	20 88 49	95 443 993	21·3775583	7·7026246	·002188184
458	20 97 64	96 071 912	21·4009346	7·7082388	·002183406
459	21 06 81	96 702 579	21·4242853	7·7138448	·002178649
460	21 16 00	97 336 000	21·4476106	7·7194426	·002173913
461	21 25 21	97 972 181	21·4709106	7·7250325	·002169197
462	21 34 44	98 611 128	21·4941853	7·7306141	·002164502
463	21 43 69	99 252 847	21·5174348	7·7361877	·002159827
464	21 52 96	99 897 344	21·5406592	7·7417532	·002155172
465	21 62 25	100 544 625	21·5638587	7·7473109	·002150538
466	21 71 56	101 194 696	21·5870331	7·7528606	·002145923
467	21 80 89	101 847 563	21·6101828	7·7584023	·002141328
468	21 90 24	102 503 232	21·6333077	7·7639361	·002136752
469	21 99 61	103 161 709	21·6564078	7·7694620	·002132196
470	22 09 00	103 823 000	21·6794834	7·7749801	·002127660
471	22 18 41	104 487 111	21·7025344	7·7804904	·002123142
472	22 27 84	105 154 048	21·7255610	7·7859928	·002118644
473	22 37 29	105 823 817	21·7485632	7·7914875	·002114165
474	22 46 76	106 496 424	21·7715411	7·7969745	·002109705
475	22 56 25	107 171 875	21·7944947	7·8024538	·002105263
476	22 65 76	107 850 176	21·8174242	7·8079254	·002100840
477	22 75 29	108 531 333	21·8403297	7·8133892	·002096436
478	22 84 84	109 215 352	21·8632111	7·8188456	·002092050
479	25 94 41	109 902 239	21·8860686	7·8242942	·002087683
480	23 04 00	110 592 000	21·9089023	7·8297353	·002083333
481	23 13 61	111 284 641	21·9317122	7·8351688	·002079002
482	23 23 24	111 980 168	21·9544984	7·8405949	·002074689
483	23 32 89	112 678 587	21·9772610	7·8460134	·002070393
484	23 42 56	113 379 904	22·0000000	7·8514244	·002066116
485	23 52 25	114 084 125	22·0227155	7·8568281	·002061856
486	23 61 96	114 791 256	22·0454077	7·8622242	002057613

TABLE XLV. (*continued*).

No.	Square	Cube	Square Root	Cube Root	Reciprocal
487	23 71 69	115 501 303	22·0680765	7·8676130	·002053388
488	23 81 44	116 214 272	22·0907220	7·8729944	·002049180
489	23 91 21	116 930 169	22·1133444	7·8783684	·002044990
490	24 01 00	117 649 000	22·1359436	7·8837352	·002040816
491	24 10 81	118 370 771	22·1585198	7·8890946	·002036660
492	24 20 64	119 095 488	22·1810730	7·8944468	·002032520
493	24 30 49	119 823 157	22·2036033	7·8997917	·002028398
494	24 40 36	120 553 784	22·2261108	7·9051294	·002024291
495	24 50 25	121 287 375	22·2485955	7·9104599	·002020202
496	24 60 16	122 023 936	22·2710575	7·9157832	·002016129
497	24 70 09	122 763 473	22·2934968	7·9210994	·002012072
498	24 80 04	123 505 992	22·3159136	7·9264085	·002008032
499	24 90 01	124 251 499	22·3383079	7·9317104	·002004008
500	25 00 00	125 000 000	22·3606798	7·9370053	·002000000
501	25 10 01	125 751 501	22·3830293	7·9422931	·001996008
502	25 20 04	126 506 008	22·4053565	7·9475739	·001992032
503	25 30 09	127 263 527	22·4276615	7·9528477	·001988072
504	25 40 16	128 024 064	22·4499443	7·9581144	·001984127
505	25 50 25	128 787 625	22·4722051	7·9633743	·001980198
506	25 60 36	129 554 216	22·4944438	7·9686271	·001976285
507	25 70 49	130 323 843	22·5166605	7·9738731	·001972387
508	25 80 64	131 096 512	22·5388553	7·9791122	·001968504
509	25 90 81	131 872 229	22·5610283	7·9843444	·001964637
510	26 01 00	132 651 000	22·5831796	7·9895697	·001960784
511	26 11 21	133 432 831	22·6053091	7·9947883	·001956947
512	26 21 44	134 217 728	22·6274170	8·0000000	·001953125
513	26 31 69	135 005 697	22·6495033	8·0052049	·001949318
514	26 41 96	135 796 744	22·6715681	8·0104032	·001945525
515	26 52 25	136 590 875	22·6936114	8·0155946	·001941748
516	26 62 56	137 388 096	22·7156334	8·0207794	·001937984
517	26 72 89	138 188 413	22·7376340	8·0259574	·001934236
518	26 83 24	138 991 832	22·7596134	8·0311287	·001930502
519	26 93 61	139 798 359	22·7815715	8·0362935	·001926782
520	27 04 00	140 608 000	22·8035085	8·0414515	·001923077
521	27 14 41	141 420 761	22·8254244	8·0466030	·001919386
522	27 24 84	142 236 648	22·8473193	8·0517479	·001915709
523	27 35 29	143 055 667	22·8691933	8·0568862	·001912046
524	27 45 76	143 877 824	22·8910463	8·0620180	·001908397
525	27 56 25	144 703 125	22·9128785	8·0671432	·001904762
526	27 66 76	145 531 576	22·9346899	8·0722620	·001901141
527	27 77 29	146 363 183	22·9564806	8·0773743	·001897533
528	27 87 84	147 197 952	22·9782506	8·0824800	·001893939
529	27 98 41	148 035 889	23·0000000	8·0875794	·001890359
530	28 09 00	148 877 000	23·0217289	8·0926723	·001886792
531	28 19 61	149 721 291	23·0434372	8·0977589	·001883239
532	28 30 24	150 568 768	23·0651252	8·1028390	·001879699
533	28 40 89	151 419 437	23·0867928	8·1079128	·001876173
534	28 51 56	152 273 304	23·1084400	8·1129803	·001872659
535	28 62 25	153 130 375	23·1300670	8·1180414	·001869159
536	28 72 96	153 990 656	23·1516738	8·1230962	·001865672
537	28 83 69	154 854 153	23·1732605	8·1281447	·001862197
538	28 94 44	155 720 872	23·1948270	8·1331870	·001858736
539	29 05 21	156 590 819	23·2163735	8·1382230	·001855288
540	29 16 00	157 464 000	23·2379001	8·1432529	·001851852
541	29 26 81	158 340 421	23·2594067	8·1482765	·001848429
542	29 37 64	159 220 088	23·2808935	8·1532939	·001845018
543	29 48 49	160 103 007	23·3023604	8·1583051	·001841621
544	29 59 36	160 989 184	23·3238076	8·1633102	·001838235
545	29 70 25	161 878 625	23·3452351	8·1683092	·001834862
546	29 81 16	162 771 336	23·3666429	8·1733020	·001831502
547	29 92 09	163 667 323	23·3880311	8·1782888	·001828154
548	30 03 04	164 566 592	23·4093998	8·1832695	·001824818
549	30 14 01	165 469 149	23·4307490	8·1882441	·001821494
550	30 25 00	166 375 000	23·4520788	8·1932127	·001818182
551	30 36 01	167 284 151	23·4733892	8·1981753	·001814882
552	30 47 04	168 196 608	23·4946802	8·2031319	·001811594
553	30 58 09	169 112 377	23·5159520	8·2080825	·001808318
554	30 69 16	170 031 464	23·5372046	8·2130271	·001805054
555	30 80 25	170 953 875	23·5584380	8·2179657	·001801802
556	30 91 36	171 879 616	23·5796522	8·2228985	·001798561
557	31 02 49	172 808 693	23·6008474	8·2278254	·001795332
558	31 13 64	173 741 112	23·6220236	8·2327463	·001792115
559	31 24 81	174 676 879	23·6431808	8·2376614	·001788909
560	31 36 00	175 616 000	23·6643191	8·2425706	·001785714
561	31 47 21	176 558 481	23·6854386	8·2474740	·001782531
562	31 58 44	177 504 328	23·7065392	8·2523715	·001779359
563	31 69 69	178 453 547	23·7276210	8·2572633	·001776199
564	31 80 96	179 406 144	23·7486842	8·2621492	·001773050
565	31 92 25	180 362 125	23·7697286	8·2670294	·001769912
566	32 03 56	181 321 496	23·7907545	8·2719039	·001766784
567	32 14 89	182 284 263	23·8117618	8·2767726	·001763668
568	32 26 24	183 250 432	23·8327506	8·2816355	·001760563
569	32 37 61	184 220 009	23·8537209	8·2864928	·001757469
570	32 49 00	185 193 000	23·8746728	8·2913444	·001754386
571	32 60 41	186 169 411	23·8956063	8·2961903	·001751313
572	32 71 84	187 149 248	23·9165215	8·3010304	·001748252
573	32 83 29	188 132 517	23·9374184	8·3058651	·001745201
574	32 94 76	189 119 224	23·9582971	8·3106941	·001742160
575	33 06 25	190 109 375	23·9791576	8·3155175	·001739130
576	33 17 76	191 102 976	24·0000000	8·3203353	·001736111
577	33 29 29	192 100 033	24·0208243	8·3251475	·001733102
578	33 40 84	193 100 552	24·0416306	8·3299542	·001730104
579	33 52 41	194 104 539	24·0624188	8·3347553	·001727116
580	33 64 00	195 112 000	24·0831891	8·3395509	·001724138
581	33 75 61	196 122 941	24·1039416	8·3443410	·001721170
582	33 87 24	197 137 368	24·1246762	8·3491256	·001718213
583	33 98 89	198 155 287	24·1453929	8·3539047	·001715266
584	34 10 56	199 176 704	24·1660919	8·3586784	·001712329

TABLE XLV. (*continued*).

No.	Square	Cube	Square Root	Cube Root	Reciprocal
585	34 22 25	200 201 625	24·1867732	8·3634466	·001709402
586	34 33 96	201 230 056	24·2074369	8·3682095	·001706485
587	34 45 69	202 262 003	24·2280829	8·3729668	·001703578
588	34 57 44	203 297 472	24·2487113	8·3777188	·005700680
589	34 69 21	204 336 469	24·2693222	8·3824653	·001697798
590	34 81 00	205 379 000	24·2899156	8·3872065	·001694915
591	34 92 81	206 425 071	24·3104916	8·3919423	·001692047
592	35 04 64	207 474 688	24·3310501	8·3966729	·001689189
593	35 16 49	208 527 857	24·3515913	8·4013981	·001686341
594	35 28 36	209 584 584	24·3721152	8·4061180	·001683502
595	35 40 25	210 644 875	24·3926218	8·4108326	·001680672
596	35 52 16	211 708 736	24·4131112	8·4155419	·001677852
597	35 64 09	212 776 173	24·4335834	8·4202460	·001675042
598	35 76 04	213 847 192	24·4540385	8·4249448	·001672241
599	35 88 01	214 921 799	24·4744765	8·4296383	·001669449
600	36 00 00	216 000 000	24·4948974	8·4343267	·001666667
601	36 12 01	217 081 801	24·5153013	8·4390098	·001663894
602	36 24 04	218 167 208	24·5356883	8·4436877	·001661130
603	36 36 09	219 256 227	24·5560583	8·4483605	·001658375
604	36 48 16	220 348 864	24·5764115	8·4530281	·001655629
605	36 60 25	221 445 125	24·5967478	8·4576906	·001652893
606	36 72 36	222 545 016	24·6170673	8·4623479	·001650165
607	36 84 49	223 648 543	24·6373700	8·4670000	·001647446
608	36 96 64	224 755 712	24·6576560	8·4716471	·001644737
609	37 08 81	225 866 529	24·6779254	8·4762892	·001642036
610	37 21 00	226 981 000	24·6981781	8·4809261	·001639344
611	37 33 21	228 099 131	24·7184142	8·4855579	·001636661
612	37 45 44	229 220 928	24·7386338	8·4901848	·001633987
613	37 57 69	230 346 397	24·7588368	8·4948065	·001631321
614	37 69 96	231 475 544	24·7790234	8·4994233	·001628664
615	37 82 25	232 608 375	24·7991935	8·5040350	·001626016
616	37 94 56	233 744 896	24·8193473	8·5086417	·001623377
617	38 06 89	234 885 113	24·8394847	8·5132435	·001620746
618	38 19 24	236 029 032	24·8596058	8·5178403	·001618123
619	38 31 61	237 176 659	24·8797106	8·5224321	·001615509
620	38 44 00	238 328 000	24·8997992	8·5270189	·001612903
621	38 56 41	239 483 061	24·9198716	8·5316009	·001610306
622	38 68 84	240 641 848	24·9399278	8·5361780	·001607717
623	38 81 29	241 804 367	24·9599679	8·5407501	·001605136
624	38 93 76	242 970 624	24·9799920	8·5453173	·001602564
625	39 06 25	244 140 625	25·0000000	8·5498797	·001600000
626	39 18 76	245 314 376	25·0199920	8·5544372	·001597444
627	39 31 29	246 491 883	25·0399681	8·5589899	·001594896
628	39 43 84	247 673 152	25·0599282	8·5635377	·001592357
629	39 56 41	248 858 189	25·0798724	8·5680807	·001589825
630	39 69 00	250 047 000	25·0998008	8·5726189	·001587302
631	39 81 61	251 239 591	25·1197134	8·5771523	·001584786
632	39 94 24	252 435 968	25·1396102	8·5816809	·001582278
633	40 06 89	253 636 137	25·1594913	8·5862047	·001579779
634	40 19 56	254 840 104	25·1793566	8·5907238	·001577287
635	40 32 25	256 047 875	25·1992063	8·5952380	·001574803
636	40 44 96	257 259 456	25·2190404	8·5997476	·001572327
637	40 57 69	258 474 853	25·2388589	8·6042525	·001569859
638	40 70 44	259 694 072	25·2586619	8·6087526	·001567398
639	40 83 21	260 917 119	25·2784493	8·6132480	·001564945
640	40 96 00	262 144 000	25·2982213	8·6177388	·001562500
641	41 08 81	263 374 721	25·3179778	8·6222248	·001560062
642	41 21 64	264 609 288	25·3377189	8·6267063	·001557632
643	41 34 49	265 847 707	25·3574447	8·6311830	·001555210
644	41 47 36	267 089 984	25·3771551	8·6356551	·001552795
645	41 60 25	268 336 125	25·3968502	8·6401226	·001550388
646	41 73 16	269 586 136	25·4165301	8·6445855	·001547988
647	41 86 09	270 840 023	25·4361947	8·6490437	·001545595
648	41 99 04	272 097 792	25·4558441	8·6534974	·001543210
649	42 12 01	273 359 449	25·4754784	8·6579465	·001540832
650	42 25 00	274 625 000	25·4950976	8·6623911	·001538462
651	42 38 01	275 894 451	25·5147016	8·6668310	·001536098
652	42 51 04	277 167 808	25·5342907	8·6712665	·001533742
653	42 64 09	278 445 077	25·5538647	8·6756974	·001531394
654	42 77 16	279 726 264	25·5734237	8·6801237	·001529052
655	42 90 25	281 011 375	25·5929678	8·6845456	·001526718
656	43 03 36	282 300 416	25·6124969	8·6889630	·001524390
657	43 16 49	283 593 393	25·6320112	8·6933759	·001522070
658	43 29 64	284 890 312	25·6515107	8·6977843	·001519757
659	43 42 81	286 191 179	25·6709953	8·7021882	·001517451
660	43 56 00	287 496 000	25·6904652	8·7065877	·001515152
661	43 69 21	288 804 781	25·7099203	8·7109827	·001512859
662	43 82 44	290 117 528	25·7293607	8·7153734	·001510574
663	43 95 69	291 434 247	25·7487864	8·7197596	·001508296
664	44 08 96	292 754 944	25·7681975	8·7241414	·001506024
665	44 22 25	294 079 625	25·7875939	8·7285187	·001503759
666	44 35 56	295 408 296	25·8069758	8·7328918	·001501502
667	44 48 89	296 740 963	25·8263431	8·7372604	·001499250
668	44 62 24	298 077 632	25·8456960	8·7416246	·001497006
669	44 75 61	299 418 309	25·8650343	8·7459846	·001494768
670	44 89 00	300 763 000	25·8843582	8·7503401	·001492537
671	45 02 41	302 111 711	25·9036677	8·7546913	·001490313
672	45 15 84	303 464 448	25·9229628	8·7590383	·001488095
673	45 29 29	304 821 217	25·9422435	8·7633809	·001485884
674	45 42 76	306 182 024	25·9615100	8·7677192	·001483680
675	45 56 25	307 546 875	25·9807621	8·7720532	·001481481
676	45 69 76	308 915 776	26·0000000	8·7763830	·001479290
677	45 83 29	310 288 733	26·0192237	8·7807084	·001477105
678	45 96 84	311 665 752	26·0384331	8·7850296	·001474926
679	46 10 41	313 046 839	26·0576284	8·7893466	·001472754
680	46 24 00	314 432 000	26·0768096	8·7936593	·001470588
681	46 37 61	315 821 241	26·0959767	8·7979679	·001468429
682	46 51 24	317 214 568	26·1151297	8·8022721	·001466276

TABLE XLV. (*continued*).

No.	Square	Cube	Square Root	Cube Root	Reciprocal
683	46 64 89	318 611 987	26·1342687	8·8065722	·001464129
684	46 78 56	320 013 504	26·1533937	8·8108681	·001461988
685	46 92 25	321 419 125	26·1725047	8·8151598	·001459854
686	47 05 96	322 828 856	26·1916017	8·8194474	·001457726
687	47 19 69	324 242 703	26·2106848	8·8237307	·001455604
688	47 33 44	325 660 672	26·2297541	8·8280099	·001453488
689	47 47 21	327 082 769	26·2488095	8·8322850	·001451379
690	47 61 00	328 509 000	26·2678511	8·8365559	·001449275
691	47 74 81	329 939 371	26·2868789	8·8408227	·001447178
692	47 88 64	331 373 888	26·3058929	8·8450854	·001445087
693	48 02 49	332 812 557	26·3248932	8·8493440	·001443001
694	48 16 36	334 255 384	26·3438797	8·8535985	·001440922
695	48 30 25	335 702 375	26·3628527	8·8578489	·001438849
696	48 44 16	337 153 536	26·3818119	8·8620952	·001436782
697	48 58 09	338 608 873	26·4007576	8·8663375	·001434720
698	48 72 04	340 068 392	26·4196896	8·8705757	·001432665
699	48 86 01	341 532 099	26·4386081	8·8748099	·001430615
700	49 00 00	343 000 000	26·4575131	8·8790400	·001428571
701	49 14 01	344 472 101	26·4764046	8·8832661	·001426534
702	49 28 04	345 948 408	26·4952826	8·8874882	·001424501
703	49 42 09	347 428 927	26·5141472	8·8917063	·001422475
704	49 56 16	348 913 664	26·5329983	8·8959204	·001420455
705	49 70 25	350 402 625	26·5518361	8·9001304	·001418440
706	49 84 36	351 895 816	26·5706605	8·9043366	·001416431
707	49 98 49	353 393 243	26·5894716	8·9085387	·001414427
708	50 12 64	354 894 912	26·6082694	8·9127369	·001412429
709	50 26 81	356 400 829	26·6270539	8·9169311	·001410437
710	50 41 00	357 911 000	26·6458252	8·9211214	·001408451
711	50 55 21	359 425 431	26·6645833	8·9253078	·001406470
712	50 69 44	360 944 128	26·6833281	8·9294902	·001404494
713	50 83 69	362 467 097	26·7020598	8·9336687	·001402525
714	50 97 96	363 994 344	26·7207784	8·9378433	·001400560
715	51 12 25	365 525 875	26·7394839	8·9420140	·001398601
716	51 26 56	367 061 696	26·7581763	8·9461809	·001396648
717	51 40 89	368 601 813	26·7768557	8·9503438	·001394700
718	51 55 24	370 146 232	26·7955220	8·9545029	·001392758
719	51 69 61	371 694 959	26·8141754	8·9586581	·001390821
720	51 84 00	373 248 000	26·8328157	8·9628095	·001388889
721	51 98 41	374 805 361	26·8514432	8·9669570	·001386963
722	52 12 84	376 367 048	26·8700577	8·9711007	·001385042
723	52 27 29	377 933 067	26·8886593	8·9752406	·001383126
724	52 41 76	379 503 424	26·9072481	8·9793766	·001381215
725	52 56 25	381 078 125	26·9258240	8·9835089	·001379310
726	52 70 76	382 657 176	26·9443872	8·9876373	·001377410
727	52 85 29	384 240 583	26·9629375	8·9917620	·001375516
728	52 99 84	385 828 352	26·9814751	8·9958829	·001373626
729	53 14 41	387 420 489	27·0000000	9·0000000	·001371742
730	53 29 00	389 017 000	27·0185122	9·0041134	·001369863
731	53 43 61	390 617 891	27·0370117	9·0082229	·001367989
732	53 58 24	392 223 168	27·0554985	9·0123288	·001366120
733	53 72 89	393 832 837	27·0739727	9·0164309	·001364256
734	53 87 56	395 446 904	27·0924344	9·0205293	·001362398
735	53 02 25	397 065 375	27·1108834	9·0246239	·001360544
736	54 16 96	398 688 256	27·1293199	9·0287149	·001358696
737	54 31 69	400 315 553	27·1477439	9·0328021	·001356852
738	54 46 44	401 947 272	27·1661554	9·0368857	·001355014
739	54 61 21	403 583 419	27·1845544	9·0409655	·001353180
740	54 76 00	405 224 000	27·2029410	9·0450417	·001351351
741	54 90 81	406 869 021	27·2213152	9·0491142	·001349528
742	55 05 64	408 518 488	27·2396769	9·0531831	·001347709
743	55 20 49	410 172 407	27·2580263	9·0572482	·001345895
744	55 35 36	411 830 784	27·2763634	9·0613098	·001344086
745	55 50 25	413 493 625	27·2946881	9·0653677	·001342282
746	55 65 16	415 160 936	27·3130006	9·0694220	·001340483
747	55 80 09	416 832 723	27·3313007	9·0734726	·001338688
748	55 95 04	418 508 992	27·3495887	9·0775197	·001336898
749	56 10 01	420 189 749	27·3678644	9·0815631	·001335113
750	56 25 00	421 875 000	27·3861279	9·0856030	·001333333
751	56 40 01	423 564 751	27·4043792	9·0896392	·001331558
752	56 55 04	425 259 008	27·4226184	9·0936719	·001329787
753	56 70 09	426 957 777	27·4408455	9·0977010	·001328021
754	56 85 16	428 661 064	27·4590604	9·1017265	·001326260
755	57 00 25	430 368 875	27·4772633	9·1057485	·001324503
756	57 15 36	432 081 216	27·4954542	9·1097669	·001322751
757	57 30 49	433 798 093	27·5136330	9·1137818	·001321004
758	57 45 64	435 519 512	27·5317998	9·1177931	·001319261
759	57 60 81	437 245 479	27·5499546	9·1218010	·001317523
760	57 76 00	438 976 000	27·5680975	9·1258053	·001315789
761	57 91 21	440 711 081	27·5862284	9·1298061	·001314060
762	58 06 44	442 450 728	27·6043475	9·1338034	·001312336
763	58 21 69	444 194 947	27·6224546	9·1377971	·001310616
764	58 36 96	445 943 744	27·6405499	9·1417874	·001308901
765	58 52 25	447 697 125	27·6586334	9·1457742	·001307190
766	58 67 56	449 455 096	27·6767050	9·1497576	·001305483
767	58 82 89	451 217 663	27·6947648	9·1537375	·001303781
768	58 98 24	452 984 832	27·7128129	9·1577139	·00 302088
769	59 13 61	454 756 609	27·7308492	9·1616869	·001300390
770	59 29 00	456 533 000	27·7488739	9·1656565	·001298701
771	59 44 41	458 314 011	27·7668868	9·1696225	·001297017
772	59 59 84	460 099 648	27·7848880	9·1735852	·001295337
773	59 75 29	461 889 917	27·8028775	9·1775445	·001293661
774	59 90 76	463 684 824	27·8208555	9·1815003	·001291990
775	60 06 25	465 484 375	27·8388218	9·1854527	·001290323
776	60 21 76	467 288 576	27·8567766	9·1894018	·001288660
777	60 37 29	469 097 433	27·8747197	9·1933474	·001287001
778	60 52 84	470 910 952	27·8926514	9·1972897	·001285347
779	60 68 41	472 729 139	27·9105715	9·2012286	·001283697
780	60 84 00	474 552 000	27·9284801	9·2051641	·001282051

TABLE XLV. (*continued*).

No.	Square	Cube	Square Root	Cube Root	Reciprocal
781	60 99 61	476 379 541	27·9463772	9·2090962	·001280410
782	61 15 24	478 211 768	27·9642629	9·2130250	·001278772
783	61 30 89	480 048 687	27·9821372	9·2169505	·001277139
784	61 46 56	481 890 304	28·0000000	9·2208726	·001275510
785	61 62 25	483 736 625	28·0178515	9·2247914	·001273885
786	61 77 96	485 587 656	28·0356915	9·2287068	·001272265
787	61 93 69	487 443 403	28·0535203	9·2326189	·001270648
788	62 09 44	489 303 872	28·0713377	9·2365277	·001269036
789	62 25 21	491 169 069	28·0891438	9·2404333	·001267427
790	62 41 00	493 039 000	28·1069386	9·2443355	·001265823
791	62 56 81	494 913 671	28·1247222	9·2482344	·001264223
792	62 72 64	496 793 088	28·1424946	9·2521300	·001262626
793	62 88 49	498 677 257	28·1602557	9·2560224	·001261034
794	63 04 36	500 566 184	28·1780056	9·2599114	·001259446
795	63 20 25	502 459 875	28·1957444	9·2637973	·001257862
796	63 36 16	504 358 336	28·2134720	9·2676798	·001256281
797	63 52 09	506 261 573	28·2311884	9·2715592	·001254705
798	63 68 04	508 169 592	28·2488938	9·2754352	·001253133
799	63 84 01	510 082 399	28·2665881	9·2793081	·001251564
800	64 00 00	512 000 000	28·2842712	9·2831777	·001250000
801	64 16 01	513 922 401	28·3019434	9·2870440	·001248439
802	64 32 04	515 849 608	28·3196045	9·2909072	·001246883
803	64 48 09	517 781 627	28·3372546	9·2947671	·001245330
804	64 64 16	519 718 464	28·3548938	9·2986239	·001243781
805	64 80 25	521 660 125	28·3725219	9·3024775	·001242236
806	64 96 36	523 606 616	28·3901391	9·3063278	·001240695
807	65 12 49	525 557 943	28·4077454	9·3101750	·001239157
808	65 28 64	527 514 112	28·4253408	9·3140190	·001237624
809	65 44 81	529 475 129	28·4429253	9·3178599	·001236094
810	65 61 00	531 441 000	28·4604989	9·3216975	·001234568
811	65 77 21	533 411 731	28·4780617	9·3255320	·001233046
812	65 93 44	535 387 328	28·4956137	9·3293634	·001231527
813	66 09 69	537 367 797	28·5131549	9·3331916	·001230012
814	66 25 96	539 353 144	28·5306852	9·3370167	·001228501
815	66 42 25	541 343 375	28·5482048	9·3408386	·001226994
816	66 58 56	543 338 496	28·5657137	9·3446575	·001225490
817	66 74 89	545 338 513	28·5832119	9·3484731	·001223990
818	66 91 24	547 343 432	28·6006993	9·3522857	·001222494
819	67 07 61	549 353 259	28·6181760	9·3560952	·001221001
820	67 24 00	551 368 000	28·6356421	9·3599016	·001219512
821	67 40 41	553 387 661	28·6530976	9·3637049	·001218027
822	67 56 84	555 412 248	28·6705424	9·3675051	·001216545
823	67 73 29	557 441 767	28·6879766	9·3713022	·001215067
824	67 89 76	559 476 224	28·7054002	9·3750963	·001213592
825	68 06 25	561 515 625	28·7228132	9·3788873	·001212121
826	68 22 76	563 559 976	28·7402157	9·3826752	·001210654
827	68 39 29	565 609 283	28·7576077	9·3864600	·001209190
828	68 55 84	567 663 552	28·7749891	9·3902419	·001207729
829	68 72 41	569 722 789	28·7923601	9·3940206	·001206273
830	68 89 00	571 787 000	28·8097206	9·3977964	·001204819
831	69 05 61	573 856 191	28·8270706	9·4015691	·001203369
832	69 22 24	575 930 368	28·8444102	9·4053387	·001201923
833	69 38 89	578 009 537	28·8617394	9·4091054	·001200480
834	69 55 56	580 093 704	28·8790582	9·4128690	·001199041
835	69 72 25	582 182 875	28·8963666	9·4166297	·001197605
836	69 88 96	584 277 056	28·9136646	9·4203873	·001196172
837	70 05 69	586 376 253	28·9309523	9·4241420	·001194743
838	70 22 44	588 480 472	28·9482297	9·4278936	·001193317
839	70 39 21	590 589 719	28·9654967	9·4316423	·001191895
840	70 56 00	592 704 000	28·9827535	9·4353880	·001190476
841	70 72 81	594 823 321	29·0000000	9·4391307	·001189061
842	70 89 64	596 947 688	29·0172363	9·4428704	·001187648
843	71 06 49	599 077 107	29·0344623	9·4466072	·001186240
844	71 23 36	601 211 584	29·0516781	9·4503410	·001184834
845	71 40 25	603 351 125	29·0688837	9·4540719	·001183432
846	71 57 16	605 495 736	29·0860791	9·4577999	·001182033
847	71 74 09	607 645 423	29·1032644	9·4615249	·001180638
848	71 91 04	609 800 192	29·1204396	9·4652470	·001179245
849	72 08 01	611 960 049	29·1376046	9·4689661	·001177856
850	72 25 00	614 125 000	29·1547595	9·4726824	·001176471
851	72 42 01	616 295 051	29·1719043	9·4763957	·001175088
852	72 59 04	618 470 208	29·1890390	9·4801061	·001173709
853	72 76 09	620 650 477	29·2061637	9·4838136	·001172333
854	72 93 16	622 835 864	29·2232784	9·4875182	·001170960
855	73 10 25	625 026 375	29·2403830	9·4912200	·001169591
856	73 27 36	627 222 016	29·2574777	9·4949188	·001168224
857	73 44 49	629 422 793	29·2745623	9·4986147	·001166861
858	73 61 64	631 628 712	29·2916370	9·5023078	·001165501
859	73 78 81	633 839 779	29·3087018	9·5059980	·001164144
860	73 96 00	636 056 000	29·3257566	9·5096854	·001162791
861	74 13 21	638 277 381	29·3428015	9·5133699	·001161440
862	74 30 44	640 503 928	29·3598365	9·5170515	·001160093
863	74 47 69	642 735 647	29·3768616	9·5207303	·001158749
864	74 64 96	644 972 544	29·3938769	9·5244063	·001157407
865	74 82 25	647 214 625	29·4108823	9·5280794	·001156069
866	74 99 56	649 461 896	29·4278779	9·5317497	·001154734
867	75 16 89	651 714 363	29·4448637	9·5354172	·001153403
868	75 34 24	653 972 032	29·4618397	9·5390818	·001152074
869	75 51 61	656 234 909	29·4788059	9·5427437	·001150748
870	75 69 00	658 503 000	29·4957624	9·5464027	·001149425
871	75 86 41	660 776 311	29·5127091	9·5500589	·001148106
872	76 03 84	663 054 848	29·5296461	9·5537123	·001146789
873	76 21 29	665 338 617	29·5465734	9·5573630	·001145475
874	76 38 76	667 627 624	29·5634910	9·5610108	·001144165
875	76 56 25	669 921 875	29·5803989	9·5646559	·001142857
876	76 73 76	672 221 376	29·5972972	9·5682982	·001141553
877	76 91 29	674 526 133	29·6141858	9·5719377	·001140251
878	77 08 84	676 836 152	29·6310648	9·5755745	·001138952

TABLE XLV. (*continued*).

No.	Square	Cube	Square Root	Cube Root	Reciprocal
879	77 26 41	679 151 439	29·6479342	9·5792085	·001137656
880	77 44 00	681 472 000	29·6647939	9·5828397	·001136364
881	77 61 61	683 797 841	29·6816442	9·5864682	·001135074
882	77 79 24	686 128 968	29·6984848	9·5900989	·001133787
883	77 96 89	688 465 387	29·7153159	9·5937169	·001132503
884	78 14 56	690 807 104	29·7321375	9·5973373	·001131222
885	78 32 25	693 154 125	29·7489496	9·6009548	·001129944
886	78 49 96	695 506 456	29·7657521	9·6045696	·001128668
887	78 67 69	697 864 103	29·7825452	9·6081817	·001127396
888	78 85 44	700 227 072	29·7993289	9·6117911	·001126126
889	79 03 21	702 595 369	29·8161030	9·6153977	·001124859
890	79 21 00	704 969 000	29·8328678	9·6190017	·001123596
891	79 38 81	707 347 971	29·8496231	9·6226030	·001122334
892	79 56 64	709 732 288	29·8663690	9·6262016	·001121076
893	79 74 49	712 121 957	29·8831056	9·6297975	·001119821
894	79 92 36	714 516 984	29·8998328	9·6333907	·001118568
895	80 10 25	716 917 375	29·9165506	9·6369812	·001117318
896	80 28 16	719 323 136	29·9332591	9·6405690	·001116071
897	80 46 09	721 734 273	29·9499583	9·6441542	·001114827
898	80 64 04	724 150 792	29·9666481	9·6477367	·001113586
899	80 82 01	726 572 699	29·9833287	9·6513166	·001112347
900	81 00 00	729 000 000	30·0000000	9·6548938	·001111111
901	81 18 01	731 432 701	30·0166620	9·6584684	·001109878
902	81 36 04	733 870 808	30·0333148	9·6620403	·001108647
903	81 54 09	736 314 327	30·0499584	9·6656096	·001107420
904	81 72 16	738 763 264	30·0665928	9·6691762	·001106195
905	81 90 25	741 217 625	30·0832179	9·6727403	·001104972
906	82 08 36	743 677 416	30·0998339	9·6763017	·001103753
907	82 26 49	746 142 643	30·1164407	9·6798604	·001102536
908	82 44 64	748 613 312	30·1330383	9·6834166	·001101322
909	82 62 81	751 089 429	30·1496269	9·6869701	·001100110
910	82 81 00	753 571 000	30·1662063	9·6905211	·001098901
911	82 99 21	756 058 031	30·1827765	9·6940694	·001097695
912	83 17 44	758 550 528	30·1993377	9·6976151	·001096491
913	83 35 69	761 048 497	30·2158899	9·7011583	·001095290
914	83 53 96	763 551 944	30·2324329	9·7046989	·001094092
915	83 72 25	766 060 875	30·2489669	9·7082369	·001092896
916	83 90 56	768 575 296	30·2654919	9·7117723	·001091703
917	84 08 89	771 095 213	30·2820079	9·7153051	·001090513
918	84 27 24	773 620 632	30·2985148	9·7188354	·001089325
919	84 45 61	776 151 559	30·3150128	9·7223631	·001088139
920	84 64 00	778 688 000	30·3315018	9·7258883	·001086957
921	84 82 41	781 229 961	30·3479818	9·7294109	·001085776
922	85 00 84	783 777 448	30·3644529	9·7329309	·001084599
923	85 19 29	786 330 467	30·3809151	9·7364484	·001083424
924	85 37 76	788 889 024	30·3973683	9·7399684	·001082251
925	85 56 25	791 453 125	30·4138127	9·7434758	·001081081
926	85 74 76	794 022 776	30·4302481	9·7469857	·001079914
927	85 93 29	796 597 983	30·4466747	9·7504930	·001078749
928	86 11 84	799 178 752	30·4630924	9·7539979	·001077586
929	86 30 41	801 765 089	30·4795013	9·7575002	·001076426
930	86 49 00	804 357 000	30·4959014	9·7610001	·001075269
931	86 67 61	806 954 491	30·5122926	9·7644974	·001074114
932	86 86 24	809 557 568	30·5286750	9·7679922	·001072961
933	87 04 89	812 166 237	30·5450487	9·7714845	·001071811
934	87 23 56	814 780 504	30·5614136	9·7749743	·001070664
935	87 42 25	817 400 375	30·5777697	9·7784616	·001069519
936	87 60 96	820 025 856	30·5941171	9·7819466	·001068376
937	87 79 69	822 656 953	30·6104557	9·7854288	·001067236
938	87 98 44	825 293 672	30·6267857	9·7889087	·001066098
939	88 17 21	827 936 019	30·6431069	9·7923861	·001064963
940	88 36 00	830 584 000	30·6594194	9·7958611	·001063830
941	88 54 81	833 237 621	30·6757233	9·7993336	·001062699
942	88 73 64	835 896 888	30·6920185	9·8028036	·001061571
943	88 92 49	838 561 807	30·7083051	9·8062711	·001060445
944	89 11 36	841 232 384	30·7245830	9·8097362	·001059322
945	89 30 25	843 908 625	30·7408523	9·8131989	·001058201
946	89 49 16	846 590 536	30·7571130	9·8166591	·001057082
947	89 68 09	849 278 123	30·7733651	9·8201169	·001055966
948	89 87 04	851 971 392	30·7896086	9·8235723	·001054852
949	90 06 01	854 670 349	30·8058436	9·8270252	·001053741
950	90 25 00	857 375 000	30·8220700	9·8304757	·001052632
951	90 44 01	860 085 351	30·8382879	9·8339238	·001051525
952	90 63 04	862 801 408	30·8544972	9·8373695	·001050420
953	90 82 09	865 523 177	30·8706981	9·8408127	·001049318
954	91 01 16	868 250 664	30·8868904	9·8442536	·001048218
955	91 20 25	870 983 875	30·9030743	9·8476920	·001047120
956	91 39 36	873 722 816	30·9192497	9·8511280	·001046025
957	91 58 49	876 467 493	30·9354166	9·8545617	·001044932
958	91 77 64	879 217 912	30·9515751	9·8579929	·001043841
959	91 96 81	881 974 079	30·9677251	9·8614218	·001042753
960	92 16 00	884 736 000	30·9838668	9·8648483	·001041667
961	92 35 21	887 503 681	31·0000000	9·8682724	·001040583
962	92 54 44	890 277 128	31·0161248	9·8716941	·001039501
963	92 73 69	893 056 347	31·0322413	9·8751135	·001038422
964	92 92 96	895 841 344	31·0483494	9·8785305	·001037344
965	93 12 25	898 632 125	31·0644491	9·8819451	·001036269
966	93 31 56	901 428 696	31·0805405	9·8853574	·001035197
967	93 50 89	904 231 063	31·0966236	9·8887673	·001034126
968	93 70 24	907 039 232	31·1126984	9·8921749	·001033058
969	93 89 61	909 853 209	31·1287648	9·8955801	·001031992
970	94 09 00	912 673 000	31·1448230	9·8989830	·001030928
971	94 28 41	915 498 611	31·1608729	9·9023835	·001029866
972	94 47 84	918 330 048	31·1769145	9·9057817	·001028807
973	94 67 29	921 167 317	31·1929479	9·9091776	·001027749
974	94 86 76	924 010 424	31·2089731	9·9125712	·001026694
975	95 06 25	926 859 375	31·2249900	9·9159624	·001025641
976	95 25 76	929 714 176	31·2409987	9·9193513	·001024590

TABLE XLV. (*continued*).

No.	Square	Cube	Square Root	Cube Root	Reciprocal
977	95 45 29	932 574 833	31·2569992	9·9227379	·001023541
978	95 64 84	935 441 352	31·2729915	9·9261222	·001022495
979	95 84 41	938 313 739	31·2889757	9·9295042	·001021450
980	96 04 00	941 192 000	31·3049517	9·9328839	·001020408
981	96 23 61	944 076 141	31·3209195	9·9362613	·001019368
982	96 43 24	946 966 168	31·3368792	9·9396363	·001018330
983	96 62 89	949 862 087	31·3528308	9·9430092	·001017294
984	96 82 56	952 763 904	31·3687743	9·9463797	·001016260
985	97 02 25	955 671 625	31·3847097	9·9497479	·001015228
986	97 21 96	958 585 256	31·4006369	9·9531138	·001014199
987	97 41 69	961 504 803	31·4165561	9·9564775	·001013171
988	97 61 44	964 430 272	31·4324673	9·9598389	·001012146
989	97 81 21	967 361 669	31·4483704	9·9631981	·001011122
990	98 01 00	970 299 000	31·4642654	9·9665549	·001010101
991	98 20 81	973 242 271	31·4801525	9·9699095	·001009082
992	98 40 64	976 191 488	31·4960315	9·9732619	·001008065
993	98 60 49	979 146 657	31·5119025	9·9766120	·001007049
994	98 80 36	982 107 784	31·5277655	9·9799599	·001006036
995	99 00 25	985 074 875	31·5436206	9·9833055	·001005025
996	99 20 16	988 047 936	31·5594677	9·9866488	·001004016
997	99 40 09	991 026 973	31·5753068	9·9899900	·001003009
998	99 60 04	994 011 992	31·5911380	9·9933289	·001002004
999	99 80 01	997 002 999	31·6069613	9·9966656	·001001001
1000	1 00 00 00	1 000 000 000	31·6227766	10·0000000	·0010000000
1001	1 00 20 01	1 003 003 001	31·6385840	10·0033322	·0009990010
1002	1 00 40 04	1 006 012 008	31·6543836	10·0066622	·0009980040
1003	1 00 60 09	1 009 027 027	31·6701752	10·0099899	·0009970090
1004	1 00 80 16	1 012 048 064	31·6859590	10·0133155	·0009960159
1005	1 01 00 25	1 015 075 125	31·7017349	10·0166389	·0009950249
1006	1 01 20 36	1 018 108 216	31·7175030	10·0199601	·0009940358
1007	1 01 40 49	1 021 147 343	31·7332633	10·0232791	·0009930487
1008	1 01 60 64	1 024 192 512	31·7490157	10·0265958	·0009920635
1009	1 01 80 81	1 027 243 729	31·7647608	10·0299104	·0009910803
1010	1 02 01 00	1 030 301 000	31·7804972	10·0332228	·0009900990
1011	1 02 21 21	1 033 364 331	31·7962262	10·0365330	·0009891197
1012	1 02 41 44	1 036 433 728	31·8119474	10·0398410	·0009881423
1013	1 02 61 69	1 039 509 197	31·8276609	10·0431469	·0009871668
1014	1 02 81 96	1 042 590 744	31·8433666	10·0464506	·0009861933
1015	1 03 02 25	1 045 678 375	31·8590646	10·0497521	·0009852217
1016	1 03 22 56	1 048 772 096	31·8747549	10·0530514	·0009842520
1017	1 03 42 89	1 051 871 913	31·8904374	10·0563485	·0009832842
1018	1 03 63 24	1 054 977 832	31·9061123	10·0596435	·0009823183
1019	1 03 83 61	1 058 089 859	31·9217794	10·0629364	·0009813543
1020	1 04 04 00	1 061 208 000	31·9374388	10·0662271	·0009803922
1021	1 04 24 41	1 064 332 261	31·9530906	10·0695156	·0009794319
1022	1 04 44 84	1 067 462 648	31·9687347	10·0728020	·0009784736
1023	1 04 65 29	1 070 599 167	31·9843712	10·0760863	·0009775171
1024	1 04 85 76	1 073 741 824	32·0000000	10·0793684	·0009765625
1025	1 05 06 25	1 076 890 625	32·0156212	10·0826484	·0009756098
1026	1 05 26 76	1 080 045 576	32·0312348	10·0859262	·0009746589
1027	1 05 47 29	1 083 206 683	32·0468407	10·0892019	·0009737098
1028	1 05 67 84	1 086 373 952	32·0624391	10·0924755	·0009727626
1029	1 05 88 41	1 089 547 389	32·0780298	10·0957469	·0009718173
1030	1 06 09 00	1 092 727 000	32·0936131	10·0990163	·0009708738
1031	1 06 29 61	1 095 912 791	32·1091887	10·1022835	·0009699321
1032	1 06 50 24	1 099 104 768	32·1247568	10·1055487	·0009689922
1033	1 06 70 89	1 102 302 937	32·1403173	10·1088117	·0009680542
1034	1 06 91 56	1 105 507 304	32·1558704	10·1120726	·0009671180
1035	1 07 12 25	1 108 717 875	32·1714159	10·1153314	·0009661836
1036	1 07 32 96	1 111 934 656	32·1869539	10·1185882	·0009652510
1037	1 07 53 69	1 115 157 653	32·2024844	10·1218428	·0009643202
1038	1 07 74 44	1 118 386 872	32·2180074	10·1250953	·0009633911
1039	1 07 95 21	1 121 622 319	32·2335229	10·1283457	·0009624639
1040	1 08 16 00	1 124 864 000	32·2490310	10·1315941	·0009615385
1041	1 08 36 81	1 128 111 921	32·2645316	10·1348403	·0009606148
1042	1 08 57 64	1 131 366 088	32·2800248	10·1380845	·0009596929
1043	1 08 78 49	1 134 626 507	32·2955105	10·1413266	·0009587728
1044	1 08 99 36	1 137 893 184	32·3109888	10·1445667	·0009578544
1045	1 09 20 25	1 141 166 125	32·3264598	10·1478047	·0009569378
1046	1 09 41 16	1 144 445 336	32·3419233	10·1510406	·0009560229
1047	1 09 62 09	1 147 730 823	32·3573794	10·1542744	·0009551098
1048	1 09 83 04	1 151 022 592	32·3728281	10·1575062	·0009541985
1049	1 10 04 01	1 154 320 649	32·3882695	10·1607359	·0009532888
1050	1 10 25 00	1 157 625 000	32·4037035	10·1639636	·0009523810
1051	1 10 46 01	1 160 935 651	32·4191301	10·1671893	·0009514748
1052	1 10 67 04	1 164 252 608	32·4345495	10·1704129	·0009505703
1053	1 10 88 09	1 167 575 877	32·4499615	10·1736344	·0009496676
1054	1 11 09 16	1 170 905 464	32·4653662	10·1768539	·0009487666
1055	1 11 30 25	1 174 241 375	32·4807635	10·1800714	·0009478673
1056	1 11 51 36	1 177 583 616	32·4961536	10·1832868	·0009469697
1057	1 11 72 49	1 180 932 193	32·5115364	10·1865002	·0009460738
1058	1 11 93 64	1 184 287 112	32·5269119	10·1897116	·0009451796
1059	1 12 14 81	1 187 648 379	32·5422802	10·1929209	·0009442871
1060	1 12 36 00	1 191 016 000	32·5576412	10·1961283	·0009433962
1061	1 12 57 21	1 194 389 981	32·5729949	10·1993336	·0009425071
1062	1 12 78 44	1 197 770 328	32·5883415	10·2025369	·0009416196
1063	1 12 99 69	1 201 157 047	32·6036807	10·2057382	·0009407338
1064	1 13 20 96	1 204 550 144	32·6190129	10·2089375	·0009398496
1065	1 13 42 25	1 207 949 625	32·6343377	10·2121347	·0009389671
1066	1 13 63 56	1 211 355 496	32·6496554	10·2153300	·0009380863
1067	1 13 84 89	1 214 767 763	32·6649659	10·2185233	·0009372071
1068	1 14 06 24	1 218 186 432	32·6802693	10·2217146	·0009363296
1069	1 14 27 61	1 221 611 509	32·6955654	10·2249039	·0009354537
1070	1 14 49 00	1 225 043 000	32·7108544	10·2280912	·0009345794
1071	1 14 70 41	1 228 480 911	32·7261363	10·2312766	·0009337068
1072	1 14 91 84	1 231 925 248	32·7414111	10·2344599	·0009328358
1073	1 15 13 29	1 235 376 017	32·7566787	10·2376413	·0009319664
1074	1 15 34 76	1 238 833 224	32·7719392	10·2408207	·0009310987

TABLE XLV. (*continued*).

No.	Square	Cube	Square Root	Cube Root	Reciprocal
1075	1 15 56 25	1 242 296 875	32·7871926	10·2439981	·0009302326
1076	1 15 77 76	1 245 766 976	32·8024389	10·2471735	·0009293680
1077	1 15 99 29	1 249 243 533	32·8176782	10·2503470	·0009285051
1078	1 16 20 84	1 252 726 552	32·8329103	10·2535186	·0009276438
1079	1 16 42 41	1 256 216 039	32·8481354	10·2566881	·0009267841
1080	1 16 64 00	1 259 712 000	32·8633535	10·2598557	·0009259259
1081	1 16 85 61	1 263 214 441	32·8785644	10·2630213	·0009250694
1082	1 17 07 24	1 266 723 368	32·8937684	10·2661850	·0009242144
1083	1 17 28 89	1 270 238 787	32·9089653	10·2693467	·0009233610
1084	1 17 50 56	1 273 760 704	32·9241553	10·2725065	·0009225092
1085	1 17 72 25	1 277 289 125	32·9393382	10·2756644	·0009216590
1086	1 17 93 96	1 280 824 056	32·9545141	10·2788203	·0009208103
1087	1 18 15 69	1 284 365 503	32·9696830	10·2819743	·0009199632
1088	1 18 37 44	1 287 913 472	32·9848450	10·2851264	·0009191176
1089	1 18 59 21	1 291 467 969	33·0000000	10·2882765	·0009182736
1090	1 18 81 00	1 295 029 000	33·0151480	10·2914247	·0009174312
1091	1 19 02 81	1 298 596 571	33·0302891	10·2945709	·0009165903
1092	1 19 24 64	1 302 170 688	33·0454233	10·2977153	·0009157509
1093	1 19 46 49	1 305 751 357	33·0605505	10·3008577	·0009149131
1094	1 19 68 36	1 309 338 584	33·0756708	10·3039982	·0009140768
1095	1 19 90 25	1 312 932 375	33·0907842	10·3071368	·0009132420
1096	1 20 12 16	1 316 532 736	33·1058907	10·3102735	·0009124088
1097	1 20 34 09	1 320 139 673	33·1209903	10·3134083	·0009115770
1098	1 20 56 04	1 323 753 192	33·1360830	10·3165411	·0009107468
1099	1 20 78 01	1 327 373 299	33·1511689	10·3196721	·0009099181
1100	1 21 00 00	1 331 000 000	33·1662479	10·3228012	·0009090909
1101	1 21 22 01	1 334 633 301	33·1813200	10·3259284	·0009082652
1102	1 21 44 04	1 338 273 208	33·1963853	10·3290537	·0009074410
1103	1 21 66 09	1 341 919 727	33·2114438	10·3321770	·0009066183
1104	1 21 88 16	1 345 572 864	33·2264955	10·3352985	·0009057971
1105	1 22 10 25	1 349 232 625	33·2415403	10·3384181	·0009049774
1106	1 22 32 36	1 352 899 016	33·2565783	10·3415358	·0009041591
1107	1 22 54 49	1 356 572 043	33·2716095	10·3446517	·0009033424
1108	1 22 76 64	1 360 251 712	33·2866339	10·3477657	·0009025271
1109	1 22 98 81	1 363 938 029	33·3016516	10·3508778	·0009017133
1110	1 23 21 00	1 367 631 000	33·3166625	10·3539880	·0009009009
1111	1 23 43 21	1 371 330 631	33·3316666	10·3570964	·0009000900
1112	1 23 65 44	1 375 036 928	33·3466640	10·3602029	·0008992806
1113	1 23 87 69	1 378 749 897	33·3616546	10·3633076	·0008984726
1114	1 24 09 96	1 382 469 544	33·3766385	10·3664103	·0008976661
1115	1 24 32 25	1 386 195 875	33·3916157	10·3695113	·0008968610
1116	1 24 54 56	1 389 928 896	33·4065862	10·3726103	·0008960573
1117	1 24 76 89	1 393 668 613	33·4215499	10·3757076	·0008952551
1118	1 24 99 24	1 397 415 032	33·4365070	10·3788030	·0008944544
1119	1 25 21 61	1 401 168 159	33·4514573	10·3818965	·0008936550
1120	1 25 44 00	1 404 928 000	33·4664011	10·3849882	·0008928571
1121	1 25 66 41	1 408 694 561	33·4813381	10·3880781	·0008920607
1122	1 25 88 84	1 412 467 848	33·4962684	10·3911661	·0008912656
1123	1 26 11 29	1 416 247 867	33·5111921	10·3942523	·0008904720
1124	1 26 33 76	1 420 034 624	33·5261092	10·3973366	·0008896797
1125	1 26 56 25	1 423 828 125	33·5410196	10·4004192	·0008888889
1126	1 26 78 76	1 427 628 376	33·5559234	10·4034999	·0008880995
1127	1 27 01 29	1 431 435 383	33·5708206	10·4065787	·0008873114
1128	1 27 23 84	1 435 249 152	33·5857112	10·4096557	·0008865248
1129	1 27 46 41	1 439 069 689	33·6005952	10·4127310	·0008857396
1130	1 27 69 00	1 442 897 000	33·6154726	10·4158044	·0008849558
1131	1 27 91 61	1 446 731 091	33·6303434	10·4188760	·0008841733
1132	1 28 14 24	1 450 571 968	33·6452077	10·4219458	·0008833922
1133	1 28 36 89	1 454 419 637	33·6600653	10·4250138	·0008826125
1134	1 28 59 56	1 458 274 104	33·6749165	10·4280800	·0008818342
1135	1 28 82 25	1 462 135 375	33·6897610	10·4311443	·0008810573
1136	1 29 04 96	1 466 003 456	33·7045991	10·4342069	·0008802817
1137	1 29 27 69	1 469 878 353	33·7194306	10·4372677	·0008795075
1138	1 29 50 44	1 473 760 072	33·7342556	10·4403267	·0008787346
1139	1 29 73 21	1 477 648 619	33·7490741	10·4433839	·0008779631
1140	1 29 96 00	1 481 544 000	33·7638860	10·4464393	·0008771930
1141	1 30 18 81	1 485 446 221	33·7786915	10·4494929	·0008764242
1142	1 30 41 64	1 489 355 288	33·7934905	10·4525448	·0008756567
1143	1 30 64 49	1 493 271 207	33·8082830	10·4555948	·0008748906
1144	1 30 87 36	1 497 193 984	33·8230691	10·4586431	·0008741259
1145	1 31 10 25	1 501 123 625	33·8378486	10·4616896	·0008733624
1146	1 31 33 16	1 505 060 136	33·8526218	10·4647343	·0008726003
1147	1 31 56 09	1 509 003 523	33·8673884	10·4677773	·0008718396
1148	1 31 79 04	1 512 953 792	33·8821487	10·4708185	·0008710801
1149	1 32 02 01	1 516 910 949	33·8969025	10·4738579	·0008703220
1150	1 32 25 00	1 520 875 000	33·9116499	10·4768955	·0008695652
1151	1 32 48 01	1 524 845 951	33·9263909	10·4799314	·0008688097
1152	1 32 71 04	1 528 823 808	33·9411255	10·4829656	·0008680556
1153	1 32 94 09	1 532 808 577	33·9558537	10·4859980	·0008673027
1154	1 33 17 16	1 536 800 264	33·9705755	10·4890286	·0008665511
1155	1 33 40 25	1 540 798 875	33·9852910	10·4920575	·0008658009
1156	1 33 63 36	1 544 804 416	34·0000000	10·4950847	·0008650519
1157	1 33 86 49	1 548 816 893	34·0147027	10·4981101	·0008643042
1158	1 34 09 64	1 552 836 312	34·0293990	10·5011337	·0008635579
1159	1 34 32 81	1 556 862 679	34·0440890	10·5041556	·0008628128
1160	1 34 56 00	1 560 896 000	34·0587727	10·5071757	·0008620690
1161	1 34 79 21	1 564 936 281	34·0734501	10·5101942	·0008613264
1162	1 35 02 44	1 568 983 528	34·0881211	10·5132109	·0008605852
1163	1 35 25 69	1 573 037 747	34·1027858	10·5162259	·0008598452
1164	1 35 48 96	1 577 098 944	34·1174442	10·5192391	·0008591065
1165	1 35 72 25	1 581 167 125	34·1320963	10·5222506	·0008583691
1166	1 35 95 56	1 585 242 296	34·1467422	10·5252604	·0008576329
1167	1 36 18 89	1 589 324 463	34·1613817	10·5282685	·0008568980
1168	1 36 42 24	1 593 413 632	34·1760150	10·5312749	·0008561644
1169	1 36 65 61	1 597 509 809	34·1906420	10·5342795	·0008554320
1170	1 36 89 00	1 601 613 000	34·2052627	10·5372825	·0008547009
1171	1 37 12 41	1 605 723 211	34·2198773	10·5402837	·0008539710
1172	1 37 35 84	1 609 840 448	34·2344855	10·5432832	·0008532423

TABLE XLV. (*continued*).

No.	Square	Cube	Square Root	Cube Root	Reciprocal
1173	1 37 59 29	1 618 964 717	34·2490875	10·5462810	·0008525149
1174	1 37 82 76	1 618 096 024	34·2636834	10·5492771	·0008517888
1175	1 38 06 25	1 622 234 375	34·2782730	10·5522715	·0008510658
1176	1 38 29 76	1 626 379 776	34·2928564	10·5552642	·0008503401
1177	1 38 53 29	1 630 532 233	34·3074336	10·5582552	·0008496177
1178	1 38 76 84	1 634 691 752	34·3220046	10·5612445	·0008488964
1179	1 39 00 41	1 638 858 339	34·3365694	10·5642322	·0008481764
1180	1 39 24 00	1 643 032 000	34·3511281	10·5672181	·0008474576
1181	1 39 47 61	1 647 212 741	34·3656805	10·5702024	·0008467401
1182	1 39 71 24	1 651 400 568	34·3802268	10·5731849	·0008460237
1183	1 39 94 89	1 655 595 487	34·3947670	10·5761658	·0008453085
1184	1 40 18 56	1 659 797 504	34·4093011	10·5791449	·0008445946
1185	1 40 42 25	1 664 006 625	34·4238289	10·5821225	·0008438819
1186	1 40 65 96	1 668 222 856	34·4383507	10·5850983	·0008431703
1187	1 40 89 69	1 672 446 203	34·4528663	10·5880725	·0008424600
1188	1 41 13 44	1 676 676 672	34·4673759	10·5910450	·0008417508
1189	1 41 37 21	1 680 914 269	34·4818793	10·5940158	·0008410429
1190	1 41 61 00	1 685 159 000	34·4963766	10·5969850	·0008403361
1191	1 41 84 81	1 689 410 871	34·5108678	10·5999525	·0008396306
1192	1 42 08 64	1 693 669 888	34·5253530	10·6029184	·0008389262
1193	1 42 32 49	1 697 936 057	34·5398321	10·6058826	·0008382230
1194	1 42 56 36	1 702 209 384	34·5543051	10·6088451	·0008375209
1195	1 42 80 25	1 706 489 875	34·5687720	10·6118060	·0008368201
1196	1 43 04 16	1 710 777 536	34·5832329	10·6147652	·0008361204
1197	1 43 28 09	1 715 072 373	34·5976879	10·6177228	·0008354219
1198	1 43 52 04	1 719 374 392	34·6121366	10·6206788	·0008347245
1199	1 43 76 01	1 723 683 599	34·6265794	10·6236331	·0008340284
1200	1 44 00 00	1 728 000 000	34·6410162	10·6265857	·0008333333
1201	1 44 24 01	1 732 323 601	34·6554469	10·6295367	·0008326395
1202	1 44 48 04	1 736 654 408	34·6698716	10·6324860	·0008319468
1203	1 44 72 09	1 740 992 427	34·6842904	10·6354338	·0008312552
1204	1 44 96 16	1 745 337 664	34·6987031	10·6383799	·0008305648
1205	1 45 00 25	1 749 690 125	34·7131099	10·6413244	·0008298755
1206	1 45 44 36	1 754 049 816	34·7275107	10·6442672	·0008291874
1207	1 45 68 49	1 758 416 743	34·7419055	10·6472085	·0008285004
1208	1 45 92 64	1 762 790 912	34·7562944	10·6501480	·0008278146
1209	1 46 16 81	1 767 172 329	34·7706773	10·6530860	·0008271299
1210	1 46 41 00	1 771 561 000	34·7850543	10·6560223	·0008264463
1211	1 46 65 21	1 775 956 931	34·7994253	10·6589570	·0008257638
1212	1 46 89 44	1 780 360 128	34·8137904	10·6618902	·0008250825
1213	1 47 13 69	1 784 770 597	34·8281495	10·6648217	·0008244023
1214	1 47 37 96	1 789 188 344	34·8425028	10·6677516	·0008237232
1215	1 47 62 25	1 793 613 375	34·8568501	10·6706799	·0008230453
1216	1 47 86 56	1 798 045 696	34·8711915	10·6736066	·0008223684
1217	1 48 10 89	1 802 485 313	34·8855271	10·6765317	·0008216927
1218	1 48 35 24	1 806 932 232	34·8998567	10·6794552	·0008210181
1219	1 48 59 61	1 811 386 459	34·9141805	10·6823771	·0008203445
1220	1 48 84 00	1 815 848 000	34·9284984	10·6852973	·0008196721
1221	1 49 08 41	1 820 316 861	34·9428104	10·6882160	·0008190008

No.	Square	Cube	Square Root	Cube Root	Reciprocal
1222	1 49 32 84	1 824 793 048	34·9571166	10·6911331	·0008183306
1223	1 49 57 29	1 829 276 567	34·9714169	10·6940486	·0008176615
1224	1 49 81 76	1 833 767 424	34·9857114	10·6969625	·0008169935
1225	1 50 06 25	1 838 265 625	35·0000000	10·6998748	·0008163265
1226	1 50 30 76	1 842 771 176	35·0142828	10·7027855	·0008156607
1227	1 50 55 29	1 847 284 083	35·0285598	10·7056947	·0008149959
1228	1 50 79 84	1 851 804 352	35·0428309	10·7086023	·0008143322
1229	1 51 04 41	1 856 331 989	35·0570963	10·7115083	·0008136696
1230	1 51 29 00	1 860 867 000	35·0713558	10·7144127	·0008130081
1231	1 51 53 61	1 865 409 391	35·0856096	10·7173155	·0008123477
1232	1 51 78 24	1 869 959 168	35·0998575	10·7202168	·0008116883
1233	1 52 02 89	1 874 516 337	35·1140997	10·7231165	·0008110300
1234	1 52 27 56	1 879 080 904	35·1283361	10·7260146	·0008103728
1235	1 52 52 25	1 883 652 875	35·1425668	10·7289112	·0008097166
1236	1 52 76 96	1 888 232 256	35·1567917	10·7318062	·0008090615
1237	1 53 01 69	1 892 819 053	35·1710108	10·7346997	·0008084074
1238	1 53 26 44	1 897 413 272	35·1852242	10·7375916	·0008077544
1239	1 53 51 21	1 902 014 919	35·1994318	10·7404819	·0008071025
1240	1 53 76 00	1 906 624 000	35·2136337	10·7433707	·0008064516
1241	1 54 00 81	1 911 240 521	35·2278299	10·7462579	·0008058018
1242	1 54 25 64	1 915 864 488	35·2420204	10·7491436	·0008051530
1243	1 54 50 49	1 920 495 907	35·2562051	10·7520277	·0008045052
1244	1 54 75 36	1 925 134 784	35·2703842	10·8549103	·0008038585
1245	1 55 00 25	1 929 781 125	35·2845575	10·7577913	·0008032129
1246	1 55 25 16	1 934 434 936	35·2987252	10·7606708	·0008025682
1247	1 55 50 09	1 939 096 223	35·3128872	10·7635488	·0008019246
1248	1 55 75 04	1 943 764 992	35·3270435	10·7664252	·0008012821
1249	1 56 00 01	1 948 441 249	35·3411941	10·7693001	·0008006405
1250	1 56 25 00	1 953 125 000	35·3553391	10·7721735	·0008000000
1251	1 56 50 01	1 957 816 251	35·3694784	10·7750453	·0007993605
1252	1 56 75 04	1 962 515 008	35·3836120	10·7779156	·0007987220
1253	1 57 00 09	1 967 221 277	35·3977400	10·7807843	·0007980846
1254	1 57 25 16	1 971 935 064	35·4118624	10·7836516	·0007974482
1255	1 57 50 25	1 976 656 375	35·4259792	10·7865173	·0007968127
1256	1 57 75 36	1 981 385 216	35·4400903	10·7893815	·0007961783
1257	1 58 00 49	1 986 121 593	35·4541958	10·7922441	·0007955449
1258	1 58 25 64	1 990 865 512	35·4682957	10·7951053	·0007949126
1259	1 58 50 81	1 995 616 979	35·4823900	10·7979649	·0007942812
1260	1 58 76 00	2 000 376 000	35·4964787	10·8008230	·0007936508
1261	1 59 01 21	2 005 142 581	35·5105618	10·8036797	·0007930214
1262	1 59 26 44	2 009 916 728	35·5246393	10·8065348	·0007923930
1263	1 59 51 69	2 014 698 447	35·5387113	10·8093884	·0007917656
1264	1 59 76 96	2 019 487 744	35·5527777	10·8122404	·0007911392
1265	1 60 02 25	2 024 284 625	35·5668385	10·8150909	·0007905138
1266	1 60 27 56	2 029 089 096	35·5808937	10·8179400	·0007898894
1267	1 60 52 89	2 033 901 163	35·5949434	10·8207876	·0007892660
1268	1 60 78 24	2 038 720 832	35·6089876	10·8236336	·0007886435
1269	1 61 03 61	2 043 548 109	35·6230262	10·8264782	·0007880221
1270	1 61 29 00	2 048 383 000	35·6370593	10·8293213	·0007874016

TABLE XLV. (*continued*).

No.	Square	Cube	Square Root	Cube Root	Reciprocal
1271	1 61 54 41	2 053 225 511	35·6510869	10·8321629	·0007867821
1272	1 61 79 84	2 058 075 648	35·6651090	10·8350030	·0007861685
1273	1 62 05 29	2 062 933 417	35·6791255	10·8378416	·0007855460
1274	1 62 30 76	2 067 798 824	35·6931366	10·8406788	·0007849294
1275	1 62 56 25	2 072 671 875	35·7071421	10·8435144	·0007843137
1276	1 62 81 76	2 077 552 576	35·7211422	10·8463485	·0007836991
1277	1 63 07 29	2 082 440 933	35·7351367	10·8491812	·0007830854
1278	1 63 32 84	2 087 336 952	35·7491258	10·8520125	·0007824726
1279	1 63 58 41	2 092 240 639	35·7631095	10·8548422	·0007818608
1280	1 63 84 00	2 097 152 000	35·7770876	10·8576704	·0007812500
1281	1 64 09 61	2 102 071 041	35·7910603	10·8604972	·0007806401
1282	1 64 35 24	2 106 997 768	35·8050276	10·8633225	·0007800312
1283	1 64 60 89	2 111 932 187	35·8189894	10·8661464	·0007794232
1284	1 64 86 56	2 116 874 304	35·8329457	10·8689687	·0007788162
1285	1 65 12 25	2 121 824 125	35·8468966	10·8717897	·0007782101
1286	1 65 37 96	2 126 781 656	35·8608421	10·8746091	·0007776050
1287	1 65 63 69	2 131 746 903	35·8747822	10·8774271	·0007770008
1288	1 65 89 44	2 136 719 872	35·8887169	10·8802436	·0007763975
1289	1 66 15 21	2 141 700 569	35·9026461	10·8830587	·0007757952
1290	1 66 41 00	2 146 689 000	35·9165699	10·8858723	·0007751938
1291	1 66 66 81	2 151 685 171	35·9304884	10·8886845	·0007745933
1292	1 66 92 64	2 156 689 088	35·9444015	10·8914952	·0007739938
1293	1 67 18 49	2 161 700 757	35·9583092	10·8943044	·0007733952
1294	1 67 44 36	2 166 720 184	35·9722115	10·8971123	·0007727975
1295	1 67 70 25	2 171 747 375	35·9861084	10·8999186	·0007722008
1296	1 67 96 16	2 176 782 336	36·0000000	10·9027235	·0007716049
1297	1 68 22 09	2 181 825 073	36·0138862	10·9055269	·0007710100
1298	1 68 48 04	2 186 875 592	36·0277671	10·9083290	·0007704160
1299	1 68 74 01	2 191 933 899	36·0416426	10·9111296	·0007698229
1300	1 69 00 00	2 197 000 000	36·0555128	10·9139287	·0007692308
1301	1 69 26 01	2 202 073 901	36·0693776	10·9167265	·0007686395
1302	1 69 52 04	2 207 155 608	36·0832371	10·9195228	·0007680492
1303	1 69 78 09	2 212 245 127	36·0970913	10·9223177	·0007674597
1304	1 70 04 16	2 217 342 464	36·1109402	10·9251111	·0007668712
1305	1 70 30 25	2 222 447 625	36·1247837	10·9279031	·0007662835
1306	1 70 56 36	2 227 560 616	36·1386220	10·9306937	·0007656968
1307	1 70 82 49	2 232 681 443	36·1524550	10·9334829	·0007651109
1308	1 71 08 64	2 237 810 112	36·1662826	10·9362706	·0007645260
1309	1 71 34 81	2 242 946 629	36·1801050	10·9390569	·0007639419
1310	1 71 61 00	2 248 091 000	36·1939221	10·9418418	·0007633588
1311	1 71 87 21	2 253 243 231	36·2077340	10·9446253	·0007627765
1312	1 72 13 44	2 258 403 328	36·2215406	10·9474074	·0007621951
1313	1 72 39 69	2 263 571 297	36·2353419	10·9501880	·0007616146
1314	1 72 65 96	2 268 747 144	36·2491379	10·9529673	·0007610350
1315	1 72 92 25	2 273 930 875	36·2629287	10·9557451	·0007604563
1316	1 73 18 56	2 279 122 496	36·2767143	10·9585215	·0007598784
1317	1 73 44 89	2 284 322 013	36·2904946	10·9612965	·0007593014
1318	1 73 71 24	2 289 529 432	36·3042697	10·9640701	·0007587253
1319	1 73 97 61	2 294 744 759	36·3180896	10·9668423	·0007581501
1320	1 74 24 00	2 299 968 000	36·3318042	10·9696131	·0007575758
1321	1 74 50 41	2 305 199 161	36·3455637	10·9723825	·0007570023
1322	1 74 76 84	2 310 438 248	36·3593179	10·9751505	·0007564297
1323	1 75 03 29	2 315 685 267	36·3730670	10·9779171	·0007558579
1324	1 75 29 76	2 320 940 224	36·3868108	10·9806823	·0007552870
1325	1 75 56 25	2 326 203 125	36·4005494	10·9834462	·0007547170
1326	1 75 82 76	2 331 473 976	36·4142829	10·9862086	·0007541478
1327	1 76 09 29	2 336 752 783	36·4280112	10·9889696	·0007535795
1328	1 76 35 84	2 342 039 552	36·4417343	10·9917293	·0007530120
1329	1 76 62 41	2 347 334 289	36·4554523	10·9944876	·0007524454
1330	1 76 89 00	2 352 637 000	36·4691650	10·9972445	·0007518797
1331	1 77 15 61	2 357 947 691	36·4828727	11·0000000	·0007513148
1332	1 77 42 24	2 363 266 368	36·4965752	11·0027541	·0007507508
1333	1 77 68 89	2 368 593 037	36·5102725	11·0055069	·0007501875
1334	1 77 95 56	2 373 927 704	36·5239647	11·0082583	·0007496252
1335	1 78 22 25	2 379 270 375	36·5376518	11·0110082	·0007490637
1336	1 78 48 96	2 384 621 056	36·5513338	11·0137569	·0007485030
1337	1 78 75 69	2 389 979 753	36·5650106	11·0165041	·0007479432
1338	1 79 02 44	2 395 346 472	36·5786823	11·0192500	·0007473842
1339	1 79 29 21	2 400 721 219	36·5923489	11·0219945	·0007468260
1340	1 79 56 00	2 406 104 000	36·6060104	11·0247377	·0007462687
1341	1 79 82 81	2 411 494 821	36·6196668	11·0274795	·0007457122
1342	1 80 09 64	2 416 893 688	36·6333181	11·0302199	·0007451565
1343	1 80 36 49	2 422 300 607	36·6469644	11·0329590	·0007446016
1344	1 80 63 36	2 427 715 584	36·6606056	11·0356967	·0007440476
1345	1 80 90 25	2 433 138 625	36·6742416	11·0384330	·0007434944
1346	1 81 17 16	2 438 569 736	36·6878726	11·0411680	·0007429421
1347	1 81 44 09	2 444 008 923	36·7014986	11·0439017	·0007423905
1348	1 81 71 04	2 449 456 192	36·7151195	11·0466339	·0007418398
1349	1 81 98 01	2 454 911 549	36·7287353	11·0493649	·0007412898
1350	1 82 25 00	2 460 375 000	36·7423461	11·0520945	·0007407407
1351	1 82 52 01	2 465 846 551	36·7559519	11·0548227	·0007401924
1352	1 82 79 04	2 471 326 208	36·7695526	11·0575497	·0007396450
1353	1 83 06 09	2 476 813 977	36·7831483	11·0602752	·0007390983
1354	1 83 33 16	2 482 309 864	36·7967390	11·0629994	·0007385524
1355	1 83 60 25	2 487 813 875	36·8103246	11·0657222	·0007380074
1356	1 83 87 36	2 493 326 016	36·8239053	11·0684437	·0007374631
1357	1 84 14 49	2 498 846 293	36·8374809	11·0711639	·0007369197
1358	1 84 41 64	2 504 374 712	36·8510515	11·0738828	·0007363770
1359	1 84 68 81	2 509 911 279	36·8646172	11·0766003	·0007358352
1360	1 84 96 00	2 515 456 000	36·8781778	11·0793165	·0007352941
1361	1 85 23 21	2 521 008 881	36·8917335	11·0820314	·0007347539
1362	1 85 50 44	2 526 569 928	36·9052842	11·0847449	·0007342144
1363	1 85 77 69	2 532 139 147	36·9188299	11·0874571	·0007336757
1364	1 86 04 96	2 537 716 544	36·9323706	11·0901679	·0007331378
1365	1 86 32 25	2 543 302 125	36·9459064	11·0928775	·0007326007
1366	1 86 59 56	2 548 895 896	36·9594372	11·0955857	·0007320644
1367	1 86 86 89	2 554 497 863	36·9729631	11·0982926	·0007315289
1368	1 87 14 24	2 560 108 032	36·9864840	11·1009982	·0007309942

TABLE XLV. (*continued*).

No.	Square	Cube	Square Root	Cube Root	Reciprocal
1369	1 87 41 61	2 565 726 409	37·0000000	11·1037025	·0007304602
1370	1 87 69 00	2 571 353 000	37·0135110	11·1064054	·0007299270
1371	1 87 96 41	2 576 987 811	37·0270172	11·1091070	·0007293946
1372	1 88 23 84	2 582 630 848	37·0405184	11·1118073	·0007288630
1373	1 88 51 29	2 588 282 117	37·0540146	11·1145064	·0007283321
1374	1 88 78 76	2 593 941 624	37·0675060	11·1172041	·0007278020
1375	1 89 06 25	2 599 609 375	37·0809924	11·1199004	·0007272727
1376	1 89 33 76	2 605 285 376	37·0944740	11·1225955	·0007267442
1377	1 89 61 29	2 610 969 633	37·1079506	11·1252893	·0007262164
1378	1 89 88 84	2 616 662 152	37·1214224	11·1279817	·0007256894
1379	1 90 16 41	2 622 362 939	37·1348893	11·1306729	·0007251632
1380	1 90 44 00	2 628 072 000	37·1483512	11·1333628	·0007246377
1381	1 90 71 61	2 633 789 341	37·1618084	11·1360514	·0007241130
1382	1 90 99 24	2 639 514 968	37·1752606	11·1387386	·0007235890
1383	1 91 26 89	2 645 248 887	37·1887079	11·1414246	·0007230658
1384	1 91 54 56	2 650 991 104	37·2021505	11·1441093	·0007225434
1385	1 91 82 25	2 656 741 625	37·2155881	11·1467926	·0007220217
1386	1 92 09 96	2 662 500 456	37·2290209	11·1494747	·0007215007
1387	1 92 37 69	2 668 267 603	37·2424489	11·1521555	·0007209805
1388	1 92 65 44	2 674 043 072	37·2558720	11·1548350	·0007204611
1389	1 92 93 21	2 679 826 869	37·2692903	11·1575133	·0007199424
1390	1 93 21 00	2 685 619 000	37·2827037	11·1601903	·0007194245
1391	1 93 48 81	2 691 419 471	37·2961124	11·1628659	·0007189073
1392	1 93 76 64	2 697 228 288	37·3095162	11·1655403	·0007183908
1393	1 94 04 49	2 703 045 457	37·3229152	11·1682134	·0007178751
1394	1 94 32 36	2 708 870 984	37·3363094	11·1708852	·0007173601
1395	1 94 60 25	2 714 704 875	37·3496988	11·1735558	·0007168459
1396	1 94 88 16	2 720 547 136	37·3630834	11·1762250	·0007163324
1397	1 95 16 09	2 726 397 773	37·3764632	11·1788930	·0007158196
1398	1 95 44 04	2 732 256 792	37·3898382	11·1815598	·0007153076
1399	1 95 72 01	2 738 124 199	37·4032084	11·1842252	·0007147963
1400	1 96 00 00	2 744 000 000	37·4165738	11·1868894	·0007142857
1401	1 96 28 01	2 749 884 201	37·4299345	11·1895523	·0007137759
1402	1 96 56 04	2 755 776 808	37·4432904	11·1922139	·0007132668
1403	1 96 84 09	2 761 677 827	37·4566416	11·1948743	·0007127584
1404	1 97 12 16	2 767 587 264	37·4699880	11·1975334	0007122507
1405	1 97 40 25	2 773 505 125	37·4833296	11·2001913	·0007117438
1406	1 97 68 36	2 779 431 416	37·4966665	11·2028479	·0007112376
1407	1 97 96 49	2 785 366 143	37·5099987	11·2055032	·0007107321
1408	1 98 24 64	2 791 309 312	37·5233261	11·2081573	·0007102273
1409	1 98 52 81	2 797 260 929	37·5366487	11·2108101	·0007097232
1410	1 98 81 00	2 803 221 000	37·5499667	11·2134617	·0007092199
1411	1 99 09 21	2 809 189 531	37·5632799	11·2161120	·0007087172
1412	1 99 37 44	2 815 166 528	37·5765885	11·2187611	·0007082153
1413	1 99 65 69	2 821 151 997	37·5898922	11·2214089	·0007077141
1414	1 99 93 96	2 827 145 944	37·6031913	11·2240554	·0007072136
1415	2 00 22 25	2 833 148 375	37·6164857	11·2267007	·0007067138
1416	2 00 50 56	2 839 159 296	37·6297754	11·2293448	·0007062147
1417	2 00 78 89	2 845 178 713	37·6430604	11·2319876	·0007057163
1418	2 01 07 24	2 851 206 632	37·6563407	11·2346292	·0007052186
1419	2 01 35 61	2 857 243 059	37·6696164	11·2372696	·0007047216
1420	2 01 64 00	2 863 288 000	37·6828874	11·2399087	·0007042254
1421	2 01 92 41	2 869 341 461	37·6961536	11·2425465	·0007037298
1422	2 02 20 84	2 875 403 448	37·7094153	11·2451831	·0007032349
1423	2 02 49 29	2 881 473 967	37·7226722	11·2478185	·0007027407
1424	2 02 77 76	2 887 553 024	37·7359245	11·2504527	·0007022472
1425	2 03 06 25	2 893 640 625	37·7491722	11·2530856	·0007017544
1426	2 03 34 76	2 899 736 776	37·7624152	11·2557173	·0007012623
1427	2 03 63 29	2 905 841 483	37·7756535	11·2583478	·0007007708
1428	2 03 91 84	2 911 954 752	37·7888873	11·2609770	·0007002801
1429	2 04 20 41	2 918 076 589	37·8021163	11·2636050	·0006997901
1430	2 04 49 00	2 924 207 000	37·8153408	11·2662318	·0006993007
1431	2 04 77 61	2 930 345 991	37·8285606	11·2688573	·0006988120
1432	2 05 06 24	2 936 493 568	37·8417759	11·2714816	·0006983240
1433	2 05 34 89	2 942 649 737	37·8549864	11·2741047	·0006978367
1434	2 05 63 56	2 948 814 504	37·8681924	11·2767266	·0006973501
1435	2 05 92 25	2 954 987 875	37·8813938	11·2793472	·0006968641
1436	2 06 20 96	2 961 169 856	37·8945906	11·2819666	·0006963788
1437	2 06 49 69	2 967 360 453	37·9077828	11·2845849	·0006958942
1438	2 06 78 44	2 973 559 672	37·9209704	11·2872019	·0006954103
1439	2 07 07 21	2 979 767 519	37·9341535	11·2898177	·0006949270
1440	2 07 36 00	2 985 984 000	37·9473319	11·2924323	·0006944444
1441	2 07 64 81	2 992 209 121	37·9605058	11·2950457	·0006939625
1442	2 07 93 64	2 998 442 888	37·9736751	11·2976579	·0006934813
1443	2 08 22 49	3 004 685 307	37·9868398	11·3002688	·0006930007
1444	2 08 51 36	3 010 936 384	38·0000000	11·3028786	·0006925208
1445	2 08 80 25	3 017 196 125	38·0131556	11·3054871	·0006920415
1446	2 09 09 16	3 023 464 536	38·0263067	11·3080945	·0006915629
1447	2 09 38 09	3 029 741 623	38·0394532	11·3107006	·0006910850
1448	2 09 67 04	3 036 027 392	38·0525952	11·3133056	·0006906078
1449	2 09 96 01	3 042 321 849	38·0657326	11·3159094	·0006901312
1450	2 10 25 00	3 048 625 000	38·0788655	11·3185119	·0006896552
1451	2 10 54 01	3 054 936 851	38·0919939	11·3211132	·0006891799
1452	2 10 83 04	3 061 257 408	38·1051178	11·3237134	·0006887052
1453	2 11 12 09	3 067 586 677	38·1182371	11·3263124	·0006882312
1454	2 11 41 16	3 073 924 664	38·1313519	11·3289102	·0006877579
1455	2 11 70 25	3 080 271 375	38·1444622	11·3315067	·0006872852
1456	2 11 99 36	3 086 626 816	38·1575681	11·3341022	·0006868132
1457	2 12 28 49	3 092 990 993	38·1706693	11·3366964	·0006863418
1458	2 12 57 64	3 099 363 912	38·1837662	11·3392894	·0006858711
1459	2 12 86 81	3 105 745 579	38·1968585	11·3418813	·0006854010
1460	2 13 16 00	3 112 136 000	38·2099463	11·3444719	·0006849315
1461	2 13 45 21	3 118 535 181	38·2230297	11·3470614	·0006844627
1462	2 13 74 44	3 124 943 128	38·2361085	11·3496497	·0006839945
1463	2 14 03 69	3 131 359 847	38·2491829	11·3522368	·0006885270
1464	2 14 32 96	3 137 785 344	38·2622529	11·3548227	·0006830601
1465	2 14 62 25	3 144 219 625	38·2753184	11·3574075	·0006825939
1466	2 14 91 56	3 150 662 696	38·2883794	11·3599911	·0006821282

TABLE XLV. (*continued*)

No.	Square	Cube	Square Root	Cube Root	Reciprocal
1467	2 15 20 89	3 157 114 563	38·3014360	11·3625735	·0006816633
1468	2 15 50 24	3 163 575 232	38·3144881	11·3651547	·0006811989
1469	2 15 79 61	3 170 044 709	38·3275358	11·3677347	·0006807352
1470	2 16 09 00	3 176 523 000	38·3405790	11·3703136	·0006802721
1471	2 16 38 41	3 183 010 111	38·3536178	11·3728914	·0006798097
1472	2 16 67 84	3 189 506 048	38·3666522	11·3754679	·0006793478
1473	2 16 97 29	3 169 010 817	38·3796821	11·3780433	·0006788866
1474	2 17 26 76	3 202 524 424	38·3927076	11·3806175	·0006784261
1475	2 17 56 25	3 209 046 875	38·4057287	11·3831906	·0006779661
1476	2 17 85 76	3 215 578 176	38·4187454	11·3857625	·0006775068
1477	2 18 15 29	3 222 118 333	38·4317577	11·3883332	·0006770481
1478	2 18 44 84	3 228 667 352	38·4447656	11·3909028	·0006765900
1479	2 18 74 41	3 235 225 239	38·4577691	11·3934712	·0006761325
1480	2 19 04 00	3 241 792 000	38·4707681	11·3960384	·0006756757
1481	2 19 33 61	3 248 367 641	38·4837627	11·3986045	·0006752194
1482	2 19 63 24	3 254 952 168	38·4967530	11·4011695	·0006747638
1483	2 19 92 89	3 261 545 587	38·5097390	11·4037332	·0006743088
1484	2 20 22 56	3 268 147 904	38·5227206	11·4062959	·0006738544
1485	2 20 52 25	3 274 759 125	38·5356977	11·4088574	·0006734007
1486	2 20 81 96	3 281 379 256	38·5486705	11·4114177	·0006729475
1487	2 21 11 69	3 288 008 303	38·5616389	11·4139769	·0006724950
1488	2 21 41 44	3 294 646 272	38·5746030	11·4165349	·0006720430
1489	2 21 71 21	3 301 293 169	38·5875627	11·4190918	·0006715917
1490	2 22 01 00	3 307 949 000	38·6005181	11·4216476	·0006711409
1491	2 22 30 81	3 314 613 771	38·6134691	11·4242022	·0006706908
1492	2 22 60 64	3 321 287 488	38·6264158	11·4267556	·0006702413
1493	2 22 90 49	3 327 970 157	38·6393582	11·4293079	·0006697924
1494	2 23 20 36	3 334 661 784	38·6522962	11·4318591	·0006693440
1495	2 23 50 25	3 341 362 375	38·6652299	11·4344092	·0006688963
1496	2 23 80 16	3 348 071 936	38·6781593	11·4369581	·0006684492
1497	2 24 10 09	3 354 790 473	38·6910843	11·4395059	·0006680027
1498	2 24 40 04	3 361 517 992	38·7040050	11·4420525	·0006675567
1499	2 24 70 01	3 368 254 499	38·7169214	11·4445980	·0006671114
1500	2 25 00 00	3 375 000 000	38·7298335	11·4471424	·0006666667
1501	2 25 30 01	3 381 754 501	38·7427412	11·4496857	·0006662225
1502	2 25 60 04	3 388 518 008	38·7556447	11·4522278	·0006657790
1503	2 25 90 09	3 395 290 527	38·7685439	11·4547688	·0006653360
1504	2 26 20 16	3 402 072 064	38·7814389	11·4573087	·0006648936
1505	2 26 50 25	3 408 862 625	38·7943294	11·4598474	·0006644518
1506	2 26 80 36	3 415 662 216	38·8072158	11·4623850	·0006640106
1507	2 27 10 49	3 422 470 843	38·8200978	11·4649215	·0006635700
1508	2 27 40 64	3 429 288 512	38·8329757	11·4674568	·0006631300
1509	2 27 70 81	3 436 115 229	38·8458491	11·4699911	·0006626905
1510	2 28 01 00	3 442 951 000	38·8587184	11·4725242	·0006622517
1511	2 28 31 21	3 449 795 831	38·8715834	11·4750562	·0006618134
1512	2 28 61 44	3 456 649 728	38·8844442	11·4775871	·0006613757
1513	2 28 91 69	3 463 512 697	38·8973006	11·4801169	·0006609385
1514	2 29 21 96	3 470 384 744	38·9101529	11·4826455	·0006605020
1515	2 29 52 25	3 477 265 875	38·9230009	11·4851731	·0006600660
1516	2 29 82 56	3 484 156 096	38·9358447	11·4876995	·0006596306
1517	2 30 12 89	3 491 055 413	38·9486841	11·4902249	·0006591958
1518	2 30 43 24	3 497 963 832	38·9615194	11·4927491	·0006587615
1519	2 30 73 61	3 504 881 359	38·9743505	11·4952722	·0006583278
1520	2 31 04 00	3 511 808 000	38·9871774	11·4977942	·0006578947
1521	2 31 34 41	3 518 743 761	39·0000000	11·5003151	·0006574622
1522	2 31 64 84	3 525 688 648	39·0128184	11·5028348	·0006570302
1523	2 31 95 29	3 532 642 667	39·0256326	11·5053535	·0006565988
1524	2 32 25 76	3 539 605 824	39·0384426	11·5078711	·0006561680
1525	2 32 56 25	3 546 578 125	39·0512483	11·5103876	·0006557377
1526	2 32 86 76	3 553 559 576	39·0640499	11·5129030	·0006553080
1527	2 33 17 29	3 560 550 183	39·0768473	11·5154173	·0006548788
1528	2 33 47 84	3 567 549 952	39·0896406	11·5179305	·0006544503
1529	2 33 78 41	3 574 558 889	39·1024296	11·5204425	·0006540222
1530	2 34 09 00	3 581 577 000	39·1152144	11·5229535	·0006535948
1531	2 34 39 61	3 588 604 291	39·1279951	11·5254634	·0006531679
1532	2 34 70 24	3 595 640 768	39·1407716	11·5279722	·0006527415
1533	2 35 00 89	3 602 686 437	39·1535439	11·5304799	·0006523157
1534	2 35 31 56	3 609 741 304	39·1663120	11·5329865	·0006518905
1535	2 35 62 25	3 616 805 375	39·1790760	11·5354920	·0006514658
1536	2 35 92 96	3 623 878 656	39·1918359	11·5379965	·0006510417
1537	2 36 23 69	3 630 961 153	39·2045915	11·5404998	·0006506181
1538	2 36 54 44	3 638 052 872	39·2173431	11·5430021	·0006501951
1539	2 36 85 21	3 645 153 819	39·2300905	11·5455033	·0006497726
1540	2 37 16 00	3 652 264 000	39·2428337	11·5480034	·0006493506
1541	2 37 46 81	3 659 383 421	39·2555728	11·5505025	·0006489293
1542	2 37 77 64	3 666 512 088	39·2683078	11·5530004	·0006485084
1543	2 38 08 49	3 673 650 007	39·2810387	11·5554973	·0006480881
1544	2 38 39 36	3 680 797 184	39·2937654	11·5579931	·0006476684
1545	2 38 70 25	3 687 953 625	39·3064880	11·5604878	·0006472492
1546	2 39 01 16	3 695 119 336	39·3192065	11·5629815	·0006468305
1547	2 39 32 09	3 702 294 323	39·3319208	11·5654740	·0006464124
1548	2 39 63 04	3 709 478 592	39·3446311	11·5679655	·0006459948
1549	2 39 94 01	3 716 672 149	39·3573373	11·5704559	·0006455778
1550	2 40 25 00	3 723 875 000	39·3700394	11·5729453	·0006451613
1551	2 40 56 01	3 731 087 151	39·3827373	11·5754336	·0006447453
1552	2 40 87 04	3 738 308 608	39·3954312	11·5779208	·0006443299
1553	2 41 18 09	3 745 539 377	39·4081210	11·5804069	·0006439150
1554	2 41 49 16	3 752 779 464	39·4208067	11·5828919	0006435006
1555	2 41 80 25	3 760 028 875	39·4334883	11·5853759	·0006430868
1556	2 42 11 36	3 767 287 616	39·4461658	11·5878588	·0006426735
1557	2 42 42 49	3 774 555 693	39·4588393	11·5903407	·0006422608
1558	2 42 73 64	3 781 833 112	39·4715087	11·5928215	·0006418485
1559	2 43 04 81	3 789 119 879	39·4841740	11·5953013	·0006414368
1560	2 43 36 00	3 796 416 000	39·4968353	11·5977799	·0006410256
1561	2 43 67 21	3 803 721 481	39·5094925	11·6002576	·0006406150
1562	2 43 98 44	3 811 036 328	39·5221457	11·6027342	·0006402049
1563	2 44 29 69	3 818 360 547	39·5347948	11·6052097	·0006397953
1564	2 44 60 96	3 825 694 144	39·5474399	11·6076841	·0006393862

TABLE XLV. (*continued*).

No.	Square	Cube	Square Root	Cube Root	Reciprocal
1565	2 44 92 25	3 833 037 125	39·5600809	11·6101575	·0006389776
1566	2 45 23 56	3 840 389 496	39·5727179	11·6126299	·0006385696
1567	2 45 54 89	3 847 751 263	39·5853508	11·6151012	·0006381621
1568	2 45 86 24	3 855 122 432	39·5979797	11·6175715	·0006377551
1569	2 46 17 61	3 862 503 009	39·6106046	11·6200407	·0006373486
1570	2 46 49 00	3 869 893 000	39·6232255	11·6225088	·0006369427
1571	2 46 80 41	3 877 292 411	39·6358424	11·6249759	·0006365372
1572	2 47 11 84	3 884 701 248	39·6484552	11·6274420	·0006361323
1573	2 47 43 29	3 892 119 517	39·6610640	11·6299070	·0006357279
1574	2 47 74 76	3 899 547 224	39·6736688	11·6323710	·0006353240
1575	2 48 06 25	3 906 984 375	39·6862696	11·6348339	·0006349206
1576	2 48 37 76	3 914 430 976	39·6988665	11·6372957	·0006345178
1577	2 48 69 29	3 921 887 033	39·7114593	11·6397566	·0006341154
1578	2 49 00 84	3 929 352 552	39·7240481	11·6422164	·0006337136
1579	2 49 32 41	3 936 827 539	39·7366329	11·6446751	·0006333122
1580	2 49 64 00	3 944 312 000	39·7492138	11·6471329	·0006329114
1581	2 49 95 61	3 951 805 941	39·7617907	11·6495895	·0006325111
1582	2 50 27 24	3 959 309 368	39·7743636	11·6520452	·0006321113
1583	2 50 58 89	3 966 822 287	39·7869625	11·6544998	·0006317119
1584	2 50 90 56	3 974 344 704	39·7994975	11·6569534	·0006313131
1585	2 51 22 25	3 981 876 625	39·8120585	11·6594059	·0006309148
1586	2 51 53 96	3 989 418 056	39·8246155	11·6618574	·0006305170
1587	2 51 85 69	3 996 969 003	39·8371686	11·6643079	·0006301197
1588	2 52 17 44	4 004 529 472	39·8497177	11·6667574	·0006297229
1589	2 52 49 21	4 012 099 469	39·8622628	11·6692058	·0006293266
1590	2 52 81 00	4 019 679 000	39·8748040	11·6716532	·0006289308
1591	2 53 12 81	4 027 268 071	39·8873413	11·6740996	·0006285355
1592	2 53 44 64	4 034 866 688	39·8998747	11·6765449	·0006281407
1593	2 53 76 49	4 042 474 857	39·9124041	11·6789892	·0006277464
1594	2 54 08 36	4 050 092 584	39·9249295	11·6814325	·0006273526
1595	2 54 40 25	4 057 719 875	39·9374511	11·6838748	·0006269592
1596	2 54 72 16	4 065 356 736	39·9499687	11·6863161	·0006265664
1597	2 55 04 09	4 073 003 173	39·9624824	11·6887563	·0006261741
1598	2 55 36 04	4 080 659 192	39·9749922	11·6911955	·0006257822
1599	2 55 68 01	4 088 324 799	39·9874980	11·6936337	·0006253909
1600	2 56 00 00	4 096 000 000	40·0000000	11·6960709	·0006250000
1601	2 56 32 01	4 103 684 801	40·0124980	11·6985071	·0006246096
1602	2 56 64 04	4 111 379 208	40·0249922	11·7009422	·0006242197
1603	2 56 96 09	4 119 083 227	40·0374824	11·7033764	·0006238303
1604	2 57 28 16	4 126 796 864	40·0499688	11·7058095	·0006234414
1605	2 57 60 25	4 134 520 125	40·0624512	11·7082417	·0006230530
1606	2 57 92 36	4 142 253 016	40·0749298	11·7106728	·0006226650
1607	2 58 24 49	4 149 995 543	40·0874045	11·7131029	·0006222775
1608	2 58 56 64	4 157 747 712	40·0998753	11·7155320	·0006218905
1609	2 58 88 81	4 165 509 529	40·1123423	11·7179601	·0006215040
1610	2 59 21 00	4 173 281 000	40·1248053	11·7203872	·0006211180
1611	2 59 53 21	4 181 062 131	40·1372645	11·7228133	·0006207325
1612	2 59 85 44	4 188 852 928	40·1497198	11·7252384	·0006203474
1613	2 60 17 69	4 196 653 397	40·1621713	11·7276625	·0006199628
1614	2 60 49 96	4 204 463 544	40·1746188	11·7300855	·0006195787
1615	2 60 82 25	4 212 283 375	40·1870626	11·7325076	·0006191950
1616	2 61 14 56	4 220 112 896	40·1995025	11·7349286	·0006188119
1617	2 61 46 89	4 227 952 113	40·2119385	11·7373487	·0006184292
1618	2 61 79 24	4 235 801 032	40·2243707	11·7397677	·0006180470
1619	2 62 11 61	4 243 659 659	40·2367990	11·7421858	·0006176652
1620	2 62 44 00	4 251 528 000	40·2492236	11·7446029	·0006172840
1621	2 62 76 41	4 259 406 061	40·2616443	11·7470190	·0006169031
1622	2 63 08 84	4 267 293 848	40·2740611	11·7494341	·0006165228
1623	2 63 41 29	4 275 191 367	40·2864742	11·7518482	·0006161429
1624	2 63 73 76	4 283 098 624	40·2988834	11·7542613	·0006157635
1625	2 64 06 25	4 291 015 625	40·3112888	11·7566734	·0006153846
1626	2 64 38 76	4 298 942 376	40·3236903	11·7590846	·0006150062
1627	2 64 71 29	4 306 878 883	40·3360881	11·7614947	·0006146282
1628	2 65 03 84	4 314 825 152	40·3484820	11·7639039	·0006142506
1629	2 65 36 41	4 322 781 189	40·3608721	11·7663121	·0006138735
1630	2 65 69 00	4 330 747 000	40·3732585	11·7687193	·0006134969
1631	2 66 01 61	4 338 722 591	40·3856410	11·7711255	·0006131208
1632	2 66 34 24	4 346 707 968	40·3980198	11·7735306	·0006127451
1633	2 66 66 89	4 354 703 137	40·4103947	11·7759349	·0006123699
1634	2 66 99 56	4 362 708 104	40·4227658	11·7783381	·0006119951
1635	2 67 32 25	4 370 722 875	40·4351332	11·7807404	·0006116208
1636	2 67 64 96	4 378 747 456	40·4474968	11·7831417	·0006112469
1637	2 67 97 69	4 386 781 853	40·4598566	11·7855420	·0006108735
1638	2 68 30 44	4 394 826 072	40·4722127	11·7879414	·0006105006
1639	2 68 63 21	4 402 880 119	40·4845649	11·7903397	·0006101281
1640	2 68 96 00	4 410 944 000	40·4969135	11·7927371	·0006097561
1641	2 69 28 81	4 419 017 721	40·5092582	11·7951335	·0006093845
1642	2 69 61 64	4 427 101 288	40·5215992	11·7975289	·0006090134
1643	2 69 94 49	4 435 194 707	40·5339364	11·7999234	·0006086427
1644	2 70 27 36	4 443 297 984	40·5462699	11·8023169	·0006082725
1645	2 70 60 25	4 451 411 125	40·5585996	11·8047094	·0006079027
1646	2 70 93 16	4 459 534 136	40·5709255	11·8071010	·0006075334
1647	2 71 26 09	4 467 667 023	40·5832477	11·8094916	·0006071645
1648	2 71 59 04	4 475 809 792	40·5955663	11·8118812	·0006067961
1649	2 71 92 01	4 483 962 449	40·6078810	11·8142698	·0006064281
1650	2 72 25 00	4 492 125 000	40·6201920	11·8166576	·0006060606
1651	2 72 58 01	4 500 297 451	40·6324993	11·8190443	·0006056935
1652	2 72 91 04	4 508 479 808	40·6448029	11·8214301	·0006053269
1653	2 73 24 09	4 516 672 077	40·6571027	11·8238149	·0006049607
1654	2 73 57 16	4 524 874 264	40·6693988	11·8261987	·0006045949
1655	2 73 90 25	4 533 086 375	40·6816912	11·8285816	·0006042296
1656	2 74 23 36	4 541 308 416	40·6939799	11·8309634	·0006038647
1657	2 74 56 49	4 549 540 393	40·7062648	11·8333444	·0006035003
1658	2 74 89 64	4 557 782 312	40·7185461	11·8357244	·0006031363
1659	2 75 22 81	4 566 034 179	40·7308237	11·8381034	·0006027728
1660	2 75 56 00	4 574 296 000	40·7430976	11·8404815	·0006024096
1661	2 75 89 21	4 582 567 781	40·7553677	11·8428586	·0006020470
1662	2 76 22 44	4 590 849 528	40·7676342	11·8452348	·0006016847

TABLE XLV. (*continued*).

No.	Square	Cube	Square Root	Cube Root	Reciprocal
1663	2 76 55 69	4 599 141 247	40·7798970	11·8476100	·0006013229
1664	2 76 88 96	4 607 442 994	40·7921561	11·8499843	·0006009615
1665	2 77 22 25	4 615 754 625	40·8044115	11·8523576	·0006006006
1666	2 77 55 56	4 624 076 296	40·8166633	11·8547299	·0006002401
1667	2 77 88 89	4 632 407 963	40·8289113	11·8571014	·0005998800
1668	2 78 22 24	4 640 749 632	40·8411557	11·8594719	·0005995204
1669	2 78 55 61	4 649 101 309	40·8533964	11·8618414	·0005991612
1670	2 78 89 00	4 657 463 000	40·8656335	11·8642100	·0005988024
1671	2 79 22 41	4 665 834 711	40·8778669	11·8665776	·0005984440
1672	2 79 55 84	4 674 216 448	40·8900966	11·8689443	·0005980861
1673	2 79 89 29	4 682 608 217	40·9023227	11·8713100	·0005977286
1674	2 80 22 76	4 691 010 024	40·9145451	11·8736748	·0005973716
1675	2 80 56 25	4 699 421 875	40·9267638	11·8760387	·0005970149
1676	2 80 89 76	4 707 843 776	40·9389790	11·8784016	·0005966587
1677	2 81 23 29	4 716 275 733	40·9511905	11·8807636	·0005963029
1678	2 81 56 84	4 724 717 752	40·9633983	11·8831246	·0005959476
1679	2 81 90 41	4 733 169 839	40·9756025	11·8854847	·0005955926
1680	2 82 24 00	4 741 632 000	40·9878031	11·8878439	·0005952381
1681	2 82 57 61	4 750 104 241	41·0000000	11·8902022	·0005948840
1682	2 82 91 24	4 758 586 568	41·0121933	11·8925595	·0005945303
1683	2 83 24 89	4 767 078 987	41·0243830	11·8949159	·0005941771
1684	2 83 58 56	4 775 581 504	41·0365691	11·8972713	·0005938242
1685	2 83 92 25	4 784 094 125	41·0487515	11·8996258	·0005934718
1686	2 84 25 96	4 792 616 856	41·0609303	11·9019793	·0005931198
1687	2 84 59 69	4 801 149 703	41·0731055	11·9043319	·0005927682
1688	2 84 93 44	4 809 692 672	41·0852772	11·9066836	·0005924171
1689	2 85 27 21	4 818 245 769	41·0974452	11·9090344	·0005920663
1690	2 85 61 00	4 826 809 000	41·1096096	11·9113843	·0005917160
1691	2 85 94 81	4 835 382 371	41·1217704	11·9137332	·0005913661
1692	2 86 28 64	4 843 965 888	41·1339276	11·9160812	·0005910165
1693	2 86 62 49	4 852 559 557	41·1460812	11·9184283	·0005906675
1694	2 86 96 36	4 861 163 384	41·1582313	11·9207744	·0005903188
1695	2 87 30 25	4 869 777 375	41·1703777	11·9231196	·0005899705
1696	2 87 64 16	4 878 401 536	41·1825206	11·9254639	·0005896226
1697	2 87 98 09	4 887 035 873	41·1946599	11·9278073	·0005892752
1698	2 88 32 04	4 895 680 392	41·2067956	11·9301497	·0005889282
1699	2 88 66 01	4 904 335 099	41·2189277	11·9324913	·0005885815
1700	2 89 00 00	4 913 000 000	41·2310563	11·9348319	·0005882353
1701	2 89 34 01	4 921 675 101	41·2431812	11·9371716	·0005878895
1702	2 89 68 04	4 930 360 408	41·2553027	11·9395104	·0005875441
1703	2 90 02 09	4 939 055 927	41·2674205	11·9418482	·0005871991
1704	2 90 36 16	4 947 761 664	41·2795349	11·9441852	·0005868545
1705	2 90 70 25	4 956 477 625	41·2916456	11·9465213	·0005865103
1706	2 91 04 36	4 965 203 816	41·3037529	11·9488564	·0005861665
1707	2 91 38 49	4 973 940 243	41·3158565	11·9511906	·0005858231
1708	2 91 72 64	4 982 686 912	41·3279566	11·9535239	·0005854801
1709	2 92 06 81	4 991 443 829	41·3400532	11·9558563	·0005851375
1710	2 92 41 00	5 000 211 000	41·3521463	11·9581878	·0005847953
1711	2 92 75 21	5 008 988 431	41·3642358	11·9605184	·0005844585
1712	2 93 09 44	5 017 776 128	41·3763217	11·9628481	·0005841121
1713	2 93 43 69	5 026 574 037	41·3884042	11·9651768	·0005837712
1714	2 93 77 96	5 035 382 344	41·4004831	11·9675047	·0005834306
1715	2 94 12 25	5 044 200 875	41·4125585	11·9698317	·0005830904
1716	2 94 46 56	5 053 029 696	41·4246304	11·9721577	·0005827506
1717	2 94 80 89	5 061 868 813	41·4366987	11·9744829	·0005824112
1718	2 95 15 24	5 070 718 232	41·4487636	11·9768071	·0005820722
1719	2 95 49 61	5 079 577 159	41·4608249	11·9791304	·0005817336
1720	2 95 84 00	5 088 448 000	41·4728827	11·9814528	·0005813953
1721	2 96 18 41	5 097 328 361	41·4849370	11·9837744	·0005810575
1722	2 96 52 84	5 106 219 048	41·4969878	11·9860950	·0005807201
1723	2 96 87 29	5 115 120 067	41·5090351	11·9884148	·0005803831
1724	2 97 21 76	5 124 031 424	41·5210790	11·9907336	·0005800464
1725	2 97 56 25	5 132 953 125	41·5331193	11·9930516	·0005797101
1726	2 97 90 76	5 141 885 176	41·5451561	11·9953686	·0005793743
1727	2 98 25 29	5 150 827 583	41·5571895	11·9976848	·0005790388
1728	2 98 59 84	5 159 780 352	41·5692194	12·0000000	·0005787037
1729	2 98 94 41	5 168 743 489	41·5812457	12·0023144	·0005783690
1730	2 99 29 00	5 177 717 000	41·5932686	12·0046278	·0005780347
1731	2 99 63 61	5 186 700 891	41·6052881	12·0069404	·0005777008
1732	2 99 98 24	5 195 695 168	41·6173041	12·0092521	·0005773672
1733	3 00 32 89	5 204 699 837	41·6293166	12·0115629	·0005770340
1734	3 00 67 56	5 213 714 904	41·6413256	12·0138728	·0005767013
1735	3 01 02 25	5 222 740 375	41·6533312	12·0161818	·0005763689
1736	3 01 36 96	5 231 776 256	41·6653333	12·0184900	·0005760369
1737	3 01 71 69	5 240 822 553	41·6773319	12·0207973	·0005757052
1738	3 02 06 44	5 249 879 272	41·6893271	12·0231037	·0005753740
1739	3 02 41 21	5 258 946 419	41·7013189	12·0254092	·0005750431
1740	3 02 76 00	5 268 024 000	41·7133072	12·0277138	·0005747126
1741	3 03 10 81	5 277 112 021	41·7252921	12·0300175	·0005743825
1742	3 03 45 64	5 286 210 488	41·7372735	12·0323204	·0005740528
1743	3 03 80 49	5 295 319 407	41·7492515	12·0346223	·0005737235
1744	3 04 15 36	5 304 438 784	41·7612260	12·0369233	·0005733945
1745	3 04 50 25	5 313 568 625	41·7731971	12·0392235	·0005730659
1746	3 04 85 16	5 322 708 936	41·7851648	12·0415229	·0005727377
1747	3 05 20 09	5 331 859 723	41·7971291	12·0438213	·0005724098
1748	3 05 55 04	5 341 020 992	41·8090899	12·0461189	·0005720824
1749	3 05 90 01	5 350 192 749	41·8210473	12·0484156	·0005717553
1750	3 06 25 00	5 359 375 000	41·8330013	12·0507114	·0005714286
1751	3 06 60 01	5 368 567 751	41·8449519	12·0530063	·0005711022
1752	3 06 95 04	5 377 771 008	41·8568991	12·0553003	·0005707763
1753	3 07 30 09	5 386 984 777	41·8688428	12·0575935	·0005704507
1754	3 07 65 16	5 396 209 064	41·8807832	12·0598859	·0005701254
1755	3 08 00 25	5 405 443 875	41·8927201	12·0621773	·0005698006
1756	3 08 35 36	5 414 689 216	41·9046537	12·0644679	·0005694761
1757	3 08 70 49	5 423 945 093	41·9165838	12·0667576	·0005691520
1758	3 09 05 64	5 433 211 512	41·9285106	12·0690464	·0005688282
1759	3 09 40 81	5 442 488 479	41·9404339	12·0713344	·0005685048
1760	3 09 76 00	5 451 776 000	41·9523539	12·0736215	·0005681818

TABLE XLV. (*continued*).

No.	Square	Cube	Square Root	Cube Root	Reciprocal
1761	3 10 11 21	5 461 074 061	41·9642705	12·0759077	·0005678592
1762	3 10 46 44	5 470 382 728	41·9761837	12·0781930	·0005675369
1763	3 10 81 69	5 479 701 947	41·9880935	12·0804775	·0005672150
1764	3 11 16 96	5 489 031 744	42·0000000	12·0827612	·0005668934
1765	3 11 52 25	5 498 372 125	42·0119031	12·0850439	·0005665722
1766	3 11 87 56	5 507 723 096	42·0238028	12·0873258	·0005662514
1767	3 12 22 89	5 517 084 663	42·0356991	12·0896069	·0005659310
1768	3 12 58 24	5 526 456 832	42·0475921	12·0918870	·0005656109
1769	3 12 93 61	5 535 839 609	42·0594817	12·0941664	·0005652911
1770	3 13 29 00	5 545 233 000	42·0713679	12·0964449	·0005649718
1771	3 13 64 41	5 554 637 011	42·0832508	12·0987226	·0005646527
1772	3 13 99 84	5 564 051 648	42·0951304	12·1009993	·0005643341
1773	3 14 35 29	5 573 476 917	42·1070065	12·1032753	·0005640158
1774	3 14 70 76	5 582 912 824	42·1188794	12·1055503	·0005636979
1775	3 15 06 25	5 592 359 375	42·1307488	12·1078245	·0005633803
1776	3 15 41 76	5 601 816 576	42·1426150	12·1100979	·0005630631
1777	3 15 77 29	5 611 284 433	42·1544778	12·1123704	·0005627462
1778	3 16 12 84	5 620 762 952	42·1663373	12·1146420	·0005624297
1779	3 16 48 41	5 630 252 139	42·1781934	12·1169128	·0005621135
1780	3 16 84 00	5 639 752 000	42·1900462	12·1191827	·0005617978
1781	3 17 19 61	5 649 262 541	42·2018957	12·1214518	·0005614823
1782	3 17 55 24	5 658 783 768	42·2137418	12·1237200	·0005611672
1783	3 17 90 89	5 668 315 687	42·2255846	12·1259874	·0005608525
1784	3 18 26 56	5 677 858 304	42·2374242	12·1282539	·0005605381
1785	3 18 62 25	5 687 411 625	42·2492603	12·1305197	·0005602241
1786	3 18 97 96	5 696 975 656	42·2610932	12·1327845	·0005599104
1787	3 19 33 69	5 706 550 403	42·2729227	12·1350485	·0005595971
1788	3 19 69 44	5 716 135 872	42·2847490	12·1373117	·0005592841
1789	3 20 05 21	5 725 732 069	42·2965719	12·1395740	·0005589715
1790	3 20 41 00	5 735 339 000	42·3083916	12·1418355	·0005586592
1791	3 20 76 81	5 744 956 671	42·3202079	12·1440961	·0005583473
1792	3 21 12 64	5 754 585 088	42·3320210	12·1463559	·0005580357
1793	3 21 48 49	5 764 224 257	42·3438307	12·1486148	·0005577245
1794	3 21 84 36	5 773 874 184	42·3556371	12·1508729	·0005574136
1795	3 22 20 25	5 783 534 875	42·3674403	12·1531302	·0005571031
1796	3 22 56 16	5 793 206 336	42·3792402	12·1553866	·0005567929
1797	3 22 92 09	5 802 888 573	42·3910368	12·1576422	·0005564830
1798	3 23 28 04	5 812 581 592	42·4028301	12·1598970	·0005561735
1799	3 23 64 01	5 822 285 399	42·4146201	12·1621509	·0005558644
1800	3 24 00 00	5 832 000 000	42·4264069	12·1644040	·0005555556
1801	3 24 36 01	5 841 725 401	42·4381903	12·1666562	·0005552471
1802	3 24 72 04	5 851 461 608	42·4499705	12·1689076	·0005549390
1803	3 25 08 09	5 861 208 627	42·4617475	12·1711582	·0005546312
1804	3 25 44 16	5 870 966 464	42·4735212	12·1734079	·0005543237
1805	3 25 80 25	5 880 735 125	42·4852916	12·1756569	·0005540166
1806	3 26 16 36	5 890 514 616	42·4970587	12·1779050	·0005537099
1807	3 26 52 49	5 900 304 943	42·5088226	12·1801522	·0005534034
1808	3 26 88 64	5 910 106 112	42·5205833	12·1823987	·0005530973
1809	3 27 24 81	5 919 918 129	42·5323406	12·1846443	·0005527916
1810	3 27 61 00	5 929 741 000	42·5440948	12·1868891	·0005524862
1811	3 27 97 21	5 939 574 731	42·5558456	12·1891331	·0005521811
1812	3 28 33 44	5 949 419 328	42·5675933	12·1913762	·0005518764
1813	3 28 69 69	5 959 274 797	42·5793377	12·1936185	·0005515720
1814	3 29 05 96	5 969 141 144	42·5910789	12·1958599	·0005512679
1815	3 29 42 25	5 979 018 375	42·6028168	12·1981006	·0005509642
1816	3 29 78 56	5 988 906 496	42·6145515	12·2003404	·0005506608
1817	3 30 14 89	5 998 805 513	42·6262829	12·2025794	·0005503577
1818	3 30 51 24	6 008 715 432	42·6380112	12·2048176	·0005500550
1819	3 30 87 61	6 018 636 259	42·6497362	12·2070549	·0005497526
1820	3 31 24 00	6 028 568 000	42·6614580	12·2092915	·0005494505
1821	3 31 60 41	6 038 510 661	42·6731766	12·2115272	·0005491488
1822	3 31 96 84	6 048 464 248	42·6848919	12·2137621	·0005488474
1823	3 32 33 29	6 058 428 767	42·6966040	12·2159962	·0005485464
1824	3 32 69 76	6 068 404 224	42·7083130	12·2182295	·0005482456
1825	3 33 06 25	6 078 390 625	42·7200187	12·2204620	·0005479452
1826	3 33 42 76	6 088 387 976	42·7317212	12·2226936	·0005476451
1827	3 33 79 29	6 098 396 283	42·7434206	12·2249244	·0005473454
1828	3 34 15 84	6 108 415 552	42·7551167	12·2271544	·0005470460
1829	3 34 52 41	6 118 445 789	42·7668095	12·2293836	·0005467469
1830	3 34 89 00	6 128 487 000	42·7784992	12·2316120	·0005464481
1831	3 35 25 61	6 138 539 191	42·7901858	12·2338396	·0005461496
1832	3 35 62 24	6 148 602 368	42·8018691	12·2360663	·0005458515
1833	3 35 98 89	6 158 676 537	42·8135492	12·2382923	·0005455537
1834	3 36 35 56	6 168 761 704	42·8252262	12·2405174	·0005452563
1835	3 36 72 25	6 178 857 875	42·8368999	12·2427418	·0005449591
1836	3 37 08 96	6 188 965 056	42·8485706	12·2449653	·0005446623
1837	3 37 45 69	6 199 083 253	42·8602380	12·2471880	·0005443658
1838	3 37 82 44	6 209 212 472	42·8719022	12·2494099	·0005440696
1839	3 38 19 21	6 219 352 719	42·8835633	12·2516310	·0005437738
1840	3 38 56 00	6 229 504 000	42·8952212	12·2538513	·0005434783
1841	3 38 92 81	6 239 666 321	42·9068759	12·2560708	·0005431831
1842	3 39 29 64	6 249 839 688	42·9185275	12·2582895	·0005428882
1843	3 39 66 49	6 260 024 107	42·9301759	12·2605074	·0005425936
1844	3 40 03 36	6 270 219 584	42·9418211	12·2627245	·0005422993
1845	3 40 40 25	6 280 426 125	42·9534632	12·2649408	·0005420054
1846	3 40 77 16	6 290 643 736	42·9651021	12·2671563	·0005417118
1847	3 41 14 09	6 300 872 423	42·9767379	12·2693710	·0005414185
1848	3 41 51 04	6 311 112 192	42·9883705	12·2715849	·0005411255
1849	3 41 88 01	6 321 363 049	43·0000000	12·2737980	·0005408329
1850	3 42 25 00	6 331 625 000	43·0116263	12·2760103	·0005405405
1851	3 42 62 01	6 341 898 051	43·0232495	12·2782218	·0005402485
1852	3 42 99 04	6 352 182 208	43·0348696	12·2804325	·0005399568
1853	3 43 36 09	6 362 477 477	43·0464865	12·2826424	·0005396654
1854	3 43 73 16	6 372 783 864	43·0581003	12·2848515	·0005393743
1855	3 44 10 25	6 383 101 375	43·0697109	12·2870598	·0005390836
1856	3 44 47 36	6 393 430 016	43·0813185	12·2892673	·0005387931
1857	3 44 84 49	6 403 769 793	43·0929228	12·2914740	·0005385030
1858	3 45 21 64	6 414 120 712	43·1045241	12·2936800	·0005382131

TABLE XLV. *(continued).*

No.	Square	Cube	Square Root	Cube Root	Reciprocal
1859	3 45 58 81	6 424 482 779	43·1161223	12·2958851	·0005379236
1860	3 45 96 00	6 434 856 000	43·1277173	12·2980895	·0005376344
1861	3 46 33 21	6 445 240 381	43·1393092	12·3002930	·0005373455
1862	3 46 70 44	6 455 635 928	43·1508980	12·3024958	·0005370569
1863	3 47 07 69	6 466 042 647	43·1624837	12·3046978	·0005367687
1864	3 47 44 96	6 476 460 544	43·1740663	12·3068990	·0005364807
1865	3 47 82 25	6 486 889 625	43·1856458	12·3090994	·0005361930
1866	3 48 19 56	6 497 329 896	43·1972221	12·3112991	·0005359057
1867	3 48 56 89	6 507 781 363	43·2087954	12·3134979	·0005356186
1868	3 48 94 24	6 518 244 032	43·2203656	12·3156959	·0005353319
1869	3 49 31 61	6 528 717 909	43·2319326	12·3178932	·0005350455
1870	3 49 69 00	6 539 203 000	43·2434966	12·3200897	·0005347594
1871	3 50 06 41	6 549 699 311	43·2550575	12·3222854	·0005344735
1872	3 50 43 84	6 560 206 848	43·2666153	12·3244803	·0005341880
1873	3 50 81 29	6 570 725 617	43·2781700	12·3266744	·0005339028
1874	3 51 18 76	6 581 255 624	43·2897216	12·3288678	·0005336179
1875	3 51 56 25	6 591 796 875	43·3012702	12·3310604	·0005333333
1876	3 51 93 76	6 602 349 376	43·3128157	12·3332522	·0005330490
1877	3 52 31 29	6 612 913 133	43·3243580	12·3354432	·0005327651
1878	3 52 68 84	6 623 488 152	43·3358973	12·3376334	·0005324814
1879	3 53 06 41	6 634 074 439	43·3474336	12·3398229	·0005321980
1880	3 53 44 00	6 644 672 000	43·3589668	12·3420116	·0005319149
1881	3 53 81 61	6 655 280 841	43·3704969	12·3441995	·0005316321
1882	3 54 19 24	6 665 900 968	43·3820239	12·3463866	·0005313496
1883	3 54 56 89	6 676 532 387	43·3935479	12·3485730	·0005310674
1884	3 54 94 56	6 687 175 104	43·4050688	12·3507586	·0005307856
1885	3 55 32 25	6 697 829 125	43·4165867	12·3529434	·0005305040
1886	3 55 69 96	6 708 494 456	43·4281015	12·3551274	·0005302227
1887	3 56 07 69	6 719 171 103	43·4396132	12·3573107	·0005299417
1888	3 56 45 44	6 729 859 072	43·4511220	12·3594932	·0005296610
1889	3 56 83 21	6 740 558 369	43·4626276	12·3616749	·0005293806
1890	3 57 21 00	6 751 269 000	43·4741302	12·3638559	·0005291005
1891	3 57 58 81	6 761 990 971	43·4856298	12·3660361	·0005288207
1892	3 57 96 64	6 772 724 288	43·4971263	12·3682155	·0005285412
1893	3 58 34 49	6 783 468 957	43·5086198	12·3703941	·0005282620
1894	3 58 72 36	6 794 224 984	43·5201103	12·3725721	·0005279831
1895	3 59 10 25	6 804 992 375	43·5315977	12·3747492	·0005277045
1896	3 59 48 16	6 815 771 136	43·5430821	12·3769255	·0005274262
1897	3 59 86 09	6 826 561 273	43·5545635	12·3791011	·0005271481
1898	3 60 24 04	6 837 362 792	43·5660418	12·3812759	·0005268704
1899	3 60 62 01	6 848 175 699	43·5775171	12·3834500	·0005265929
1900	3 61 00 00	6 859 000 000	43·5889894	12·3856233	·0005263158
1901	3 61 38 01	6 869 835 701	43·6004587	12·3877959	·0005260389
1902	3 61 76 04	6 880 682 808	43·6119249	12·3899676	·0005257624
1903	3 62 14 09	6 891 541 327	43·6233882	12·3921386	·0005254861
1904	3 62 52 16	6 902 411 264	43·6348485	12·3943089	·0005252101
1905	3 62 90 25	6 913 292 625	43·6463057	12·3964784	·0005249344
1906	3 63 28 36	6 924 185 416	43·6577599	12·3986471	·0005246590
1907	3 63 66 49	6 935 089 643	43·6692111	12·4008151	·0005243838
1908	3 64 04 64	6 946 005 312	43·6806593	12·4029823	·0005241090
1909	3 64 42 81	6 956 932 429	43·6921045	12·4051488	·0005238345
1910	3 64 81 00	6 967 871 000	43·7035467	12·4073145	·0005235602
1911	3 65 19 21	6 978 821 031	43·7149860	12·4094794	·0005232862
1912	3 65 57 44	6 989 782 528	43·7264222	12·4116436	·0005230126
1913	3 65 95 69	7 000 755 497	43·7378554	12·4138070	·0005227392
1914	3 66 33 96	7 011 739 944	43·7492857	12·4159697	·0005224660
1915	3 66 72 25	7 022 735 875	43·7607129	12·4181316	·0005221932
1916	3 67 10 56	7 033 743 296	43·7721373	12·4202928	·0005219207
1917	3 67 48 89	7 044 762 213	43·7835585	12·4224533	·0005216484
1918	3 67 87 24	7 055 792 632	43·7949768	12·4246129	·0005213764
1919	3 68 25 61	7 066 834 559	43·8063922	12·4267719	·0005211047
1920	3 68 64 00	7 077 888 000	43·8178046	12·4289300	·0005208333
1921	3 69 02 41	7 088 952 961	43·8292140	12·4310875	·0005205622
1922	3 69 40 84	7 100 029 448	43·8406204	12·4332441	·0005202914
1923	3 69 79 29	7 111 117 467	43·8520239	12·4354001	·0005200208
1924	3 70 17 76	7 122 217 024	43·8634244	12·4375552	·0005197505
1925	3 70 56 25	7 133 328 125	43·8748219	12·4397097	·0005194805
1926	3 70 94 76	7 144 450 776	43·8862165	12·4418634	·0005192108
1927	3 71 33 29	7 155 584 983	43·8976081	12·4440163	·0005189414
1928	3 71 71 84	7 166 730 752	43·9089968	12·4461685	·0005186722
1929	3 72 10 41	7 177 888 089	43·9203825	12·4483200	·0005184033
1930	3 72 49 00	7 189 057 000	43·9317652	12·4504707	·0005181347
1931	3 72 87 61	7 200 237 491	43·9431451	12·4526206	·0005178664
1932	3 73 26 24	7 211 429 568	43·9545220	12·4547699	·0005175983
1933	3 73 64 89	7 222 633 237	43·9658959	12·4569184	·0005173306
1934	3 74 03 56	7 233 848 504	43·9772668	12·4590661	·0005170631
1935	3 74 42 25	7 245 075 375	43·9886349	12·4612131	·0005167959
1936	3 74 80 96	7 256 313 856	44·0000000	12·4633594	·0005165289
1937	3 75 19 69	7 267 563 953	44·0113622	12·4655049	·0005162623
1938	3 75 58 44	7 278 825 672	44·0227214	12·4676497	·0005159959
1939	3 75 97 21	7 290 099 019	44·0340777	12·4697937	·0005157298
1940	3 76 36 00	7 301 384 000	44·0454311	12·4719370	·0005154639
1941	3 76 74 81	7 312 680 621	44·0567815	12·4740796	·0005151984
1942	3 77 13 64	7 323 988 888	44·0681291	12·4762214	·0005149331
1943	3 77 52 49	7 335 308 807	44·0794737	12·4783625	·0005146680
1944	3 77 91 36	7 346 640 384	44·0908154	12·4805029	·0005144033
1945	3 78 30 25	7 357 983 625	44·1021541	12·4826426	·0005141388
1946	3 78 69 16	7 369 338 536	44·1134900	12·4847815	·0005138746
1947	3 79 08 09	7 380 705 123	44·1248229	12·4869197	·0005136107
1948	3 79 47 04	7 392 083 392	44·1361530	12·4890571	·0005133470
1949	3 79 86 01	7 403 473 349	44·1474801	12·4911938	·0005130836
1950	3 80 25 00	7 414 875 000	44·1588043	12·4933298	·0005128205
1951	3 80 64 01	7 426 288 351	44·1701256	12·4954651	·0005125577
1952	3 81 03 04	7 437 713 408	44·1814441	12·4975995	·0005122951
1953	3 81 42 09	7 449 150 177	44·1927596	12·4997333	·0005120328
1954	3 81 81 16	7 460 598 664	44·2040722	12·5018664	·0005117707
1955	3 82 20 25	7 472 058 875	44·2153819	12·5039988	·0005115090
1956	3 82 59 36	7 483 530 816	44·2266888	12·5061304	·0005112474

TABLE XLV. (*continued*).

No.	Square	Cube	Square Root	Cube Root	Reciprocal
1957	3 82 98 49	7 495 014 498	44·2379927	12·5082612	·0005109862
1958	3 83 37 64	7 506 509 912	44·2492938	12·5108914	·0005107252
1959	3 83 76 81	7 518 017 079	44·2605919	12·5125208	·0005104645
1960	3 84 16 00	7 529 536 000	44·2718872	12·5146495	·0005102041
1961	3 84 55 21	7 541 066 681	44·2831797	12·5167775	·0005099439
1962	3 84 94 44	7 552 609 128	44·2944692	12·5189047	·0005096840
1963	3 85 33 69	7 564 163 347	44·3057558	12·5210313	·0005094244
1964	3 85 72 96	7 575 729 344	44·3170396	12·5231571	·0005091650
1965	3 86 12 25	7 587 307 125	44·3283205	12·5252822	·0005089059
1966	3 86 51 56	7 598 896 696	44·3395985	12·5274065	·0005086470
1967	3 86 90 89	7 610 498 063	44·3508737	12·5295302	·0005083884
1968	3 87 30 24	7 622 111 232	44·3621460	12·5316531	·0005081301
1969	3 87 69 61	7 633 736 209	44·3734155	12·5337753	·0005078720
1970	3 88 09 00	7 645 373 000	44·3846820	12·5358968	·0005076142
1971	3 88 48 41	7 657 021 611	44·3959457	12·5380176	·0005073567
1972	3 88 87 84	7 668 682 048	44·4072066	12·5401377	·0005070994
1973	3 89 27 29	7 680 354 317	44·4184646	12·5422570	·0005068424
1974	3 89 66 76	7 692 038 424	44·4297198	12·5443757	·0005065856
1975	3 90 06 25	7 703 734 375	44·4409720	12·5464936	·0005063291
1976	3 90 45 76	7 715 442 176	44·4522215	12·5486107	·0005060729
1977	3 90 85 29	7 727 161 833	44·4634681	12·5507272	·0005058169
1978	3 91 24 84	7 738 893 352	44·4747119	12·5528430	·0005055612
1979	3 91 64 41	7 750 636 739	44·4859528	12·5549580	·0005053057
1980	3 92 04 00	7 762 392 000	44·4971909	12·5570723	·0005050505
1981	3 92 43 61	7 774 159 141	44·5084262	12·5591860	·0005047956
1982	3 92 83 24	7 785 938 168	44·5196586	12·5612989	·0005045409
1983	3 93 22 89	7 797 729 087	44·5308881	12·5634111	·0005042864
1984	3 93 62 56	7 809 531 904	44·5421149	12·5655226	·0005040323
1985	3 94 02 25	7 821 346 625	44·5533388	12·5676334	·0005037788
1986	3 94 41 96	7 833 173 256	44·5645599	12·5697435	·0005035247
1987	3 94 81 69	7 845 011 803	44·5757781	12·5718529	·0005032713
1988	3 95 21 44	7 856 862 272	44·5869936	12·5739615	·0005030181
1989	3 95 61 21	7 868 724 669	44·5982062	12·5760695	·0005027652
1990	3 96 01 00	7 880 599 000	44·6094160	12·5781767	·0005025126
1991	3 96 40 81	7 892 485 271	44·6206230	12·5802832	·0005022602
1992	3 96 80 64	7 904 383 488	44·6318272	12·5823891	·0005020080
1993	3 97 20 49	7 916 293 657	44·6430286	12·5844942	·0005017561
1994	3 97 60 36	7 928 215 784	44·6542271	12·5865987	·0005015045
1995	3 98 00 25	7 940 149 875	44·6654228	12·5887024	·0005012531
1996	3 98 40 16	7 952 095 936	44·6766158	12·5908054	·0005010020
1997	3 98 80 09	7 964 053 973	44·6878059	12·5929078	·0005007511
1998	3 99 20 04	7 976 023 992	44·6989933	12·5950094	·0005005005
1999	3 99 60 01	7 988 005 999	44·7101778	12·5971103	·0005002501
2000	4 00 00 00	8 000 000 000	44·7213596	12·5992105	·0005000000
2001	4 00 40 01	8 012 006 001	44·7325385	12·6013101	·0004997501
2002	4 00 80 04	8 024 024 008	44·7437146	12·6034089	·0004995005
2003	4 01 20 09	8 036 054 027	44·7548880	12·6055070	·0004992511
2004	4 01 60 16	8 048 096 064	44·7660586	12·6076044	·0004990020
2005	4 02 00 25	8 060 150 125	44·7772264	12·6097011	·0004987531
2006	4 02 40 36	8 072 216 216	44·7883913	12·6117971	·0004985045
2007	4 02 80 49	8 084 294 343	44·7995535	12·6138924	·0004982561
2008	4 03 20 64	8 096 384 512	44·8107130	12·6159870	·0004980080
2009	4 03 60 81	8 108 486 729	44·8218697	12·6180810	·0004977601
2010	4 04 01 00	8 120 601 000	44·8330235	12·6201743	·0004975124
2011	4 04 41 21	8 132 727 331	44·8441746	12·6222669	·0004972650
2012	4 04 81 44	8 144 865 728	44·8553230	12·6243587	·0004970179
2013	4 05 21 69	8 157 016 197	44·8664685	12·6264499	·0004967710
2014	4 05 61 96	8 169 178 744	44·8776113	12·6285404	·0004965243
2015	4 06 02 25	8 181 353 375	44·8887514	12·6306301	·0004962779
2016	4 06 42 56	8 193 540 096	44·8998886	12·6327192	·0004960317
2017	4 06 82 89	8 205 738 913	44·9110231	12·6348076	·0004957858
2018	4 07 23 24	8 217 949 832	44·9221549	12·6368953	·0004955401
2019	4 07 63 61	8 230 172 859	44·9332839	12·6389823	·0004952947
2020	4 08 04 00	8 242 408 000	44·9444101	12·6410687	·0004950495
2021	4 08 44 41	8 254 655 261	44·9555336	12·6431543	·0004948046
2022	4 08 84 84	8 266 914 648	44·9666543	12·6452393	·0004945598
2023	4 09 25 29	8 279 186 167	44·9777723	12·6473235	·0004943154
2024	4 09 65 76	8 291 469 824	44·9888875	12·6494071	·0004940711
2025	4 10 06 25	8 303 765 625	45·0000000	12·6514900	·0004938272
2026	4 10 46 76	8 316 073 576	45·0111097	12·6535722	·0004935834
2027	4 10 87 29	8 328 393 683	45·0222167	12·6556538	·0004933399
2028	4 11 27 84	8 340 725 952	45·0333210	12·6577346	·0004930966
2029	4 11 68 41	8 353 070 389	45·0444225	12·6598148	·0004928536
2030	4 12 09 00	8 365 427 000	45·0555213	12·6618943	·0004926108
2031	4 12 49 61	8 377 795 791	45·0666173	12·6639731	·0004923683
2032	4 12 90 24	8 390 176 768	45·0777107	12·6660512	·0004921260
2033	4 13 30 89	8 402 569 937	45·0888013	12·6681286	·0004918839
2034	4 13 71 56	8 414 975 304	45·0998891	12·6702053	·0004916421
2035	4 14 12 25	8 427 392 875	45·1109743	12·6722814	·0004914005
2036	4 14 52 96	8 439 822 656	45·1220567	12·6743567	·0004911591
2037	4 14 93 69	8 452 264 653	45·1331364	12·6764314	·0004909180
2038	4 15 34 44	8 464 718 872	45·1442134	12·6785054	·0004906771
2039	4 15 75 21	8 477 185 319	45·1552876	12·6805788	·0004904365
2040	4 16 16 00	8 489 664 000	45·1663592	12·6826514	·0004901961
2041	4 16 56 81	8 502 154 921	45·1774280	12·6847234	·0004899559
2042	4 16 97 64	8 514 658 088	45·1884941	12·6867947	·0004897160
2043	4 17 38 49	8 527 173 507	45·1995575	12·6888654	·0004894762
2044	4 17 79 36	8 539 701 184	45·2106182	12·6909354	·0004892368
2045	4 18 20 25	8 552 241 125	45·2216762	12·6930047	·0004889976
2046	4 18 61 16	8 564 793 336	45·2327315	12·6950733	·0004887586
2047	4 19 02 09	8 577 357 823	45·2437841	12·6971412	·0004885198
2048	4 19 43 04	8 589 934 592	45·2548340	12·6992084	·0004882813
2049	4 19 84 01	8 602 523 649	45·2658812	12·7012750	·0004880429
2050	4 20 25 00	8 615 125 000	45·2769257	12·7033409	·0004878049
2051	4 20 66 01	8 627 738 651	45·2879675	12·7054061	·0004875670
2052	4 21 07 04	8 640 364 608	45·2990066	12·7074707	·0004873294
2053	4 21 48 09	8 653 002 877	45·3100430	12·7095346	·0004870921
2054	4 21 89 16	8 665 653 464	45·3210768	12·7115978	·0004868549

TABLE XLV. (*continued*).

No.	Square	Cube	Square Root	Cube Root	Reciprocal
2055	4 22 30 25	8 678 316 375	45·3321078	12·7136603	·0004866180
2056	4 22 71 36	8 690 991 616	45·3431562	12·7157222	·0004863813
2057	4 23 12 49	8 703 679 193	45·3541619	12·7177835	·0004861449
2058	4 23 53 64	8 716 379 112	45·3651849	12·7198441	·0004859086
2059	4 23 94 81	8 729 091 379	45·3762052	12·7219040	·0004856727
2060	4 24 36 00	8 741 816 000	45·3872229	12·7239632	·0004854369
2061	4 24 77 21	8 754 552 981	45·3982378	12·7260218	·0004852014
2062	4 25 18 44	8 767 302 328	45·4092501	12·7280797	·0004849661
2063	4 25 59 69	8 780 064 047	45·4202598	12·7301370	·0004847310
2064	4 26 00 96	8 792 838 144	45·4312668	12·7321935	·0004844961
2065	4 26 42 25	8 805 624 625	45·4422711	12·7342494	·0004842615
2066	4 26 83 56	8 818 423 496	45·4532727	12·7363046	·0004840271
2067	4 27 24 89	8 831 234 763	45·4642717	12·7383592	·0004837929
2068	4 27 66 24	8 844 058 432	45·4752680	12·7404131	·0004835590
2069	4 28 07 61	8 856 894 509	45·4862616	12·7424664	·0004833253
2070	4 28 49 00	8 869 743 000	45·4972526	12·7445189	·0004830918
2071	4 28 90 41	8 882 603 911	45·5082410	12·7465709	·0004828585
2072	4 29 31 84	8 895 477 248	45·5192267	12·7486222	·0004826255
2073	4 29 73 29	8 908 363 017	45·5302097	12·7506728	·0004823927
2074	4 30 14 76	8 921 261 224	45·5411901	12·7527227	·0004821601
2075	4 30 56 25	8 934 171 875	45·5521679	12·7547721	·0004819277
2076	4 30 97 76	8 947 094 976	45·5631430	12·7568207	·0004816956
2077	4 31 39 29	8 960 030 533	45·5741155	12·7588687	·0004814636
2078	4 31 80 84	8 972 978 552	45·5850853	12·7609160	·0004812320
2079	4 32 22 41	8 985 939 039	45·5960525	12·7629627	·0004810005
2080	4 32 64 00	8 998 912 000	45·6070170	12·7650087	·0004807692
2081	4 33 05 61	9 011 897 441	45·6179789	12·7670540	·0004805382
2082	4 33 47 24	9 024 895 368	45·6289382	12·7690987	·0004803074
2083	4 33 88 89	9 037 905 787	45·6398948	12·7711427	·0004800768
2084	4 34 30 56	9 050 928 704	45·6508488	12·7731861	·0004798464
2085	4 34 72 25	9 063 964 125	45·6618002	12·7752288	·0004796163
2086	4 35 13 96	9 077 012 056	45·6727490	12·7772709	·0004793864
2087	4 35 55 69	9 090 072 503	45·6836951	12·7793123	·0004791567
2088	4 35 97 44	9 103 145 472	45·6946386	12·7813531	·0004789272
2089	4 36 39 21	9 116 230 969	45·7055795	12·7833932	·0004786979
2090	4 36 81 00	9 129 329 000	45·7165178	12·7854326	·0004784689
2091	4 37 22 81	9 142 439 571	45·7274534	12·7874714	·0004782401
2092	4 37 64 64	9 155 562 688	45·7383865	12·7895096	·0004780115
2093	4 38 06 49	9 168 698 357	45·7493169	12·7915471	·0004777831
2094	4 38 48 36	9 181 846 584	45·7602447	12·7935840	·0004775549
2095	4 38 90 25	9 195 007 375	45·7711699	12·7956202	·0004773270
2096	4 39 32 16	9 208 180 736	45·7820926	12·7976558	·0004770992
2097	4 39 74 09	9 221 366 673	45·7930126	12·7996907	·0004768717
2098	4 40 16 04	9 234 565 192	45·8039299	12·8017250	·0004766444
2099	4 40 58 01	9 247 776 299	45·8148447	12·8037586	·0004764173
2100	4 41 00 00	9 261 000 000	45·8257569	12·8057916	·0004761905
2101	4 41 42 01	9 274 236 301	45·8366665	12·8078239	·0004759638
2102	4 41 84 04	9 287 485 208	45·8475735	12 8098556	·0004757374
2103	4 42 26 09	9 300 746 727	45·8584779	12·8118866	·0004755112
2104	4 42 68 16	9 314 020 864	45·8693798	12·8139170	·0004752852
2105	4 43 10 25	9 327 307 625	45·8802790	12·8159468	·0004750594
2106	4 43 52 36	9 340 607 016	45·8911756	12·8179759	·0004748338
2107	4 43 94 49	9 353 919 043	45·9020696	12·8200044	·0004746084
2108	4 44 36 64	9 367 243 712	45·9129611	12·8220323	·0004743833
2109	4 44 78 81	9 380 581 029	45·9238500	12·8240595	·0004741584
2110	4 45 21 00	9 393 931 000	45·9347363	12·8260861	·0004739336
2111	4 45 63 21	9 407 293 631	45·9456200	12·8281120	·0004737091
2112	4 46 05 44	9 420 668 928	45·9565012	12·8301373	·0004734848
2113	4 46 47 69	9 434 056 897	45·9673798	12·8321620	·0004732608
2114	4 46 89 96	9 447 457 544	45·9782557	12·8341860	·0004730369
2115	4 47 32 25	9 460 870 875	45·9891291	12·8362094	·0004728132
2116	4 47 74 56	9 474 296 896	46·0000000	12·8382321	·0004725898
2117	4 48 16 89	9 487 735 613	46 0108683	12·8402542	·0004723666
2118	4 48 59 24	9 501 187 032	46·0217340	12·8422756	·0004721435
2119	4 49 01 61	9 514 651 159	46·0325971	12·8442964	·0004719207
2120	4 49 44 00	9 528 128 000	46·0434577	12·8463166	·0004716981
2121	4 49 86 41	9 541 617 561	46·0543158	12·8483361	·0004714757
2122	4 50 28 84	9 555 119 848	46·0651712	12·8503551	·0004712535
2123	4 50 71 29	9 568 634 867	46·0760241	12·8523733	·0004710316
2124	4 51 13 76	9 582 162 624	46·0868745	12·8543910	·0004708098
2125	4 51 56 25	9 595 703 125	46·0977223	12·8564080	·0004705882
2126	4 51 98 76	9 609 256 376	46·1085675	12·8584243	·0004703669
2127	4 52 41 29	9 622 822 383	46·1194102	12·8604401	·0004701457
2128	4 52 83 84	9 636 401 152	46·1302504	12·8624552	·0004699248
2129	4 53 26 41	9 649 992 689	46·1410880	12·8644697	·0004697041
2130	4 53 69 00	9 663 597 000	46·1519230	12·8664835	·0004694836
2131	4 54 11 61	9 677 214 091	46·1627555	12·8684967	·0004692633
2132	4 54 54 24	9 690 843 968	46·1735855	12·8705093	·0004690432
2133	4 54 96 89	9 704 486 637	46·1844130	12·8725213	·0004688233
2134	4 55 39 56	9 718 142 104	46·1952378	12·8745326	·0004686036
2135	4 55 82 25	9 731 810 375	46·2060602	12·8765433	·0004683841
2136	4 56 24 96	9 745 491 456	46·2168800	12·8785534	·0004681648
2137	4 56 67 69	9 759 185 353	46·2276973	12·8805628	·0004679457
2138	4 57 10 44	9 772 892 072	46·2385121	12·8825717	·0004677268
2139	4 57 53 21	9 786 611 619	46·2493243	12·8845799	·0004675082
2140	4 57 96 00	9 800 344 000	46·2601340	12·8865874	·0004672897
2141	4 58 38 81	9 814 089 221	46·2709412	12·8885944	·0004670715
2142	4 58 81 64	9 827 847 288	46·2817459	12·8906007	·0004668534
2143	4 59 24 49	9 841 618 207	46·2925480	12·8926064	·0004666356
2144	4 59 67 36	9 855 401 984	46·3033476	12·8946115	·0004664179
2145	4 60 10 25	9 869 198 625	46·3141447	12·8966159	·0004662005
2146	4 60 53 16	9 883 008 136	46·3249393	12·8986197	·0004659832
2147	4 60 96 09	9 896 830 523	46·3357314	12·9006229	·0004657662
2148	4 61 39 04	9 910 665 792	46·3465209	12·9026255	·0004655493
2149	4 61 82 01	9 924 513 949	46·3573079	12·9046275	·0004653327
2150	4 62 25 00	9 938 375 000	46·3680924	12·9066288	·0004651163
2151	4 62 68 01	9 952 248 951	46·3788745	12·9086295	·0004649000
2152	4 63 11 04	9 966 135 808	46·3896540	12·9106296	·0004646840

TABLE XLV. (*continued*).

No.	Square	Cube	Square Root	Cube Root	Reciprocal
2153	4 63 54 09	9 980 085 577	46·4004310	12·9126291	·0004644682
2154	4 63 97 16	9 993 948 264	46·4112055	12·9146279	·0004642526
2155	4 64 40 25	10 007 878 875	46·4219775	12·9166262	·0004640371
2156	4 64 83 36	10 021 812 416	46·4327471	12·9186238	·0004638219
2157	4 65 26 49	10 035 768 893	46·4435141	12·9206208	·0004636069
2158	4 65 69 64	10 049 728 312	46·4542786	12·9226172	·0004633920
2159	4 66 12 81	10 063 705 679	46·4650406	12·9246129	·0004631774
2160	4 66 56 00	10 077 696 000	46·4758002	12·9266081	·0004629630
2161	4 66 99 21	10 091 699 281	46·4865572	12·9286027	·0004627487
2162	4 67 42 44	10 105 715 528	46·4973118	12·9305966	·0004625347
2163	4 67 85 69	10 119 744 747	46·5080638	12·9325899	·0004623209
2164	4 68 28 96	10 133 786 944	46·5188134	12·9345827	·0004621072
2165	4 68 72 25	10 147 842 125	46·5295605	12·9365747	·0004618938
2166	4 69 15 56	10 161 910 296	46·5403051	12·9385662	·0004616805
2167	4 69 58 89	10 175 991 463	46·5510472	12·9405570	·0004614675
2168	4 70 02 24	10 190 085 632	46·5617869	12·9425472	·0004612546
2169	4 70 45 61	10 204 192 809	46·5725241	12·9445369	·0004610420
2170	4 70 89 00	10 218 313 000	46·5832588	12·9465259	·0004608295
2171	4 71 32 41	10 232 446 211	46·5939910	12·9485143	·0004606172
2172	4 71 75 84	10 246 592 448	46·6047208	12·9505021	·0004604052
2173	4 72 19 29	10 260 751 717	46·6154481	12·9524893	·0004601933
2174	4 72 62 76	10 274 924 024	46·6261729	12·9544759	·0004599816
2175	4 73 06 25	10 289 109 375	46·6368953	12·9564618	·0004597701
2176	4 73 49 76	10 303 307 776	46·6476152	12·9584472	·0004595588
2177	4 73 93 29	10 317 519 233	46·6583326	12·9604319	·0004593477
2178	4 74 36 84	10 331 743 752	46·6690476	12·9624161	·0004591368
2179	4 74 80 41	10 345 981 339	46·6797601	12·9643996	·0004589261
2180	4 75 24 00	10 360 232 000	46·6904701	12·9663826	·0004587156
2181	4 75 67 61	10 374 495 741	46·7011777	12·9683649	·0004585053
2182	4 76 11 24	10 388 772 568	46·7118829	12·9703466	·0004582951
2183	4 76 54 89	10 403 062 487	46·7225855	12·9723277	·0004580852
2184	4 76 98 56	10 417 365 504	46·7332858	12·9743082	·0004578755
2185	4 77 42 25	10 431 681 625	46·7439836	12·9762881	·0004576659
2186	4 77 85 96	10 446 010 856	46·7546789	12·9782674	·0004574565
2187	4 78 29 69	10 460 353 203	46·7653718	12·9802461	·0004572474
2188	4 78 73 44	10 474 708 672	46·7760623	12·9822242	·0004570384
2189	4 79 17 21	10 489 077 269	46·7867503	12·9842017	·0004568296
2190	4 79 61 00	10 503 459 000	46·7974358	12·9861786	·0004566210
2191	4 80 04 81	10 517 853 871	46·8081189	12·9881549	·0004564126
2192	4 80 48 64	10 532 261 888	46·8187996	12·9901306	·0004562044
2193	4 80 92 49	10 546 683 057	46·8294779	12·9921057	·0004559964
2194	4 81 36 36	10 561 117 384	46·8401537	12·9940802	·0004557885
2195	4 81 80 25	10 575 564 875	46·8508271	12·9960540	·0004555809
2196	4 82 24 16	10 590 025 536	46·8614981	12·9980273	·0004553734
2197	4 82 68 09	10 604 499 373	46·8721666	13·0000000	·0004551661
2198	4 83 12 04	10 618 986 392	46·8828327	13·0019721	·0004549591
2199	4 83 56 01	10 633 486 599	46·8934963	13·0039436	·0004547522
2200	4 84 00 00	10 648 000 000	46·9041576	13·0059145	·0004545455
2201	4 84 44 01	10 662 526 601	46·9148164	13·0078848	·0004543389

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TABLE XLVI.

Squares, Cubes, and Fourth Powers of Fractions.

No.	Square.	Cube.	Fourth Power.	No.	Square.	Cube.	Fourth Power.
1/64	0·0002441	0·000003815	0·00000005961	33/64	0·2659	0·1371	0·07069
1/32	0·0009766	0·00003052	0·0000009537	17/32	0·2822	0·1499	0·07965
3/64	0·002197	0·0001030	0·0000001922	35/64	0·2991	0·1636	0·08944
1/16	0·003906	0·0002441	0·00001526	9/16	0·3164	0·1780	0·1001
5/64	0·006104	0·0004768	0·00003725	37/64	0·3342	0·1932	0·1117
3/32	0·008789	0·0008240	0·00007725	19/32	0·3526	0·2093	0·1243
7/64	0·01196	0·001308	0·0001431	39/64	0·3713	0·2263	0·1379
1/8	0·01563	0·001953	0·0002441	5/8	0·3906	0·2441	0·1526
9/64	0·01978	0·002781	0·0003911	41/64	0·4104	0·2629	0·1684
5/32	0·02441	0·003815	0·0005961	21/32	0·4307	0·2825	0·1855
11/64	0·02954	0·005077	0·0008727	43/64	0·4514	0·3033	0·2038
3/16	0·03516	0·006592	0·001236	11/16	0·4727	0·3250	0·2234
13/64	0·04126	0·008381	0·001702	45/64	0·4944	0·3476	0·2444
7/32	0·04785	0·01047	0·002290	23/32	0·5166	0·3713	0·2669
15/64	0·05493	0·01287	0·003018	47/64	0·5393	0·3961	0·2909
1/4	0·06250	0·01563	0·003906	3/4	0·5625	0·4219	0·3164
17/64	0·07056	0·01874	0·004978	49/64	0·5862	0·4488	0·3436
9/32	0·07910	0·02225	0·006257	25/32	0·6104	0·4768	0·3725
19/64	0·08813	0·02617	0·007768	51/64	0·6350	0·5060	0·4032
5/16	0·09766	0·03052	0·009537	13/16	0·6602	0·5364	0·4358
21/64	0·1077	0·03533	0·01159	55/64	0·6858	0·5679	0·4703
11/32	0·1182	0·04062	0·01396	27/32	0·7119	0·6007	0·5068
23/64	0·1292	0·04641	0·01668	57/64	0·7385	0·6347	0·5454
3/8	0·1406	0·05273	0·01978	7/8	0·7656	0·6699	0·5862
25/64	0·1526	0·05960	0·02328	59/64	0·7932	0·7065	0·6290
13/32	0·1650	0·06705	0·02724	29/32	0·8213	0·7443	0·6745
27/64	0·1780	0·07508	0·03168	61/64	0·8499	0·7885	0·7223
7/16	0·1914	0·08374	0·03664	15/16	0·8789	0·8240	0·7725
29/64	0·2053	0·09304	0·04216	63/64	0·9084	0·8659	0·8253
15/32	0·2197	0·1030	0·04828	31/32	0·9385	0·9091	0·8807
31/64	0·2346	0·1136	0·05505	65/64	0·9690	0·9539	0·9390
1/2	0·2500	0·1250	0·06250	1	1·000	1·000	1·000

Squares, Cubes, and Fourth Powers of Fractions—(continued).

No.	Square.	Cube.	Fourth Power.	No.	Square.	Cube.	Fourth Power.
1 1/64	1·031	1·048	1·064	1 33/64	2·297	3·482	5·277
1 1/32	1·063	1·097	1·131	1 17/32	2·345	3·590	5·498
1 3/64	1·096	1·147	1·201	1 35/64	2·393	3·701	5·726
1 1/16	1·129	1·199	1·274	1 9/16	2·441	3·815	5·961
1 5/64	1·162	1·253	1·351	1 37/64	2·490	3·930	6·203
1 3/32	1·196	1·308	1·431	1 19/32	2·540	4·048	6·452
1 7/64	1·231	1·365	1·515	1 39/64	2·590	4·168	6·709
1 1/8	1·266	1·424	1·602	1 5/8	2·641	4·291	6·973
1 9/64	1·301	1·484	1·693	1 41/64	2·692	4·416	7·245
1 5/32	1·337	1·546	1·787	1 21/32	2·743	4·543	7·525
1 11/64	1·373	1·609	1·996	1 43/64	2·795	4·673	7·813
1 3/16	1·410	1·675	1·989	1 11/16	2·848	4·805	8·109
1 13/64	1·448	1·742	2·095	1 45/64	2·901	4·940	8·414
1 7/32	1·485	1·810	2·206	1 23/32	2·954	5·077	8·727
1 15/64	1·524	1·881	2·322	1 47/64	3·008	5·217	9·048
1 1/4	1·563	1·953	2·441	1 3/4	4·063	5·359	9·379
1 17/64	1·602	2·027	2·566	1 49/64	3·117	5·504	9·718
1 9/32	1·642	2·103	2·695	1 25/32	3·173	5·652	10·07
1 19/64	1·682	2·181	2·829	1 51/64	3·229	5·802	10·43
1 5/16	1·723	2·261	2·968	1 13/16	3·285	5·954	10·79
1 21/64	1·764	2·343	3·111	1 55/64	3·342	6·110	11·17
1 11/32	1·806	2·426	3·260	1 27/32	3·399	6·268	11·56
1 23/64	1·848	2·512	3·415	1 57/64	3·457	6·428	11·95
1 3/8	1·891	2·600	3·575	1 7/8	3·516	6·592	12·36
1 25/64	1·934	2·689	3·740	1 59/64	3·574	6·758	12·78
1 13/32	1·978	2·781	3·911	1 29/32	3·634	6·927	13·20
1 27/64	2·022	2·875	4·087	1 61/64	3·694	7·099	13·64
1 7/16	2·066	2·970	4·270	1 15/16	3·754	7·273	14·09
1 29/64	2·112	3·068	4·459	1 63/64	3·815	7·451	14·55
1 15/32	2·157	3·168	4·654	1 31/32	3·876	7·631	15·02
1 31/64	2·203	3·271	4·855	1 65/64	3·938	7·814	15·51
1 1/2	2·250	3·375	5·063	2	4·000	8·000	16·00

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